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PRACTICALLY STABILITY AND ULAM-HYERS STABILITY OF FUZZY CONTROL VOLTERRA INTEGRO DIFERENTIAL SYSTEM UNDER GRANULAR DIFFERENTIABILITY: A STABILITY COMPARISON

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Abstract. In this study, the stability analysis of the solutions of a fuzzy control Volterra Integro differential system under granular differentiability has been examined for the first time in literature. Mainly, practical stability and Ulam-Hyers-Rassias stability have been investigated, and classical Lyapunov and Ulam-Hyers-Rassias stability have been compared. The comparison and stability aspects of the study are further illustrated by an example of a fuzzy differential equation solved using fuzzy granular Laplace transformation.

Keywords: Granular Differentiability, Controllability, Initial Time Difference, Perturbed Systems

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1. Introduction

Fuzzy differential systems (FDS) is an intriguing and captivating method of pure and applied sciences for modeling dynamic systems [1] subject to uncertainty and for processing uncertain or candidate data in mathematical models. They have been used in a vast range of applied areas, including quantum optics [2], gravity, population models, engineering applications [3], and population models.

While Chang and Zadeh [4], initially proposed the concept of fuzzy derivatives, the term "fuzzy differential equation" was first used in 1978 [5]. FDEs, as we know them today, are based on a concept of fuzzy derivative put forth by Dubois-Prade in 1982 [6]. The Hukuhara derivative (the Puri Ralescu derivative) [7] was proposed in 1983 and is one of the most popular derivative definitions. The Hukuhara derivative-based FDE was processed with care by Kaleva [8],[9] and it served as the basis for several studies looking at the behavior of FDEs.

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Numerous drawbacks of the Hukuhara derivative have come to light over time, including always not existence of the H-difference of two type-1 fuzzy numbers (T1FNs), the diameter of the fuzzy function necessarily monotonically non-decrease or non-increase as time increases, the multiplicity of solution and unnatural behavior in modeling (UBM phenomenon), etc [10], [11]. Granular differentiability was implemented in 2018 [12] to address these drawbacks. As a result, study into it is growing daily [13], [14].

The concept that the control function can be fuzzy has been developed in dynamic control systems [15] much earlier than these advancements in FDEs. Mamdani and Assilian [16] developed the fundamental architecture of the fuzzy controller, which was first used to manage a steam engine in 1973. Takagi and Sugeno wrote an essay in 1985 [17] that claimed their fuzzy models could accurately and highly represent practically all nonlinear dynamical systems. Therefore, it has been demonstrated by this research that the fuzzy controller is simple to develop and performs very well [18], [19], [20] and [21].

FDEs have been studied with the idea that in addition to the control function, the entire differential system, from the derivative to the initial state, may have a fuzzy structure. With this viewpoint, studies have focused on the fuzzy control differential equation (FCDE) [22]; the existence and uniqueness of the solution involving fuzzy control, the accessibility, stability, and controllability of fuzzy control systems have emerged as the primary issues for fuzzy control problems [23], [24]. It has been studied about the stability of the fuzzy control and fuzzy differential equation in [25], [26], [27], [28] and the stability and controllability of the fuzzy control system have been examined together in [15].

Like FDS, another essential technique to represent dynamical systems subject to uncertainties is fuzzy integro-differential equations (FIDE). The existence and uniqueness problem of nonlinear set, fuzzy and fuzzy control Volterra integro-differential equations are studied in [29], [30], [31].

Using the second Lyapunov method [32]-[37], which is a critical tool for stability analysis, one may predict the qualitative behavior and examine the stability of differential equations in nonlinear systems. It is sufficient to comprehend how the comparison system's solution behaves without knowing the exact solution. This method successfully demonstrates the system's stability by identifying the proper Lyapunov function.

Ulam-Hyers-Rassias stability remains another sensitive stability analysis that has also been used to study the behavior of fuzzy differential equations in [13], [38], [34], [30], [39]. For the first time, Obloza compared Ulam-Hyers stability with classical Lyapunov stability in the ordinary differential equation [40]. This comparison subsequently sparked a lot of research.

This study applied Lyapunov's second method to fuzzy control integro-differential system under granular differentiability via a Lyapunov-like function and comparison method and studied the practical stability of the system. It investigated the Ulam-Hyers-Rassias stability of a fuzzy control Volterra Integro differential system and compared it with classical Lyapunov stability. An illustration of a fuzzy differential equation solved using fuzzy granular Laplace transformation further illustrates the comparison and stability aspect under study.

2. Preliminaries

Definition 2.1. *Define* $E^n = \{x : R^n \to [0,1] \mid x(t)$ *is onto, fuzzy convex,* $[x]^\alpha$ *is compact subset and,* $[x]^0$ *is bounded subset*$\}$ *where* $\alpha-$*level set* $x_\alpha = [x]^\alpha = \{z \in \mathbb{R}^n \mid x(z) \geq \alpha\}$.

