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FUZZY PARAMETERIZED FUZZY BINARY SOFT SETS AND THEIR APPLICATION IN DECISION MAKING

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ABSTRACT. This paper aims to define fuzzy parameterized fuzzy binary soft sets and there operations. Further, defined the decision-making algorithm under the fuzzy parameterized binary soft sets and illustrate an example to show the process of the algorithm provide the solution for decision making problems based on fuzzy parameterized fuzzy binary soft sets and compare the proposed decision-making algorithm with the existing method in the literature.

Keywords: Fuzzy soft sets; Binary soft sets; Fuzzy binary soft sets; Fuzzy parameterized fuzzy binary soft sets; Decision making.

AMS Subject Classification: 03E72, 90Bxx

1. INTRODUCTION

In daily life, we encounter many circumstances comprising uncertainty and complete data. Many theories have been put forward by researchers to model vagueness. Some of the theories are fuzzy set, rough set, hesitant fuzzy set, intuitionistic fuzzy set and vague set theories are well known and often useful mathematical approaches to model vagueness. Each of these theories lack an appropriate parameterized tool. To overcome this limitation, Molodtsov [18] introduced the concept of soft set theory. After, defining the concept of the soft set, many researchers studied set-theoretical operations of the soft sets and application in decision-making [9],[16],[17],[26]. The theory of soft set is based on adequate parameters and provides a more comprehensive description of objects. While soft sets are useful in many applications, they do not assign membership values to elements of a universal set. In some cases, soft sets be inefficient in handling real-time applications.

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Also, many researchers have worked on the expansions of soft sets. [15] Maji et al. who introduced the notion of fuzzy soft sets. A fuzzy soft set is a mapping from the parameter set to a fuzzy subset of the universal set. In fuzzy soft sets, it is considered that the importance of the degree of the parameters are same but this approach may not be applicable to some problems. Therefore, the concept of FP-soft set and FPF- soft set(fuzzy parameterized soft and fuzzy parameterized fuzzy soft sets) were introduced by Çağman [9]. By similar approach, many researchers worked on decision making problems, MPFS-set (multi parameter fuzzy soft set) approach to decision making problem, Intuitionistic FPF-soft sets, Bipolar FPF-soft sets, Hesitant FPFH-soft sets, NPS-set (neturosophic parameterized soft set), Interval valued NPS-set. [5], [4],[14],[23],[11],[12],[6],[7][24],[20].

Ahu Acikgoz et al. [1] defined and studied some properties of binary soft sets, which deal with two universal sets and a parameter set. Binary soft sets solve some applications that cannot be solved by soft sets and also do not assign membership values to the elements of universal sets. To address this limitation, Metilda et al. [19] defined fuzzy binary soft sets involving two universal sets and a parameter set, assigning membership values to the elements of universal sets. Patil et al.[21] studied applications of fuzzy binary soft sets. In this set the degrees of the parameters are the same. In this paper, we defined the fuzzy parameterized fuzzy binary soft sets (briefly fpfb-soft sets) and provide a solution for decision-making problems based on fpfb-soft sets.

2. Preliminary

Definition 2.1. [25]
Let A be a universe. Then, fuzzy set X over A is a function defined as follows:
$X = \{(\mu_X(x)/x) : x \in A\}$, *where,* $\mu_X : A \to [0, 1]$.

Here, μ_X called the membership function of X, and the value $\mu_X(x)$ is called the membership grade of $x \in A$. The value represents the degree of x belonging to the fuzzy set X.

Definition 2.2. [10]
Let U be the universal set, E be a set of parameters and $A \subset E$. Let $P(U)$ denote the power set of U. A soft set F_A over U is a set represented by the set of ordered pairs

$$F_A = \{(x, f_A(x)) : x \in E, f_A(x) \in P(U)\},$$

where, $f_A : E \to P(U)$ such that $f_A(x) = \phi$ if $x \neq A$.

and f_A is called the approximate function of the soft set F_A, and the value $f_A(x)$ is a set called x-element of the soft set for all $x \in E$.

