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# INVESTIGATION OF INTUITIONISTIC FUZZY CONTRA $G_\delta$-$e$-LOCALLY CONTINUOUS AND IRRESOLUTE FUNCTIONS

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Abstract. This paper aims to present the concepts of intuitionistic fuzzy contra $G_\delta$-$e$-locally continuous and intuitionistic fuzzy contra $G_\delta$-$e$-locally irresolute functions within the framework of intuitionistic fuzzy topological spaces. Additionally, various noteworthy properties of these functions are explored and elucidated.

Keywords:intuitionistic fuzzy contra $G_\delta$-$e$-locally continuous, intuitionistic fuzzy contra $G_\delta$-$e$-locally irresolute function.

AMS Subject Classification: 54A40, 54A99, 03E72, 03E99

## 1. Introduction

The genesis of fuzzy set theory can be traced back to Zadeh [11], with Atanassov [1] later extending the concept to intuitionistic fuzzy($\mathbb{I}.\mathbb{F}$) sets. $\mathbb{I}.\mathbb{F}$ topological spaces and related notions, including $\mathbb{I}.\mathbb{F}$ continuity, were introduced by Coker [3]. Sobana et al. [9] introduced the notion of $\mathbb{I}.\mathbb{F}$ $e$-closed sets, while Ganster and Relly [5] utilized locally closed sets to define LC-continuity and LC-irresoluteness. Balasubramanian [2] delved into fuzzy $G_\delta$ sets in fuzzy topological spaces.

Building upon these foundational works, this paper introduces the concepts of $\mathbb{I}.\mathbb{F}$ contra $G_\delta$-$e$-locally continuous and $\mathbb{I}.\mathbb{F}$ contra $G_\delta$-$e$-locally irresolute functions in $\mathbb{I}.\mathbb{F}$ topological spaces, along with the establishment of various intriguing properties.

## 2. Preliminaries

**Definition 2.1.** [1] Let $\Im$ be a nonempty, predetermined set $I$ be [0, 1]. An $\mathbb{I}.\mathbb{F}$ set ($\mathbb{I}.\mathbb{F}.\mathbb{S}$)$\mathit{\Gamma}$ is an entity of the subsequent structure $\mathit{\Gamma} = \{\langle \varkappa,\ \theta_{\mathit{\Gamma}}(\varkappa),\ \varsigma_{\mathit{\Gamma}}(\varkappa)\rangle : \varkappa \in \Im\}$, where the function $\theta_{\mathit{\Gamma}} : \Im \to I$ & $\varsigma_{\mathit{\Gamma}} : \Im \to I$ represent the degree of inclusion ($\theta_{\mathit{\Gamma}}(\varkappa)$) and the degree of exclusion ($\varsigma_{\mathit{\Gamma}}(\varkappa)$) $\forall\ \varkappa \in \Im$ in regard to the set $\mathit{\Gamma}$, as appropriate, and $0 \leq \theta_{\mathit{\Gamma}}(\varkappa) + \varsigma_{\mathit{\Gamma}}(\varkappa) \leq 1\ \forall\ \varkappa \in \Im$.

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**Definition 2.2.** [1] Let $\Im$ be a nonempty predetermined set and the $\mathbb{I}.\mathbb{F}.\mathbb{S}$'s $\Gamma$ and ß is an entity of the subsequent structure $\Gamma = \{\langle \varkappa,\ \theta_\Gamma(\varkappa),\ \varsigma_\Gamma(\varkappa)\rangle : \varkappa \in \Im\}$, $ß = \{\langle \varkappa,\ \theta_ß(\varkappa),\ \varsigma_ß(\varkappa)\rangle : \varkappa \in \Im\}$. Then