Definition 2.2. [12] *Let* $x : [a, b] \subseteq \mathbb{R} \to [0, 1]$ *be a fuzzy number and* $[x]^\alpha = [\underline{x^\alpha}, \overline{x^\alpha}]$*. The horizontal membership function (HMF)* $x^{gr} : [0,1]\times[0,1] \to [a, b]$ *is defined as* $x^{gr}(\alpha, \mu_x) = \underline{x^\alpha} + (\overline{x^\alpha} - \underline{x^\alpha})\mu_x$ *where* $\mu_x \in [0, 1]$ *is called relative-distance-measure (RDM) variable. The horizontal membership function of* $x(t) \in E^1$*, i.e.* $x^{gr}(\alpha, \mu_x)$ *is also represented by* $\hat{H}(x)$*. Moreover,* $\alpha-$*level set of* x *can be given by*

$$\hat{H}^{-1}(x^{gr}(\alpha, \mu_x)) = [x]^\alpha = \left[\inf_{\gamma\geq\alpha}\min_{\mu_x} x^{gr}(\gamma, \mu_x), \sup_{\gamma\geq\alpha}\max_{\mu_x} x^{gr}(\gamma, \mu_x)\right].$$

Definition 2.3. [12] *Let* $x, y \in E^1$*. The relations* $x = y$ *and* $x \geq y$ *hold, respectively, whenever* $\hat{H}(x) = \hat{H}(y)$ *and* $\hat{H}(x) \geq \hat{H}(y)$ *for all* $\mu_x = \mu_y \in [0, 1]$*.*

Definition 2.4. [12] *Let* $x, y \in E^1$ *and* $\odot$ *denote one of the addition, subtraction, division and, multiplication operations. Therefore* $x \odot y$ *is equal to* $z \in E^1$ *if and only if* $\hat{H}(z) = \hat{H}(x) \odot \hat{H}(y)$*.*

Remark 2.1. [12] *Let* $x, y, z \in E^1$*, the we have* $x - y = -(y - x)$*,* $x - x = 0$ *,* $x \div x = 1$ *and,* $(x + y)z = xz + yz$*.*

Definition 2.5. [12] *Let* $f : E^n \to E^1$ *and* $x_i : [a, b] \subseteq \mathbb{R} \to E^1$ *for all* $i = 1, 2, .., n$*. The HMF of* $f(x_1(t), x_2(t), ..., x_n(t))$*is given by*

$$\hat{H}\left(f(\hat{H}\left(x_1(t)\right), \hat{H}\left(x_2(t)\right), ..., \hat{H}\left(x_n(t)\right))\right).$$

Definition 2.6. [12] *Let* $x, y \in E^1$*. The granular metric* $D_{gr} : E^1 \times E^1 \to \mathbb{R}^+ \cup \{0\}$ *is defined as* $D_{gr}(x, y) = \sup_{\alpha}\max_{\mu_x,\mu_y} |x^{gr}(\alpha, \mu_x) - y^{gr}(\alpha, \mu_y)|$*.*

Definition 2.7. [12] *Let* $f : [a, b] \subseteq \mathbb{R} \to E^1$*. If a fuzzy number* $\check{D}_{gr}$ *exist such that* $\check{D}_{gr}f(t) = \lim_{h\to0}\frac{f(t+h)-f(t)}{h}$*. Then* f *is called granular differentiable (gr-diffentiable) at* $t \in [a, b]$*.*

Theorem 2.1. [12] *The fuzzy function* $f(t)$ *is gr-diffentiable if and only if the horizontal membership function of* $f(t)$ *i.e.* $\hat{H}(f(t))$ *is differentiable with respect to* t*. Futhermore,* $\hat{H}\left(\check{D}_{gr}f(t)\right) = \frac{\partial}{\partial t}\hat{H}\left(f(t)\right)$*.*

Definition 2.8. [12] *Let* $f : [a, b] \subseteq \mathbb{R} \to E^1$*be continuous fuzzy function whose horizontal membership function* $f^{gr}(t, \alpha, \mu_x)$ *is integrable on* $[a, b]$*. Let* $\int_a^b f(t)dt$ *denote the integral of* $f(t)$ *on* $[a, b]$*. Then the fuzzy function* $f(t)$ *is said to be granular integrable on* $[a, b]$ *if there exists a fuzzy number* $m = \int_a^b f(t)dt$ *such that* $\hat{H}\left(m\right) = \hat{H}\left(\int_a^b f(t)dt\right)$*.*

Definition 2.9. [13] *Let* $f(t)$ *be continuous fuzzy function and* $f(t)e^{-st}$ *be improper fuzzy integrable function on* $[0, +\infty)$*. Thus the fuzzy Laplace transform of* $f(t)$ *denoted by* $\mathbf{L}\left[f(t)\right] = \int_0^{+\infty} f(t)e^{-st}dt$ *(*$s > 0$ *and integer).*

Remark 2.2. *It should be note that* $[\mathbf{L}\left[f(t)\right]]^\alpha = \left[\mathbf{L}\left[\underline{f^\alpha}(t)\right], \mathbf{L}\left[\overline{f^\alpha}(t)\right]\right]$*, where* $\mathbf{L}\left[\underline{f^\alpha}(t)\right]$ *and* $\mathbf{L}\left[\overline{f^\alpha}(t)\right]$ *are classical Laplace transform of* $\underline{f^\alpha}(t)$ *and* $\overline{f^\alpha}(t)$ *respectively.*