Definition 2.3. [15]
Let U be a universal set, E be a set of parameters and $A \subset E$. Let $P(U)$ denote the fuzzy subsets of U. A fuzzy soft set F_A over U is a set that can be represented by the set of ordered pairs,

$$F_A = \{(x, f_A(x)) : x \in E, f_A(x) \in P(U)\},$$

where,

$$f_A : E \to P(U) \text{ such that } f_A(x) = \phi \text{ if } x \neq A,$$

and f_A is called the fuzzy approximate function of the fuzzy soft set F_A and the value $f_A(x)$ is a fuzzy set called x-element of the fuzzy soft set for all $x \in E$.

Definition 2.4. [9]
Let U be a universal set, $P(U)$ denote the fuzzy subsets of U, E be a set of parameters and X be a fuzzy set over E. Then, a fuzzy parameterized fuzzy soft set (briefly, FPF-soft set) F_X over U can be represented by the set of order pairs as $F_X = \{(\mu_X(x)/x), f_X(x) : x \in E\}$

where, $\mu_X : E \to [0,1]$ and $f_X : E \to P(U)$ such that $f_X(x) = \phi$ if $\mu_X(x) = 0$.
Here, f_X is called the fuzzy approximate function and μ_X called the membership function of FPF-soft sets.

Example 2.1. *Assume that $U = \{u_1, u_2, u_3, u_4\}$ be universal set and $E = \{x_1, x_2, x_3, x_4\}$ be set of parameters. If $X = \{0.1/x_1, 0.2/x_2, 0.4/x_3, 1/x_4\}$ is a fuzzy set over E, then we can write the following FPF-soft set as,*
$F_X = \{(0.1/x_1, \{u_1/0.1, u_2/0.2, u_3/0.5\}), (0.2/x_2, \{u_1/0.3, u_2/0.4, u_3/0.2\}),$
$(0.4/x_3\{u_1/0.3, u_2/0.4, u_3/0.2\}), (1/x_4, \{u_1/0.3, u_2/0.4, u_3/0.2\})\}.$

Definition 2.5. [1]
Let U_1 and U_2 be two universal sets, E be a set of parameters and $A \subseteq E$. Let F be a function defined by $F : A \to P(U_1) \times P(U_2)$, where, $P(U_1), P(U_2)$ denote the power sets of U_1, U_2. Then, the set F_A is called binary soft set over U_1 and U_2.

Definition 2.6. [21]
Let U_1 and U_2 be two universal sets. E be a set of parameters and $A \subseteq E$. Let F be a function defined by $F : A \to P(U_1) \times P(U_2)$, where, $P(U_1)$, $P(U_2)$ are a set of all fuzzy sets of U_1, U_2. Then, the set F_A is called Fuzzy binary soft set over U_1 and U_2.

3. Fuzzy Parameterized Fuzzy Binary Soft Sets

In this section, the concept of fuzzy parameterized fuzzy binary soft sets (briefly fpfb-soft sets) along with their operations, are introduced, and established their properties by examples.

Definition 3.1. *Let U_1 and U_2 be two universal sets, E be a set of all parameters and X be a fuzzy set over E. Then, a fpfb-soft set F_x over U_1 and U_2 is defined as*
$F_X = \{(\mu_X(x)/x, f_X(x)) : x \in E\}$ *where,* $\mu_X : E \to [0,1]$, $f_X : E \to P(U_1) \times P(U_2)$, *here, f_X is the fuzzy approximation function and μ_X is the membership function of fpfb-soft set.*