(i) $\Gamma \subseteq ß$ if and only if $\theta_\Gamma(\varkappa) \leq \theta_ß(\varkappa)$ and $\varsigma_\Gamma(\varkappa) \geq \varsigma_ß(\varkappa)$ for all $\varkappa \in \Im$;
(ii) $\overline{\Gamma} = \{\langle \varkappa,\ \varsigma_\Gamma(\varkappa), \theta_\Gamma(\varkappa)\rangle : \varkappa \in \Im\}$;
(iii) $\Gamma \cap ß = \{\langle \varkappa,\ \theta_\Gamma(\varkappa) \wedge \theta_ß(\varkappa), \varsigma_\Gamma(\varkappa) \vee \varsigma_ß(\varkappa)\rangle : \varkappa \in \Im\}$;
(iv) $\Gamma \cup ß = \{\langle \varkappa,\ \theta_\Gamma(\varkappa) \vee \theta_ß(\varkappa), \varsigma_\Gamma(\varkappa) \wedge \varsigma_ß(\varkappa)\rangle : \varkappa \in \Im\}$;

**Definition 2.3.** [1] The $\mathbb{I}.\mathbb{F}.\mathbb{S}$'s $\mathfrak{0}$ and $\mathfrak{1}$ are formulated by , $\mathfrak{0} = \{\langle ,\ 0,\ 1\rangle : \varkappa \in \Im\}$ and $\mathfrak{1} = \{\langle \varkappa,\ 1,\ 0\rangle : \varkappa \in \Im\}$.

**Definition 2.4.** [3] An $\mathbb{I}.\mathbb{F}$ topology ($\mathbb{I}.\mathbb{F}.\mathbb{T}$) n Coker's sense, given a nonempty set, express concisely. $\Im$ is a family $\top$ of $\mathbb{I}.\mathbb{F}.\mathbb{S}$s in $\Im$ Meeting the following axioms.:

(i) $\mathfrak{0},\ \mathfrak{1} \in \top$;
(ii) $G_1 \cap G_2 \in \top$, for any $G_1,\ G_2 \in \top$;
(iii) $\cup G_i \in \top$ for arbitrary family $\{G_i : i \in J\} \subseteq \top$.

Within this article, authored by $(\Im,\ \top)$ Simply through $\Im$ We'll signify Coker's $\mathbb{I}.\mathbb{F}$ topological space ($\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$). Each $\mathbb{I}.\mathbb{F}.\mathbb{S}$ that pertains to $\top$ is called an $\mathbb{I}.\mathbb{F}$ open set ($\mathbb{I}.\mathbb{F}_{\mathbb{OS}}$) in $\Im$. The complement thereof $\overline{\Gamma}$ of an $\mathbb{I}.\mathbb{F}_{\mathbb{OS}}$ $\Gamma$ in $\Im$ is referred to as a $\mathbb{I}.\mathbb{F}$ closed set ($\mathbb{I}.\mathbb{F}_{\mathbb{CS}}$) in $\Im$.

**Definition 2.5.** [3] Let $(\Im,\ \top)$ represent $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$ and $\Gamma = \{\langle \varkappa,\ \theta_\Gamma,\ \vartheta_\Gamma\rangle : \varkappa \in \Im\}$ be an $\mathbb{I}.\mathbb{F}.\mathbb{S}$ in $\Im$. Then the $\mathbb{I}.\mathbb{F}$ closure and $\mathbb{I}.\mathbb{F}$ interior of $\Gamma$ are delineated by

(i) $\mathbb{I}.\mathbb{F}_{cl}(\Gamma) = \bigcap\{C : C$ is an $\mathbb{I}.\mathbb{F}_{\mathbb{CS}}$ in $\Im$and $C \supseteq \Gamma\}$;
(ii) $\mathbb{I}.\mathbb{F}_{int}(\Gamma) = \bigcup\{D : D$ is an $\mathbb{I}.\mathbb{F}_{\mathbb{OS}}$ in $\Im$and $D \subseteq \Gamma\}$;