Definition 2.10. [13] *Let $f:[a,\infty)\to E^1$. For any fixed $\alpha,\mu_x\in[0,1]$ assume $\hat{H}(f(t))=f^{gr}(t,\alpha,\mu_x)$ is integrable on $[a,b]$ for every $b\geq a$. Moreover, suppose there is a positive constant $M(\alpha,\mu_M)$ such that $\int_a^b |f^{gr}(t,\alpha,\mu_x)|\,dt\leq M(\alpha,\mu_M)$. Then $f(t)$ is called granular improper fuzzy integrable on $[a,\infty)$ and it is denoted by $\int_a^\infty f(t)dt$. Moreover, $\hat{H}\left(\int_a^\infty f(t)dt\right)=\int_a^\infty \hat{H}(f(t))\,dt$.*

Definition 2.11. [13] *Let $f(t)$ be continuous fuzzy function. Suppose that $f(t)e^{-st}$ granular improper fuzzy integrable on $[0,\infty)$. Then $\int_0^\infty f(t)e^{-st}dt$ is called granular fuzzy Laplace transform of $f(t)$ and it is denoted by $\mathbf{L}_{gr}[f(t)]$.*

Remark 2.3. *It is clear that $\hat{H}(\mathbf{L}_{gr}[f(t)])=\mathbf{L}\left[\hat{H}(f(t))\right]$. Also, the granular fuzzy Laplace transform is a linear operator.*

Remark 2.4. *Suppose that $f(t)$ is a granular continuous fuzzy function on $[a,\infty)$ and $a\geq 0$. Then $L_{gr}\left[\breve{D}_{gr}f(t)\right]=sL_{gr}[f(t)]-f(a)$.*

3. Initial Value Problem for Fuzzy Control Volterra Integro-Differential Systems

Let's take a look at the initial valued problems for fuzzy control Volterra integro-diffrerential equations (IVP for FCVIDEs) in fuzzy metric space E^n:

$$\begin{aligned}\breve{D}_{gr}x(t) &= f(t,x(t),u(t))+\int_{t_0}^{t} g(t,s,x(s),u(s))ds \quad x(t_0)=x_0\in E^n, \qquad (1)\\ u(t_0) &= u_0\in E^p \text{ and } t\in[t_0,T] \quad t_0\geq 0,\end{aligned}$$

where $x(t)\in E^n$, $f:[t_0,T]\times E^n\times E^p\to E^n$ and $g:[t_0,T]\times[t_0,T]\times E^n\times E^p\to E^n$ the admissible control [29] $u(t)\in\Omega$ where $\Omega=\{u(t)\in E^p; U(t,u(t))\leq v(t)$ for $t\geq t_0$ and $U(t,u)\in C[[t_0,T]\times E^p,\mathbb{R}_+]$, $v(t)\in\mathbb{R}_+\}$.

A fuzzy mapping $x:[t_0,T]\to E^n$ is a *solution* of IVP for FCVIDEs (1) on $[t_0,T]$ if $f:[t_0,T]\times E^n\times E^p\to E^n$ and $g:[t_0,T]\times[t_0,T]\times E^n\times E^p\to E^n$ are integrable on $[t_0,T]$, moreover satisfying the subsequent fuzzy integral equations [29] :

$$x(t)=x_0+\int_{t_0}^{t} f(s,x(s),u(s))ds+\int_{t_0}^{t}\int_{t_0}^{s} g(s,r,x(r),u(r))drds,$$

$$\hat{H}(x(t))=\hat{H}\left(x_0+\int_{t_0}^{t} f(s,x(s),u(s))ds+\int_{t_0}^{t}\int_{t_0}^{s} g(s,r,x(r),u(r))drds\right)\quad t\in[t_0,T].$$

4. Stability of Fuzzy Control Volterra Integro-Differential Systems

Definition 4.1. [34] *We suppose that trivial solution exists for FCVIDE in equation (1), where $f(t,\theta^n,u(t))=\theta^n$ and $g(t,s,\theta^n,u(t))=\theta^n$ and any solution $x(t)=x(t,t_0,x_0,u(t))$ of equation (1) through (t_0,x_0).*

i) *If for each* $\varepsilon > 0$ *and* $t_0 > 0$ *there exist a* $\delta = \delta(t_0, \varepsilon)$ *such that* $D_{gr}(x_0, \theta^n) < \delta$ *implies* $D_{gr}(x(t), \theta^n) < \varepsilon$ *for* $t \geq t_0$, then *the trivial solution* $x = \theta^n$ *is stable by Lyapunov's mean,*

ii) *If* δ *in* i) *is independent of* $t_0 \in \mathbb{R}_+$*, then the trivial solution* is uniformly stable.

iii) *If given scalar numbers* (λ, A) *with* $0 < \lambda < A$ *there exists a* $(\lambda, A) \geq 0$ *such that* $D_{gr}(x_0, \theta^n) < \lambda$ *implies* $D_{gr}(x(t), \theta^n) < A$, $t \geq t_0$ *for some* $t_0 \in \mathbb{R}_+$*, then the trivial solution* is *practical stable.*

iv) *If* iii) *holds for every* $t_0 \in \mathbb{R}_+$*, then the trivial solution* is *uniformly practical stable.*

It is easy to see that the solution $x(t) = x(t, t_0, x_0, u(t))$ is trivial solution of equation (1) through (t_0, x_0), if and only if $\hat{H}(x(t))$ is trivial solution of following dynamical system

$$\frac{\partial}{\partial t}\hat{H}(x(t)) = \hat{H}(f(t, x(t), u(t))) + \hat{H}\Big(\int_{t_0}^{t} g(t, s, x(s), u(s))ds\Big), \quad \hat{H}(x(t_0)) = \hat{H}(x_0).$$

Definition 4.2. *The Dini derivatives for FCVIDE in equation (1) is defined as follows for a real-valued function* $V(t, x(t)) \in C[\mathbb{R}_+ \times E^n, \mathbb{R}_+]$

$$D^+V(t,x) \equiv \lim_{h \to 0^+} \sup \frac{1}{h}\left[V\left(t+h, x+h\left[(f(t,x,u) + \int_{t_0}^{t} g(t,s,x(s),u(s))ds)\right]\right) - V(t,x)\right],$$

for $(t, x) \in \mathbb{R}_+ \times E^n$.