Example 3.1. *Let consider the universal sets $U_1 = \{u_1, u_2, u_3\}$ be the colleges, $U_2 = \{v_1, v_2, v_3\}$ be the courses and $E = \{e_1, e_2, e_3\}$ set of parameters. $E = \{0.2/e_1, 0.5/e_2, 0.6/e_3\}$ and*
$f_X(e_1) = \{u_1/0.5, u_2/0.4, u_3/0.7\}, \{v_1/0.2, v_2/0.6, v_3/0.9\}$
$f_X(e_2) = \{u_2/0.3, u_3/0.8\}, \{v_1/0.5, v_3/0.7\}$
$f_X(e_3) = \{u_1/0.2, u_2/0.6, u_3/0.5\}, \{v_1/0.3, v_2/0.1, v_3/0.9\}$
Then, the fpfb-soft set F_X is written as

$$F_X = \{(0.2/e_1, \{u_1/0.5, u_2/0.4, u_3/0.7\}, \{v_1/0.2, v_2/0.6, v_3/0.9\}),$$
$$(0.5/e_2, \{u_2/0.3, u_3/0.8\}, \{v_1/0.5, v_3/0.7\}),$$
$$(0.6/e_3, \{u_1/0.2, u_2/0.6, u_3/0.5\}, \{v_1/0.3, v_2/0.1, v_3/0.9\})\}.$$

Definition 3.2. *Let F_X, F_Y be fuzzy parameterized fuzzy binary soft sets. Then, F_X is fpfb-soft set subset of F_Y, denoted by $F_X \subseteq F_Y$ if*
(i) $\mu_X(x) \leq \mu_Y(x)$ and
(ii) $f_X(x) \subseteq f_Y(x)\ \forall\ x \in E$.

Example 3.2. *Let consider universal sets $U_1 = \{u_1, u_2, u_3\}$ be the colleges, $U_2 = \{v_1, v_2, v_3\}$ be the courses and $E = \{e_1, e_2, e_3\}$ set of parameters.*

$$F_X = \{(0.2/e_1, \{u_1/0.5, u_2/0.2, u_3/0.5\}, \{v_1/0.2, v_2/0.4, v_3/0.3\}),$$

$$(0.5/e_2, \{u_1/0.2, u_2/0.3, u_3/0.5\}, \{v_1/0.2, v_2/0.4, v_3/0.7\}),$$
$$(0.6/e_3, \{u_1/0.2, u_2/0.6, u_3/0.5\}, \{v_1/0.3, v_2/0.1, v_3/0.7\})\}$$

$$F_Y = \{(0.2/e_1, \{u_1/0.5, u_2/0.4, u_3/0.7\}, \{v_1/0.2, v_2/0.6, v_3/0.9\}),$$
$$(0.9/e_2, \{u_1/0.6, u_2/0.3, u_3/0.8\}, \{v_1/0.5, v_2/0.4, v_3/0.7\}),$$
$$(0.8/e_3, \{u_1/0.2, u_2/0.6, u_3/0.5\}, \{v_1/0.7, v_2/0.4, v_3/0.9\})\}$$

Here, $F_X \subseteq F_Y$.

Definition 3.3. *Let* F_X, F_Y *be fuzzy parameterized fuzzy binary soft sets. Then, the union of* F_X *and* F_Y*, denoted by* $F_X \cup F_Y$ *is defined as,*
(i) $\mu_{X\cup Y}(x) = max\{\mu_X(x), \mu_Y(x)\}$ *and*
(ii) $f_{X\cup Y}(x) = f_X(x) \cup f_Y(x)$.

Definition 3.4. *Let* F_X, F_Y *be fuzzy parameterized fuzzy binary soft sets. Then, the intersection of* F_X *and* F_Y*, denoted by* $F_X \cap F_Y$ *is defined as,*
(i) $\mu_{X\cap Y}(x) = min\{\mu_X(x), \mu_Y(x)\}$ *and*
(ii) $f_{X\cap Y}(x) = f_X(x) \cap f_Y(x)$.