**Definition 2.6.** [4] Let $\Im$ constitute a nonempty set, and $\varkappa \in \Im$ a constant element within $\Im$. If $r \in I_0$, $s \in I_1$ are predetermined real numbers such that $r + s \leq 1$, then the $\mathbb{I}.\mathbb{F}.\mathbb{S}$ $\varkappa_{r,s} = \langle \varkappa, \varkappa_r, 1 - \varkappa_{1-s}\rangle$ is termed as $\mathbb{I}.\mathbb{F}$ point ($\mathbb{I}.\mathbb{F}.\mathbb{P}$) in $\Im$, where $r$ denotes the degree of inclusion of $\varkappa_{r,s}$, $s$ denotes the degree of exclusion of $\varkappa_{r,s}$ and $\varkappa \in \Im$ the support of $\varkappa_{r,s}$. The $\mathbb{I}.\mathbb{F}.\mathbb{P}$ $\varkappa_{r,s}$ resides within $\mathbb{I}.\mathbb{F}.\mathbb{S}$ $\Gamma(\varkappa_{r,s} \in \Gamma)$ if and only if $r < \theta_\Gamma(\varkappa)$, $s > \varsigma_\Gamma(\varkappa)$.

**Definition 2.7.** [6] An $\mathbb{I}.\mathbb{F}.\mathbb{S}$ $U$ of an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$ $\Im$ is called

(i) neighborhood of an $\mathbb{I}.\mathbb{F}.\mathbb{P}$ $c(a,b)$, if there's a $\mathbb{I}.\mathbb{F}_{\mathbb{OS}}$ $G$ in $\Im$ such that $c(a,b) \in G \leq U$.
(ii) $q$-neighborhood of an $\mathbb{I}.\mathbb{F}.\mathbb{P}$ $c(a,b)$, if there's a $\mathbb{I}.\mathbb{F}_{\mathbb{OS}}$ $G$ in $\Im$ such that $c(a,b)qG \leq U$.

**Definition 2.8.** [10] Let $\Gamma$ be $\mathbb{I}.\mathbb{F}.\mathbb{S}$ in an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$ $(\Im,\ \top)$. $\Gamma$ is termed as

(i) $\mathbb{I}.\mathbb{F}$ regular open set ( $\mathbb{I}.\mathbb{F}_{\mathbb{ROS}}$) if $\Gamma = \mathbb{I}.\mathbb{F}_{int}\mathbb{I}.\mathbb{F}_{cl}(\Gamma)$
(ii) $\mathbb{I}.\mathbb{F}$ regular closed set ( $\mathbb{I}.\mathbb{F}_{\mathbb{RCS}}$) if $\Gamma = \mathbb{I}.\mathbb{F}_{cl}\mathbb{I}.\mathbb{F}_{int}(\Gamma)$

**Definition 2.9.** [2] Let $(\Im,\ \top)$ be a fuzzy topological space and $\lambda$ be a fuzzy set in $\Im$. $\lambda$ is called $G_\delta$ set if $\lambda = \bigwedge_{i=1}^{\infty} \lambda_i$ where each $\lambda_i \in \top$. The complement of fuzzy $G_\lambda$ is fuzzy $F_\sigma$

**Definition 2.10.** [10] Let $(\Im,\ \top)$ be an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$ and $\Gamma = \langle \varkappa, \theta_\Gamma(\varkappa), \vartheta_\Gamma(\varkappa)\rangle$ be a $\mathbb{I}.\mathbb{F}.\mathbb{S}$ in $\Im$. Then the $\mathbb{I}.\mathbb{F}$ $\delta$ closure and interior of $\Gamma$ are represented and delineated by $\mathbb{I}.\mathbb{F}_{cl_\delta}(\Gamma) = \cap\{K : K$ is an $\mathbb{I}.\mathbb{F}_{\mathbb{RCS}}$ in $\Im$ and $\Gamma \subseteq K\}$ and $\mathbb{I}.\mathbb{F}_{int_\delta}(\Gamma) = \cup\{G : G$ is an $\mathbb{I}.\mathbb{F}_{\mathbb{ROS}}$ in $\Im$ and $G \subseteq \Gamma\}$.