We take the comparison theorem into consideration to forecast the stability characteristics of the solution $x(t, t_0, x_0, u(t))$ of the system (1).

Theorem 4.1. *i)Let Lyapunov-like function* $V(t, x(t)) \in C(\mathbb{R}_+ \times E^n, \mathbb{R}_+)$*,*

$\mid V(t, x) - V(t, y) \mid \leq LD_{gr}(x, y)$ $L > 0$ *bounded Lipschitz constant and for* $(t, x) \in \mathbb{R}_+ \times S_\rho$ *where* $S_\rho = [x \in E^n : D_{gr}(x, \theta^n) < \rho]$ *such that*

$$D^+V(t,x) \leq F(t, D_{gr}(x,\theta^n), D_{gr}(u,\theta^n)) + \int_{t_0}^{t} G(t, s, D_{gr}(x,\theta^n), D_{gr}(u,\theta^n))ds,$$

where $F \in C([t_0, t_0 + p] \times [0, b_1] \times \mathbb{R}_+, \mathbb{R}_+)$ and $G \in C([t_0, t_0 + p] \times [t_0, t_0 + p] \times [0, b_1] \times \mathbb{R}_+, \mathbb{R}_+)$.

ii) $r(t) = r(t, t_0, w_0, v)$ *is maximal solution of the scalar integro-differential equation exists on* $[t_0, T]$,

$$\frac{dw}{dt} = F(t, w(t), v(t)) + \int_{t_0}^{t} G(t, s, w(s), v(t))ds, \quad w(t_0) = w_0 = 0 \text{ for } t \geq t_0.$$

Then if $x(t)$ *is solution of* (1) *through* (t_0, x_0) *on* $[t_0, T]$. *We have*

$$V(t, x) \leq r(t, t_0, w_0, v) \text{ provided that } V(t_0, x_0) \leq r_0.$$

Proof. We can prove with similar methods as in [41]. □

The function $F(t, w, v) \equiv 0$ and $G(t, s, w, v) \equiv 0$ *is admissible in Theorem 7 to yield the estimate* $V(t, x) \leq V(t_0, x_0)$.

5. Practical Stability of Fuzzy Control Volterra Integro-Differential Equations

Theorem 5.1. *Assume that admissible control $u(t) \in \Omega$ and the following hold,*

i) Let Lyapunov-like function $V(t, x(t)) \in C(R_+ \times E^n, \mathbb{R}_+)$,
$| V(t,x) - V(t,y) | \leq LD_{gr}(x,y)$ $L > 0$ *bounded Lipschitz constant and for* $(t, x \in \mathbb{R}_+ \times S_\rho$ *where* $S_\rho = [x \in E^n : D_{gr}(x, \theta^n) < \rho]$ *such that*

$$D^+V(t,x) \ \leq 0. \tag{2}$$

ii)Let $V(t, x(t)) \in C(R_+ \times E^n, \mathbb{R}_+)$ and a, b belong to the class K,

$$b(D_{gr}(x(t,x,u(t)), \theta^n)) \leq V(t, x(t)) \leq a(t, D_{gr}(x(t,x,u(t)), \theta^n)). \tag{3}$$

Then trivial solution $x(t, t_0, x_0, u(t))$ of FCVIDE (1) is practically stable *for $t \geq t_0$.*

Proof. We can prove with similar methods as in [42]. □

For the practical stability of fuzzy control integro-differential systems via scalar integro-differential equation, we employ the comprasion theorem in this section.

Theorem 5.2. *Assume that admissible control $u(t) \in \Omega$ and the following hold,*

i) *Let $V(t, x(t)) \in C(R_+ \times E^n, \mathbb{R}_+)$ $\quad | V(t,x) - V(t,y) | \leq LD_{gr}(x,y)$*
$L > 0$ and

$$b(D_{gr}(x(t), \theta^n)) \leq V(t, x(t)) \leq a(t, D_{gr}(x(t), \theta^n)) \quad a, b \in \kappa. \tag{4}$$

Dini derivatives of Lyapunov functions and comprasion of the scalar integro-differential equation

$$D^+V(t,x) \ \leq F(t, V(t, x(t), U(t, u(t))) + \int_{t_0}^{t} G(t, s, V(t, x(t), U(t, u(t)))ds. \tag{5}$$