Definition 3.5. *If* F_A *and* F_B *are two fpfb-soft sets then "*F_A *AND* F_B*" is a fpfb-soft set denoted by* $F_A \wedge F_B$ *and defined as,*
$F_A \wedge F_B = H_{A\times B}$*, where* $H_{A\times B}$ *calculated as,*
$\mu_{A\times B}(e_{\alpha\beta}) = \mu_A(e_\alpha) \wedge \mu_B(e_\beta)$
$f_{A\times B}(e_{\alpha\beta}) = f_A(e_\alpha) \wedge f_B(e_\beta)$.

Example 3.3. *Let consider universal sets* $U_1 = \{u_1, u_2, u_3\}$ *be colleges,* $U_2 = \{v_1, v_2, v_3\}$ *be the courses and* $E = \{e_1, e_2, e_3, e_4, e_5\}$ *set of parameters.*

$$F_A = \{(0.2/e_1, \{u_1/0.5, u_2/0.4, u_3/0.2\}, \{v_1/0.2, v_2/0.1, v_3/0.9\}),$$
$$(0.5/e_2, \{u_1/0.6, u_2/0.6, u_3/0.8\}, \{v_1/0.5, v_2/0.4, v_3/0.3\}),$$
$$(0.6/e_3, \{u_1/0.6, u_2/0.6, u_3/0.5\}, \{v_1/0.7, v_2/0.1, v_3/0.8\})\}$$

$$F_B = \{(0.2/e_4, \{u_1/0.3, u_2/0.4, u_3/0.7\}, \{v_1/0.2, v_2/0.6, v_3/0.9\}),$$
$$(0.9/e_5, \{u_1/0.6, u_2/0.3, u_3/0.8\}, \{v_1/0.8, v_2/0.6, v_3/0.7\})\}$$

$$F_A \wedge F_B = \{(0.2/e_{14}, \{u_1/0.3, u_2/0.4, u_3/0.2\}, \{v_1/0.2, v_2/0.1, v_3/0.9\})$$
$$(0.2/e_{15}, \{u_1/0.5, u_2/0.3, u_3/0.2\}, \{v_1/0.2, v_2/0.1, v_3/0.7\})$$
$$(0.2/e_{24}, \{u_1/0.3, u_2/0.4, u_3/0.7\}, \{v_1/0.2, v_2/0.4, v_3/0.3\})$$
$$(0.5/e_{25}, \{u_1/0.6, u_2/0.3, u_3/0.8\}, \{v_1/0.5, v_2/0.4, v_3/0.3\})$$
$$(0.2/e_{34}, \{u_1/0.3, u_2/0.4, u_3/0.5\}, \{v_1/0.2, v_2/0.1, v_3/0.8\})$$
$$(0.6/e_{35}, \{u_1/0.6, u_2/0.3, u_3/0.5\}, \{v_1/0.7, v_2/0.1, v_3/0.7\})\}.$$

Definition 3.6. *If* F_A *and* F_B *are two fpfb-soft sets then "*F_A *OR* F_B*" is a fpfb-soft set denoted by* $F_A \vee F_B$ *and defined as,*
$F_A \vee F_B = H_{A\times B}$*, where* $H_{A\times B}$ *calculated as,*
$\mu_{A\times B}(e_{\alpha\beta}) = \mu_A(e_\alpha) \vee \mu_B(e_\beta)$
$f_{A\times B}(e_{\alpha\beta}) = f_A(e_\alpha) \vee f_B(e_\beta)$.

Example 3.4. *Let consider universal sets $U_1 = \{u_1, u_2, u_3\}$ be a colleges, $U_2 = \{v_1, v_2, v_3\}$ be the courses and $E = \{e_1, e_2, e_3, e_4, e_5\}$ set of parameters.*

$$\begin{aligned} F_A = \{&(0.2/e_1, \{u_1/0.5, u_2/0.4, u_3/0.7\}, \{v_1/0.2, v_2/0.6, v_3/0.9\}), \\ &(0.5/e_2, \{u_1/0.6, u_2/0.3, u_3/0.8\}, \{v_1/0.5, v_2/0.4, v_3/0.7\}), \\ &(0.6/e_3, \{u_1/0.2, u_2/0.6, u_3/0.5\}, \{v_1/0.3, v_2/0.1, v_3/0.9\})\} \end{aligned}$$