**Definition 2.11.** [9] Let $\varGamma$ be an $\mathbb{I}.\mathbb{F}.\mathbb{S}$ in an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$ $(\Im,\ \top)$. $\varGamma$ is termed as an $\mathbb{I}.\mathbb{F}$ $e$-open set ($\mathbb{I}.\mathbb{F}e\mathbb{O}\mathbb{S}$) in $\Im$ if $\varGamma \subseteq \mathbb{I}.\mathbb{F}_{cl}\mathbb{I}.\mathbb{F}_{int_\delta}(\varGamma) \cup \mathbb{I}.\mathbb{F}_{int}\mathbb{I}.\mathbb{F}_{cl_\delta}(\varGamma)$

**Definition 2.12.** [7] Let $(\Im,\ \top)$ be an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$. Let $\varGamma = \{\langle \varkappa,\ \theta_\varGamma(\varkappa),\ \varsigma_\varGamma(\varkappa)\rangle : \varkappa \in \Im\}$ be an $\mathbb{I}.\mathbb{F}.\mathbb{S}$ on an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$ $(\Im,\ \top)$. Then $\varGamma$ is deemed $\mathbb{I}.\mathbb{F}$ $e$- locally closed set ($\mathbb{I}.\mathbb{F}-e$-$\mathbb{L}\mathbb{C}\mathbb{S}$) if $\varGamma = C \cap D$, where $C = \{\langle \varkappa,\ \theta_C(\varkappa),\ \varsigma_C(\varkappa)\rangle : \varkappa \in \Im\}$ is an $\mathbb{I}.\mathbb{F}e\mathbb{O}\mathbb{S}$ and $D = \{\langle \varkappa,\ \theta_D(\varkappa),\ \varsigma_D(\varkappa)\rangle : \varkappa \in \Im\}$ is an $\mathbb{I}.\mathbb{F}e\mathbb{C}\mathbb{S}$ in $(\Im,\ \top)$.

**Definition 2.13.** [7] Let $(\Im,\ \top)$ be an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$. Let $\varGamma = \{\langle \varkappa,\ \theta_\varGamma(\varkappa),\ \varsigma_\varGamma(\varkappa)\rangle : \varkappa \in \Im\}$ be an $\mathbb{I}.\mathbb{F}.\mathbb{S}$ on an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$ $\Im$. Then $\varGamma$ is deemed an $\mathbb{I}.\mathbb{F}$ $eG_\delta$- set if $\varGamma = \bigcap_{i=1}^{\infty} \varGamma_i$, where $\varGamma_i = \{\langle \varkappa,\ \theta_{\varGamma_i}(\varkappa),\ \varsigma_{\varGamma_i}(\varkappa)\rangle : \varkappa \in \Im\}$ is an $\mathbb{I}.\mathbb{F}e\mathbb{O}\mathbb{S}$ in an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$ $(\Im,\ \top)$.

**Definition 2.14.** [7] Let $(\Im,\ \top)$ be an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$. Let $\varGamma = \{\langle \varkappa,\ \theta_\varGamma(\varkappa),\ \varsigma_\varGamma(\varkappa)\rangle : \varkappa \in \Im\}$ be an $\mathbb{I}.\mathbb{F}.\mathbb{S}$ on an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$ $(\Im,\ \top)$. Then $\varGamma$ is deemed an $\mathbb{I}.\mathbb{F}$ $eG_\delta$-locally closed set ($\mathbb{I}.\mathbb{F}-eG_\delta$-$\mathbb{L}\mathbb{C}\mathbb{S}$) if $\varGamma = C \cap D$, where $C = \{\langle \varkappa,\ \theta_C(\varkappa),\ \varsigma_C(\varkappa)\rangle : \varkappa \in \Im\}$ is an $\mathbb{I}.\mathbb{F}$ $eG_\delta$ set and $D = \{\langle \varkappa,\ \theta_D(\varkappa),\ \varsigma_D(\varkappa)\rangle : \varkappa \in \Im\}$ is an $\mathbb{I}.\mathbb{F}e\mathbb{C}\mathbb{S}$ in $(\Im,\ \top)$.