$F(t, V(t, x(t), U(t, u(t))) \in C[\mathbb{R}^2_+ \ , \mathbb{R}]$ and $G(t, s, V(t, x(t), U(t, u(t))) \in C[\mathbb{R}^2_+, \mathbb{R}^2_+ \ , \mathbb{R}]$,

ii) *Let $r(t) = r(t, t_0, w_0, v)$ be the maximal solution of the scalar integro-differential equation*

$$\frac{dw}{dt} = F(t, w(t), v(t)) + \int_{t_0}^{t} G(t, s, w(s), v(t))ds \ \ w(t_0) = w_0 = 0 \text{ for } t \geq t_0. \tag{6}$$

Then the practical stability properties of the comparison differential equation imply the corresponding practical stability properties of $x(t, t_0, x_0, u(t))$ solution of the system (1) *for $t \geq t_0$.*

Proof. Suppose that comparison equation is practically stable, then for given any scalar numbers (λ, A) with $0 < \lambda < A$ and there exists $b = b(A)$ and $b \in \kappa$ such that

$$w(t, t_0, w_0, v) < b(A) \text{ provided that } d_s[w_0, 0] < \lambda, \ t \geq t_0. \tag{7}$$

We claim that with this λ, practical stability holds such that

$$D_{gr}(x(t, t_0, x_0, u(t)), \theta^n) < A \text{ provided that } D_{gr}(x_0, \theta^n) < \lambda \text{ for } t \geq t_0. \tag{8}$$

If the solution is not practically stable and then there would exist solution of fuzzy control integro-differential equation; for $t \geq t_0$ exist a $t_1 > t_0$ and with $D_{gr}(x_0, \theta^n) < \lambda$ for $t \geq t_0$ satisfying

$$D_{gr}(x(t_1, t_0, x_0, u(t)), \theta^n) = A, \tag{9}$$

for $t \in [t_0, t_1]$. *Choose* $w_0 = a(t_0, D_{gr}(x_0, \theta^n))$, *we get the inequality*

$$V(t, x(t)) \leq r(t, t_0, w_0, v) \quad t \in [t_0, t_1]. \tag{10}$$

So that we have, because of (4) and (9)

$$b(A) \leq V(t_1, x(t_1, t_0, x_0, u(t))) \quad t_1 > t_0. \tag{11}$$

This means that $D_{gr}(x(t), \theta^n) < \rho$ *for* $t \in [t_0, t_1]$ *and hence we have the inequality*

$$V(t_1, x(t_1, t_0, x_0, u(t))) \leq r(t_1, t_0, w_0, v) \quad t \geq t_0. \tag{12}$$

By using (7) , (8), (9) and (10), we get

$$\begin{aligned} b(A) &= b(D_{gr}(x(t_1, t_0, x_0, u(t)), \theta^n)) \\ &\leq V(t_1, x(t_1, t_0, x_0, u(t))) \\ &\leq r(t_1, t_0, w_0, v) \\ &\leq r(t_1, t_0, a(t_0, D_{gr}(x_0, \theta^n)), v) \\ &\leq r(t_1, t_0, a(t_0, \lambda_1), v) \\ &< b(A). \end{aligned}$$

This contradiction gives us practical stability properties of $x(t, t_0, x_0, u(t))$ *solution of the system (1) for* $t \geq t_0$. □

6. Ulam-Hyers Stability of Fuzzy Control Volterra Integro-Differential Systems

Definition 6.1. *We say that problem (1) is Ulam-Hyers stable if there exists a real number* K_f *such that for* $\varepsilon > 0$ *and for each* $\nu \in C([t_0,T] \times E^n \times E^p, E^n)$ *to the problem*

$$D_{gr}(\check{D}_{gr}\nu(t), f(t, \nu(t), u(t)) + \int_{t_0}^{t} g(t, s, \nu(s), u(s))ds) \leq \varepsilon. \tag{13}$$

There exist a solution to problem (1) with $D_{gr}(\nu(t), x(t)) \leq K_f \varepsilon$ for all $t \in [t_0,T]$. We call a K_f a *Ulam-Hyers stability constant* of (1).

Definition 6.2. *We say that problem (1) is Ulam-Hyers-Rassias stable if there exists a real number* C_f *such that for* $\varepsilon > 0$ *and for each* $\nu \in C([t_0,T] \times E^n \times E^p, E^n)$ *to the problem*

$$D_{gr}(\check{D}_{gr}\nu(t), f(t, \nu(t), u(t)) + \int_{t_0}^{t} g(t, s, \nu(s), u(s))ds) \leq \varphi(t).$$

There exist a solution to problem (1) with $D_{gr}(\nu(t), x(t)) \leq C_f \varphi(t)$ *for all* $t \in [t_0,T]$. *We call a* C_f *a Ulam-Hyers-Rassias stability constant of (1).*

Definition 6.3. *We say that a function* $\nu \in C([t_0,T] \times E^n \times E^p, E^n)$ *is soluntion of (13) if and only if there exists a function,*

i) $D_{gr}(\delta(t), \theta^n) \leq \varepsilon$ *for any* $t \in [t_0,T]$.

ii) $\check{D}_{gr}\nu(t) = f(t, \nu(t), u(t)) + \int_{t_0}^{t} g(t, s, \nu(s), u(s))ds + \delta(t)$ *for* $t \in [t_0,T]$.