$$\begin{aligned} F_B = \{&(0.2/e_4, \{u_1/0.6, u_2/0.4, u_3/0.3\}, \{v_1/0.2, v_2/0.5, v_3/0.9\}), \\ &(0.9/e_5, \{u_1/0.8, u_2/0.3, u_3/0.8\}, \{v_1/0.5, v_2/0.6, v_3/0.8\})\} \end{aligned}$$

$$\begin{aligned} F_A \vee F_B = \{&(0.2/e_{14}, \{u_1/0.6, u_2/0.4, u_3/0.7\}, \{v_1/0.2, v_2/0.6, v_3/0.9\}) \\ &(0.9/e_{15}, \{u_1/0.8, u_2/0.4, u_3/0.8\}, \{v_1/0.2, v_2/0.6, v_3/0.9\}) \\ &(0.5/e_{24}, \{u_1/0.6, u_2/0.4, u_3/0.8\}, \{v_1/0.5, v_2/0.5, v_3/0.9\}) \\ &(0.9/e_{25}, \{u_1/0.8, u_2/0.3, u_3/0.8\}, \{v_1/0.5, v_2/0.6, v_3/0.8\}) \\ &(0.6/e_{34}, \{u_1/0.6, u_2/0.6, u_3/0.5\}, \{v_1/0.3, v_2/0.5, v_3/0.9\})\} \\ &(0.9/e_{35}, \{u_1/0.8, u_2/0.6, u_3/0.8\}, \{v_1/0.5, v_2/0.6, v_3/0.9\})\}. \end{aligned}$$

4. An Application of Fuzzy Parameterized Fuzzy Binary Soft Sets in Decision Making

In this section, solved an application of fpfb-soft sets in decision making problem.

Example 4.1. *A farmer who is intended to purchase a tractor for the cultivation of his land. He is very much concerned about the tractors available in the market with standard quality. During the survey, it is observed that there are many companies or models with different sets of engines. Suppose that there are three companies tractors T_1, T_2 and T_3. The set of tractors is denoted by $U_1 = \{T_1, T_2, T_3\}$ consider another set as engines $U_2 = \{E_1, E_2\}$ and their features as set of parameters $E = \{e_1, e_2, e_3, e_4, e_5\}$, where e_1 stands for fuel consumption, e_2 stands for service, e_3 stands for 4 Wheel Drive, e_4 stands for cheap and e_5 stands for expensive.*

We define an algorithm to solve the given problem as follows:
***Step 1**: Construct a fpfb-soft sets F_A and F_B.*
***Step 2**: Apply AND operation to fpfb-soft sets F_A and F_B then we get fpfb-soft set F_C for desired set of parameters.*
***Step 3**: Construct a table for fpfb-soft set F_C.*
***Step 4**: Calculate the Column-sum.*
***Step 5**: Select the alternative with the maximum value.*

Construct a fpfb-soft sets F_A, let $A = \{0.2/e_1, 0.9/e_2, 0.4/e_3\}$ and $B = \{0.3/e_4, 0.8/e_5\}$ be the fuzzy subsets of E, fuzzy set A represents the features of tractors and fuzzy set B represents the prizes of tractors.

$$\begin{aligned} F_A = \{&(0.2/e_1, \{T_1/0.4, T_2/0.5, T_3/0.7\}, \{E_1/0.6, E_2/0.2, \}), \\ &(0.9/e_2, \{T_1/0.6, T_2/0.2, T_3/0.5\}, \{E_1/0.1, E_2/0.4\}), \\ &(0.4/e_3, \{T_1/0.2, T_2/0.6, T_3/0.5\}, \{E_1/0.3, E_2/0.1\})\} \\ F_B = \{&(0.3/e_4, \{T_1/0.2, T_2/0.5, T_3/0.3\}, \{E_1/0.7, E_2/0.6\}), \end{aligned}$$