The complement of an $\mathbb{I}.\mathbb{F}$ $eG_\delta$-$\mathbb{L}\mathbb{C}\mathbb{S}$ is said to be an $\mathbb{I}.\mathbb{F}$ $eG_\delta$-$\mathbb{L}\mathbb{O}\mathbb{S}$.

**Definition 2.15.** [7] Let $(\Im,\ \top)$ be an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$. Let $\varGamma = \{\langle \varkappa,\ \theta_\varGamma(\varkappa),\ \varsigma_\varGamma(\varkappa)\rangle : \varkappa \in \Im\}$ be an $\mathbb{I}.\mathbb{F}.\mathbb{S}$ on an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$ $(\Im,\ \top)$. Then $\varGamma$ is deemed an $\mathbb{I}.\mathbb{F}$ $G_\delta$-$e$-locally closed set ($\mathbb{I}.\mathbb{F}G_\delta$-$e$-$\mathbb{L}\mathbb{C}\mathbb{S}$) if $\varGamma = \text{ß} \cap C$, where $\text{ß} = \{\langle \varkappa,\ \theta_{\text{ß}}(\varkappa),\ \varsigma_{\text{ß}}(\varkappa)\rangle : \varkappa \in \Im\}$ is an $\mathbb{I}.\mathbb{F}$ $G_\delta$ set and $C = \{\langle \varkappa,\ \theta_C(\varkappa),\ \varsigma_C(\varkappa)\rangle : \varkappa \in \Im\}$ is an $\mathbb{I}.\mathbb{F}e\mathbb{C}\mathbb{S}$ in $(\Im,\ \top)$.

The complement of an $\mathbb{I}.\mathbb{F}$ $G_\delta$-$e$-$\mathbb{L}\mathbb{C}\mathbb{S}$ is said to be an $\mathbb{I}.\mathbb{F}$ $G_\delta$-$e$-$\mathbb{L}\mathbb{O}\mathbb{S}$.

**Definition 2.16.** [7] Let $(\Im,\ \top)$ be an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$. Let $\varGamma = \{\langle \varkappa,\ \theta_\varGamma(\varkappa),\ \varsigma_\varGamma(\varkappa)\rangle : \varkappa \in \Im\}$ be an $\mathbb{I}.\mathbb{F}.\mathbb{S}$ on an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$ $(\Im,\ \top)$. The $\mathbb{I}.\mathbb{F}$ $G_\delta$-$e$-locally closure of $\varGamma$ are represented and delineated by $\mathbb{I}.\mathbb{F}G_\delta$-$e$-$l_{cl}(\varGamma) = \bigcap\{\text{ß} : \text{ß} = \langle \varkappa,\ \theta_{\text{ß}}(\varkappa),\ \varsigma_{\text{ß}}(\varkappa)\rangle : \varkappa \in \Im$ is an $\mathbb{I}.\mathbb{F}$ $G_\delta$-$e$-$\mathbb{L}\mathbb{C}\mathbb{S}$ in $\Im$ and $\varGamma \subseteq \text{ß}\}$.

**Definition 2.17.** [7] Let $(\Im,\ \top)$ be an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$ Let $\varGamma = \{\langle \varkappa,\ \theta_\varGamma(\varkappa),\ \varsigma_\varGamma(\varkappa)\rangle : \varkappa \in \Im\}$ be an $\mathbb{I}.\mathbb{F}.\mathbb{S}$ on an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$ $(\Im,\ \top)$. The $\mathbb{I}.\mathbb{F}$ $G_\delta$-$e$-locally interior of $\varGamma$ are represented and delineated by $\mathbb{I}.\mathbb{F}G_\delta$-$e$-$l_{int}(\varGamma) = \bigcup\{\text{ß} : \text{ß} = \{\langle \varkappa,\ \theta_{\text{ß}}(\varkappa),\ \varsigma_{\text{ß}}(\varkappa)\rangle : \varkappa \in \Im\}$ is an $\mathbb{I}.\mathbb{F}$ $G_\delta$-$e$-$\mathbb{L}\mathbb{O}\mathbb{S}$ in $\Im$ and $\text{ß} \subseteq \varGamma$.