Theorem 6.1. *i)* $f \in C(Q, E^n)$ *is levelwise continuous, there exists* $L_f \succ 0$ *such that* $D_{gr}(f(t, x, u), f(t, y, v)) \leq L_f(D_{gr}(x, y) + D_{gr}(u, v))$ *for all* $x, y \in E^n$, $u, v \in E^p$ *and* $t \in [t_0,T]$.

ii) $g \in C(Q_1, E^n)$ *is levelwise continuous, there exists* $L_g \succ 0$ *such that* $D_{gr}(g(t,s,x,u), g(t,s,y,v)) \leq L_g(D_{gr}(x,y) + D_{gr}(u,v))$ *for all* $x, y \in E^n$, $u, v \in E^p$ *and* $t \in [t_0, T]$.

iii) $L_{fg} = Max\{L_f, L_g\} > 0$.

iv) There is a function $\nu \in C([t_0,T] \times E^n \times E^p, E^n)$ *is soluntion of (13) then there exists a solution to problem (1) with* $D_{gr}(\nu(t), x(t)) \leq K_f \varepsilon$ *and* $x_0 = \nu_0$ *where*

$K_f = (T - t_0)\left[1 + \frac{L_{fg}}{L_{fg}+1}(e^{(L_{fg}+1)(T-t_0)} - 1)\right]$.

That is the solution of problem (1) is Ulam-Hyers stable.

Proof. *It can be proved in a similar way as in* [45]. □

7. Comparing Lyapunov Stability and Ulam-Hyers Stability of Fuzzy Control Volterra Integro-Differential Systems

In chapter 4, we gave our definitions of stability over null solution. Because Lyapunov second method works on null solution. But we will rearrange the concept of Lyapunov stability so that it is easier to compare the concepts of Ulam-Hyper stability and Lyapunov stability.

Definition 7.1. [44] *We have any solution* $x(t) = x(t, t_0, x_0, u(t))$ *of equation (1) through* (t_0, x_0) *is said to be stable by Lyapunov's mean if: for each* $\varepsilon > 0$ *and* $t_0 > 0$ *there exist a* $\delta = \delta(t_0, \varepsilon)$ *such that* $D_{gr}(x_0, x_1) < \delta$ *implies* $D_{gr}(x(t, t_0, x_0, u(t)), x(t, t_0, x_1, u(t))) < \varepsilon$ *for* $t \geq t_0$.

If each solution is stable by Lyapunov's mean, then the equation (1) is stable by Lyapunov's mean.

Theorem 7.1. *i)* $f \in C(Q, E^n)$ *is levelwise continuous, there exists* $L_f \succ 0$ *such that* $D_{gr}(f(t,x,u), f(t,y,v)) \leq L_f(D_{gr}(x,y) + D_{gr}(u,v))$ *for all* $x, y \in E^n$, $u, v \in E^p$ *and* $t \in [t_0, T]$.

ii) $x^0(t) = x^0(t, t_0, x_0, u(t))$ *and* $x^1(t) = x^1(t, t_0, x_1, u(t))$ *solution of problem (1) with* $x_0 = x^0(t_0)$ *and* $x_1 = x^1(t_0)$ *such that* $D_{gr}(x_0, x_1) < \delta$.

Then for all $t \in [t_0, T]$; $D_{gr}(x^0, x^1) \leq 3\delta\left(1 + 3L_{fg}e^{3L_{fg}+1}(e^{T-t_0} - 1)\right)$.

Proof. *It can be proved in a similar way as in* [40]. □

Theorem 7.2. *i)* $f \in C(Q, E^n)$ *is levelwise continuous, there exists* $L_f \succ 0$ *such that* $D_{gr}(f(t,x,u), f(t,y,v)) \leq L_f(D_{gr}(x,y) + D_{gr}(u,v))$ *for all* $x, y \in E^n$, $u, v \in E^p$ *and* $t \in [t_0, T]$.

ii) $x^0(t) = x^0(t, t_0, x_0, u(t))$ *and* $x^1(t) = x^1(t, t_0, x_1, u(t))$ *solution of problem (1) with* $x_0 = x^0(t_0)$ *and* $x_1 = x^1(t_0)$ *such that* $D_{gr}(x_0, x_1) < \delta$.

Then there exist $d > 0$ *such that* $D_{gr}(x^0, x^1) < 2\delta$ *holds for all* $t \in [t_0 - d, t_0 + d]$.

Proof. *It can be proved in a similar way as in* [40]. □

Theorem 7.3. *The problem (1) is Ulam-Hyers stable, then it is stable by Lyapunov's mean.*

Proof. If the problem (1) is *Ulam-Hyers stable,* then assumption of Theorem 14 is valid. □

Let us denote $x^0(t) = x^0(t, t_0, x_0, u(t))$ *and* $x^1(t) = x^1(t, t_0, x_1, u(t))$ *solution of problem (1) with* $D_{gr}(x_0, x_1) < \delta$.

Since f satisfies $D_{gr}(f(t,x,u), f(t,y,v)) \le L_f(D_{gr}(x,y)+D_{gr}(u,v))$, by Teorem 18 we have $D_{gr}(x^0,x^1) < 12\delta$ for $t \in [t_0-d, t_0+d]$.

We take the function $p:\mathbb{R}\to[0,1]$ satisfying following conditions;

i) $p \in C^1(\mathbb{R})$.

ii) $p(t)=0$ for $t \le t_0$

$p(t)=1$ for $t \ge t_0+d \quad d>0$.

iii) $p'(t) \ge 0$ for $\forall\ t \in \mathbb{R}$.