$(0.8/e_5, \{T_1/0.1, T_2/0.3, T_3/0.2\}, \{E_1/0.5, E_2/0.4\})\}$

Apply And operation to F_A and F_B then, we get

$$\begin{aligned} F_C = \{&(0.2/e_{14}, \{T_1/0.2, T_2/0.5, T_3/0.3\}, \{E_1/0.6, E_2/0.2\}),\\ &(0.2/e_{15}, \{T_1/0.1, T_2/0.3, T_3/0.2\}, \{E_1/0.5, E_2/0.2\}),\\ &(0.3/e_{24}, \{T_1/0.2, T_2/0.2, T_3/0.3\}, \{E_1/0.1, E_2/0.4\})\\ &(0.8/e_{25}, \{T_1/0.1, T_2/0.5, T_3/0.2\}, \{E_1/0.1, E_2/0.4\})\\ &(0.3/e_{34}, \{T_1/0.2, T_2/0.5, T_3/0.3\}, \{E_1/0.3, E_2/0.1\})\\ &(0.4/e_{35}, \{T_1/0.1, T_2/0.3, T_3/0.2\}, \{E_1/0.3, E_2/0.1\})\}. \end{aligned}$$

TABLE 1. Tabular depiction of the fpfb-soft set F_C

	(T_1, E_1)	(T_1, E_2)	(T_2, E_1)	(T_2, E_2)	(T_3, E_1)	(T_3, E_2)
$0.2/e_{14}$	*(0.2, 0.6)*	*(0.2, 0.2)*	*(0.5, 0.6)*	*(0.5, 0.2)*	*(0.3, 0.6)*	*(0.3, 0.2)*
$0.2/e_{15}$	*(0.1, 0.5)*	*(0.1, 0.2)*	*(0.3, 0.5)*	*(0.3, 0.2)*	*(0.2, 0.5)*	*(0.2, 0.2)*
$0.3/e_{24}$	*(0.2, 0.1)*	*(0.2, 0.4)*	*(0.2, 0.1)*	*(0.2, 0.4)*	*(0.3, 0.1)*	*(0.3, 0.4)*
$0.8/e_{25}$	*(0.1, 0.1)*	*(0.1, 0.4)*	*(0.2, 0.1)*	*(0.2, 0.4)*	*(0.2, 0.1)*	*(0.2, 0.4)*
$0.3/e_{34}$	*(0.2, 0.3)*	*(0.2, 0.1)*	*(0.5, 0.3)*	*(0.5, 0.1)*	*(0.3, 0.3)*	*(0.3, 0.1)*
$0.4/e_{35}$	*(0.1, 0.3)*	*(0.1, 0.1)*	*(0.3, 0.3)*	*(0.3, 0.1)*	*(0.2, 0.3)*	*(0.2, 0.1)*

Calculate the column-sum as $\sum_{e_{\alpha\beta}} T_iE_j$ for the particular column for all the parameters:

$\sum_{e_{\alpha\beta}} T_1E_1 = (0.2\times0.6)+(0.1\times0.5)+(0.2\times0.1)+(0.1\times0.1)+(0.2\times0.3)+(0.1\times0.3) = 0.29.$
$\sum_{e_{\alpha\beta}} T_1E_2 = (0.2\times0.2)+(0.1\times0.2)+(0.2\times0.4)+(0.1\times0.4)+(0.2\times0.1)+(0.1\times0.1) = 0.21.$
$\sum_{e_{\alpha\beta}} T_2E_1 = (0.5\times0.6)+(0.3\times0.5)+(0.2\times0.1)+(0.2\times0.1)+(0.5\times0.3)+(0.3\times0.3) = 0.73.$
$\sum_{e_{\alpha\beta}} T_2E_2 = (0.5\times0.2)+(0.3\times0.2)+(0.2\times0.4)+(0.2\times0.4)+(0.5\times0.1)+(0.3\times0.1) = 0.40$
$\sum_{e_{\alpha\beta}} T_3E_1 = (0.3\times0.6)+(0.2\times0.5)+(0.3\times0.1)+(0.2\times0.1)+(0.3\times0.3)+(0.2\times0.3) = 0.48.$
$\sum_{e_{\alpha\beta}} T_3E_2 = (0.3\times0.2)+(0.2\times0.2)+(0.3\times0.4)+(0.2\times0.4)+(0.3\times0.1)+(0.2\times0.1) = 0.35.$