**Definition 2.18.** [7] Let $(\Im,\ \top)$ be an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$. Let $\varGamma = \langle \varkappa,\ \theta_\varGamma,\ \varsigma_\varGamma\rangle$ be an $\mathbb{I}.\mathbb{F}.\mathbb{S}$ in an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$ $(\Im,\ \top)$. Then $\varGamma$ is deemed an $\mathbb{I}.\mathbb{F}$ $G_\delta$-$e$-locally neighbourhood of an $\mathbb{I}.\mathbb{F}.\mathbb{P}$ $x_{r,s}$ if there exists an $\mathbb{I}.\mathbb{F}$ $G_\delta$-$e$-$\mathbb{L}\mathbb{O}\mathbb{S}$ ß in an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$ $(\Im,\ \top)$ such that $x_{r,s} \in \text{ß},\ \text{ß} \subseteq \varGamma$. It is indicated by $\mathbb{I}.\mathbb{F}G_\delta$-$e$-$lnbd$.

**Definition 2.19.** [7] Let $(\Im,\ \top)$ be an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$. Let $\varGamma = \langle \varkappa,\ \theta_\varGamma,\ \varsigma_\varGamma\rangle$ be an $\mathbb{I}.\mathbb{F}.\mathbb{S}$ in an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$ $(\Im,\ \top)$. Then $\varGamma$ is deemed an $\mathbb{I}.\mathbb{F}$ $G_\delta$-$e$-locally quasi neighbourhood of an $\mathbb{I}.\mathbb{F}.\mathbb{P}$ $x_{r,s}$ if there exists an $\mathbb{I}.\mathbb{F}$ $G_\delta$-$e$-$loc$ ß in an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$ $(\Im,\ \top)$ such that $x_{r,s}q\text{ß},\ \text{ß} \subseteq \varGamma$. It is indicated by $\mathbb{I}.\mathbb{F}G_\delta$-$e$-$lqnbd$.

**Remark 2.1.** [7]

(i) The family of all $\mathbb{I}.\mathbb{F}$ $G_\delta$-$e$-$lnbd$ of an $\mathbb{I}.\mathbb{F}.\mathbb{P}$ $x_{r,s}$ It is indicated by $N^{\mathbb{I}.\mathbb{F}G_\delta\text{-}e\text{-}l}(x_{r,s})$.
(i) The family of all $\mathbb{I}.\mathbb{F}$ $G_\delta$-$e$-$lqnbd$ of an $\mathbb{I}.\mathbb{F}.\mathbb{P}$ $x_{r,s}$ It is indicated by $N^{\mathbb{I}.\mathbb{F}G_\delta\text{-}e\text{-}lq}(x_{r,s})$.

## 3. Intuitionistic fuzzy contra $G_\delta$-$e$-locally continuous functions

**Definition 3.1.** Let $(\Im,\ \top)$ and $(\wp,\ S)$ be any two $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$'s. Let $\mathfrak{f} : (\Im,\ \top) \to (\wp,\ S)$ be an $\mathbb{I}.\mathbb{F}$ mapping ($\mathbb{I}\mathbb{F}\mathbb{M}$). Then $\mathfrak{f}$ is said to be an $\mathbb{I}.\mathbb{F}$ contra $G_\delta$-$e$-locally continuous function($\mathbb{I}.\mathbb{F}Cont.G_\delta$-$e$-$l.cts.fun$), if $\mathfrak{f}^{-1}(\varGamma)$ is $\mathbb{I}.\mathbb{F}G_\delta$-$e$-$\mathbb{L}\mathbb{C}\mathbb{S}$ in $\Im$ for every $\mathbb{I}.\mathbb{F}OS$ $\varGamma$ in $\wp$

**Theorem 3.1.** Let $(\Im,\ \top)$ and $(\wp,\ S)$ be any two $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$'s. Let $\mathfrak{f}:(\Im,\ \top)\to(\wp,\ S)$ be an $\mathbb{IFM}$. Then the following are equivalent.