We define the function $\nu \in C([t_{0,}T]\times E^n \times E^p, E^n)$

$$\nu(t) = x^0(t) + p(t)[x^1 - x^0] \qquad t \in \mathbb{R}$$

where $x^1 - x^0$ is Hukuhara distance and exist. Note that $\nu(t) = x^0(t)$ for $t \le t_0$ and $\nu(t) = x^1(t)$ for $t \ge t_0+d$. We want to find that for $t \in [t_0, t_0+d]$,

$$I_v = D_{gr}(D_{gr}v(t), f(t,\nu(t),u(t)) + \int_{t_0}^{t} g(t,s,\nu(s),u(s))ds).$$

If we look at each term one by one,

$$D_{_{gr}}\nu(t) = D_{_{gr}}x^0(t) + p^{'}(t)[x^1-x^0] + p(t)D_{_{gr}}\left([x^1-x^0]\right).$$

$$\begin{aligned} & f(t,\nu(t),u(t)) + \int_{t_0}^{t} g(t,s,\nu(s),u(s))ds \\ = \; & f(t,x^0(t)+p(t)[x^1-x^0],u(t)) + \int_{t_0}^{t} g(t,s,x^0(t)+p(t)[x^1-x^0],u(s))ds. \end{aligned}$$

We need the following for $D_{_{gr}}\nu(t)$,

$$D_{_{gr}}x^0(t) = D_{gr}((f(t,x^0(t),u(t)) + \int_{t_0}^{t} g(t,s,x^0(s),u(s))ds),$$

$$\begin{aligned} D_{_{gr}}\left([x^1-x^0]\right) \;\; = \;\; & f(t,x^1(t),u(t)) + \int_{t_0}^{t} g(t,s,x^1(s),u(s)) \\ & -f(t,x^0(t),u(t)) + \int_{t_0}^{t} g(t,s,x^0(s),u(s))ds. \end{aligned}$$

With some calculation by Lemma 3.1 in [29] and definition as $\sup\limits_{t\in[t_0,t_0+d]} p(t) = P$ and $\sup\limits_{t\in[t_0,t_0+d]} p^{'}(t) = P'$,

$$\begin{aligned} I_v &\leq L_f D_{gr}(x^0(t),\ x^0(t)+p(t)(x^1-x^0)) \\ &\quad + L_g\int_{t_0}^{t} D_{gr}(x^0(s),x^0(s)+p(s)[x^1-x^0])ds \\ &\quad + P^{'} D_{gr}(x^1,x^0) + P D_{gr}(\check{D}_{gr}x^1(t)-\check{D}_{gr}x^0(t),\theta^n), \end{aligned}$$

where

$$D_{gr}(\check{D}_{gr}x^1(t)-\check{D}_{gr}x^0(t),\theta^n) = D_{gr}(D_{x^1-x^0},\theta^n).$$

$$D_{x^1-x^0} = f(t,x^1(t),u(t))+\int_{t_0}^{t} g(t,s,x^1(s),u(s))ds - f(t,x^0(t),u(t))+\int_{t_0}^{t} g(t,s,x^0(s),u(s))ds.$$

With some calculation on last Hukuhara distance;

$$I_v \leq 4PL_{fg}(\delta+\delta') + 2P^{'}\delta = \varepsilon.$$

Consequently, $\nu(t)$ is ε -approximate solution of (1) and there exists a solution to problem (1) with $D_{gr}(\nu(t),x(t)) \leq K_f\varepsilon$ and $x_0=\nu_0$. That is the solution of problem (1) is Ulam-Hyers stable.

We have $\nu(t_0)=x^0(t_0)$ and $\nu(t_0+d)=x^1(t_0+d)$ So, $D_{gr}(\nu(t),x^0(t)) \leq K_f\varepsilon_0$ and $D_{gr}(\nu(t),x^1(t)) \leq K_f\varepsilon_1$.

$D_{gr}(x^0(t),x^1(t)) \leq D_{gr}(\nu(t),x^0(t)) + D_{gr}(\nu(t),x^1(t)) \leq K_f\varepsilon_0 + K_f\varepsilon_1 = \varepsilon^*$. Consequently solution of (1) is said to be stable by Lyapunov's mean.

8. Application

Example 8.1. *We consider the initial valued problems of fuzzy control Volterra integro-diffrerential equations (IVP for FCVIDEs) like as equation (1).*

$$\check{D}_{gr}x(t) = -3x+\int_{0}^{t} e^{s-t}x(s)ds\ ,\ x(t_0)=x_0, \tag{14}$$

$$[x_0]^\alpha = \varphi(\alpha,t) = [\alpha-1, 1-\alpha] \quad t\in[0,3]. \tag{15}$$

The α-level set of equation (14) as following;

$$\begin{aligned} s\mathbf{L}_{gr}\left[\underline{x}^\alpha(t)\right] - \underline{x}_0^\alpha &= -3\mathbf{L}_{gr}\left[\underline{x}^\alpha(t)\right] + \frac{\mathbf{L}_{gr}\left[\underline{x}^\alpha(t)\right]}{s}, \\ s\mathbf{L}_{gr}\left[\overline{x}^\alpha(t)\right] - \overline{x}_0^\alpha &= -3\mathbf{L}_{gr}\left[\overline{x}^\alpha(t)\right] + \frac{\mathbf{L}_{gr}\left[\overline{x}^\alpha(t)\right]}{s}. \end{aligned}$$

By granular fuzzy laplace transform [45], we can find α-level set of solution of (14)

$$\begin{aligned} \underline{x}^\alpha(t) &= (\alpha-1)\varphi(t), \\ \overline{x}^\alpha(t) &= (1-\alpha)\varphi(t), \end{aligned}$$

where $\varphi(t) = \left[\left(\frac{\sqrt{13}+3}{2\sqrt{13}}\right)e^{-(3/2+\sqrt{13}/2)t} + \left(\frac{\sqrt{13}-3}{2\sqrt{13}}\right)e^{(\sqrt{13}/2-3/2)t}\right]$.