Clearly, a farmer chooses the T_2 Tractor with the E_1 Engine, since it has the maximum value 0.73 among the others.

5. Comparison of Results

In this section, the proposed method is validated by comparing it with the existing methods Metilda and Subhashini [19].

5.1. **Comparative Study.** In this subsection, the example discussed by Metilda and Subhashini [19] is considered and the experimental result of the proposed method is compared with their method.

Example 5.1. *Let us consider the colleges $U_1 = \{c_1, c_2, c_3, c_4\}$ and courses $U_2 = \{d_1, d_2, d_3, d_4\}$ with the parameter set $E = \{1/e_1, 1/e_2, 1/e_3, 1/e_4\}$ we are considered all parameters with membership grade as 1.*

$$\begin{aligned} F_A = \{&(1/e_1, \{c_1/0.5, c_2/0.4, c_3/0.1, c_4/0.7\}, \{d_1/0.3, d_2/0.4, d_3/0.6, d_4/0.2\}),\\ &(1/e_2, \{c_1/0.7, c_2/0.6, c_3/0.3, c_4/0.5\}, \{d_1/0.5, d_2/0.4, d_3/0.7, d_4/0.9\}),\\ &(1/e_3, \{c_1/0.1, c_2/0.3, c_3/0.2, c_4/0.4\}, \{d_1/0.2, d_2/0.5, d_3/0.6, d_4/0.3\}), \end{aligned}$$

$$(1/e_4, \{c_1/0.4, c_2/0.2, c_3/0.7, c_4/0.3\}, \{d_1/0.4, d_2/0.3, d_3/0.1, d_4/0.5\})\}$$

$$F_B = \{(1/e_1, \{c_1/0.5, c_2/0.3, c_3/0.6, c_4/0.7\}, \{d_1/0.5, d_2/0.1, d_3/0.7, d_4/0.3\}),$$
$$(1/e_2, \{c_1/0.3, c_2/0.2, c_3/0.5, c_4/0.8\}, \{d_1/0.4, d_2/0.9, d_3/0.6, d_4/0.2\}),$$
$$(1/e_3, \{c_1/0.5, c_2/0.2, c_3/0.6, c_4/0.7\}, \{d_1/0.8, d_2/0.3, d_3/0.4, d_/0.5\}),$$
$$(1/e_4, \{c_1/0.4, c_2/0.7, c_3/0.8, c_4/0.3\}, \{d_1/0.6, d_2/0.4, d_3/0.8, d_4/0.2\})\}.$$

The result obtained by proposed method suggests that the college c_4 and course d_3 is better option. Also, the proposed method gives the best college for each course d_1, d_2, d_3 and d_4 and also the best course in each college c_1, c_2, c_3 and c_4.

But the results of Metilda and subhashini suggest the c_4 college and d_1 course . Their method limits by suggesting only a single college (c_k) and single course (d_m). It fails to suggest the best college for each course and the best course in each college. The absence of weightage to the parameter set, which is the advantage of the proposed method. Which gives the best method.

6. Conclusions

This article mainly focus on solving the DM problem by fuzzy parameterized fuzzy binary soft set which can be achieved by assigning the membership value to each parameter. An algorithm to solve DM problems is discussed and an application of fpfb-soft sets in DM is discussed using a practical example. To show the advantage of the proposed method, the comparison has been done with the existing methods. The limitations of existing methods are pointed out and the advantage of the proposed method is shown.

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