For Ulam-Hyers stability, we can easily find that $L_f=3$ and $L_g=1$, so $L_{fg}=3$.

$$D_{gr}(\check{D}_{gr}\nu(t),-3\nu(t))+\int_{t_0}^{t} e^{s-t}\nu(s)\ ds) \leq \varepsilon \text{ and } D_{gr}(\nu(t),x(t)) \leq K_f\varepsilon,$$

for $x_0 = \nu_0$ *where* $K_f = 3[1 + 3(e^{10} - e^4)]$.

For Lyapunov stability, We can take $x_1 = \psi(\alpha, t) = 6\,[\alpha - 1, 1 - \alpha]$ *while* $x_0 = \varphi(\alpha, t) = [\alpha - 1, 1 - \alpha]$ *and such that* $D_{gr}(x_0, x_1) < \delta$ *then we show* $D_{gr}(x^0, x^1) < \varepsilon$ *provided* $\delta = \delta(\varepsilon)$ *where* $x^0 = x(t, t_0, x_0), \quad x^1 = x(t, t_0, x_1)$,

$$D_{gr}(x_0, x_1) = \sup_{\alpha\in[0,1]} \max\{|5(\alpha - 1)|, |5(1 - \alpha)|\} < \delta,$$

and for $D_{gr}(x^0, x^1)$,

$$\begin{aligned} D_{gr}(x^0, x^1) &= \sup_{\alpha\in[0,1]} \max\{|6(\alpha - 1)\varphi(t) - (\alpha - 1)\varphi(t)|, |6(1 - \alpha)\varphi(t) - (1 - \alpha)\varphi(t)|\} \\ &\le \sup_{\alpha\in[0,1]} \max\{k\,|5(\alpha - 1)|, k\,|5(1 - \alpha)|\} \\ &= kD_{gr}(x_0, x_1) = k\delta \ = \varepsilon. \end{aligned}$$

So we find that the equation is Ulam-Hyers stable and stable by Lyapunov's mean too.

Example 8.2. *We want to show that Example 1 is practically stable with help of Theorem 10. We can choose* $V(t, x(t)) = D_{gr}(x(t), \theta^n)$ *and* $U(t, u(t)) = D_{gr}(u(t), \theta^n)$ *calculate* $D^+V(t, x) \le 4V(t, x) + U(t, u(t))$. *After this inequality, the scalar differential equation can be chosen as follows,*

$$\frac{dw}{dt} = F(t, w(t), v(t)) = 4w(t) + v(t) \quad w(t_0) = w_0 \text{ for } t \ge t_0, \tag{16}$$

$v(t) \in \mathbb{R}_+$ *is control function. Suppose that* $Y(t) = w_0 e^{4(t-t_0)}$ *is the fundemental solution of* $w' = 4w$. *We shall show that we can find suitable admissble controls* $v(t)$*to assure practically stable of the system (16). The transformation* $w = Y(t)z$ *reduces (16) to*

$$z' = 4\,(w_0)^{-1}\, e^{-4(t-t_0)} v(t) \quad z(t_0) = w_0. \tag{17}$$

Then with necessary processes and assumptions, we can find easily that $|z(t)| < A$ $t \ge t_0$, *provided* $|w_0| < \lambda$.

But $|w| = |Y(t)|\,|z(t)|$ *and therefore, if* $|Y(t)| \le 1 \quad t \ge t_0$ *and there exists a* $T > 0$ *such that* $|Y(t)| \le \frac{\beta}{A} < 1$, *then we have* $|w(t)| < A$ *which shows that the system (16) is practically stable. Let* $b(.), a(t, .) \in \kappa$ *and be chosen so that* $a(t, \lambda) < b(A)$ *and* $b(D_{gr}(x(t), \theta^n)) \le V(t, x) \le a(t, D_{gr}(x(t), \theta^n))$.

We can choose, $a(t, V(t, x)) = 2V(t, x)$ *and* $b(V(t, x)) = \frac{1}{2}V(t, x)$ *is satisfied that* $2\lambda < \frac{1}{2}A$.

Consequently, all assumption of Theorem 10 hold and IVP for NLFCDEs (14) is said to be practically stable like linear control differential equation (16).

9. Conclusions

This paper gave the necessary conditions for the solution of fuzzy control Volterra Integro differential system under granular differentiability to be practically stable for the first time in literature. This given method used especially Lyapunov-like function and comparison system. The paper investigated Ulam-Hyers-Rassias' stability and gave the relation between them by comparing it with classical Lyapunov stability. It supported this comparison and practical stability property with a numerical examples solved by fuzzy granular Laplace transform.

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