(i) $\mathfrak{f}$ is an $\mathbb{I}.\mathbb{F}Cont.G_\delta\text{-}e\text{-}l.cts.fun$.
(ii) $\mathfrak{f}^{-1}(\text{ß})$ is an $\mathbb{I}.\mathbb{F}\ G_\delta\text{-}e\text{-}\mathbb{LCS}$ in an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$ $(\Im,\ \top)$, for each $\mathbb{I}.\mathbb{F}_{\mathbb{OS}}$ ß in an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$ $(\wp,\ S)$.
(iii) $\mathfrak{f}^{-1}(\varGamma)$ is an $\mathbb{I}.\mathbb{F}\ G_\delta\text{-}e\text{-}\mathbb{LOS}$ in an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$ $(\Im,\ \top)$, for each $\mathbb{I}.\mathbb{F}_{\mathbb{CS}}$ $\varGamma$ in an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$ $(\wp,\ S)$.
(iv) $\mathfrak{f}^{-1}(\mathbb{I}.\mathbb{F}_{int}(\varGamma))\subseteq\mathbb{I}.\mathbb{F}G_\delta\text{-}e\text{-}l_{int}(\mathfrak{f}^{-1}(\varGamma))$, for each $\mathbb{I}.\mathbb{F}_{\mathbb{CS}}$ $\varGamma$ in an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$ $(\wp,\ S)$.
(v) $\mathbb{I}.\mathbb{F}G_\delta\text{-}e\text{-}l_{cl}(\mathfrak{f}^{-1}(\varGamma))\subseteq\mathfrak{f}^{-1}(\mathbb{I}.\mathbb{F}_{cl}(\varGamma))$, for each $\mathbb{I}.\mathbb{F}_{\mathbb{OS}}$ $\varGamma$ in an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$ $(\wp,\ S)$.

*Proof.* (i)⇒ (ii): Let $\varGamma$ be an $\mathbb{I}.\mathbb{F}_{\mathbb{CS}}$ in an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$ $(\wp,\ S)$. Let $x_{r,s}$ be an $\mathbb{I}.\mathbb{F}.\mathbb{P}$ in an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$ $(\Im,\ \top)$ such that $x_{r,s}qf^{-1}(\varGamma)$. Since $\mathfrak{f}$ is an $\mathbb{I}.\mathbb{F}Cont.G_\delta\text{-}e\text{-}l.cts.fun$, there exists $\text{ß}\in N^{\mathbb{I}.\mathbb{F}G_\delta\text{-}e\text{-}lq}(x_{r,s})$ such that $\mathfrak{f}(\text{ß})\subseteq\varGamma$. Then

$$x_{r,s}\in\text{ß} \tag{1}$$

$$\text{ß}\subseteq\mathfrak{f}^{-1}(\mathfrak{f}(\text{ß})) \tag{2}$$

Thus, $x_{r,s}\in\text{ß}\subseteq\mathfrak{f}^{-1}(\mathfrak{f}(\text{ß}))\subseteq\mathfrak{f}^{-1}(\varGamma)$. This implies $x_{r,s}\in\mathfrak{f}^{-1}(\varGamma)$ which shows that $\mathfrak{f}^{-1}(\varGamma)\in N^{\mathbb{I}.\mathbb{F}G_\delta\text{-}e\text{-}lq}(x_{r,s})$. Hence $\mathfrak{f}^{-1}(\varGamma)$ is an $\mathbb{I}.\mathbb{F}\ G_\delta\text{-}e\text{-}\mathbb{LOS}$ in an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$ $(\Im,\ \top)$.

(ii)⇒ (i): This can be proved by taking complement of (i)

(iii)⇒ (iv): Let $\varGamma$ be an $\mathbb{I}.\mathbb{F}_{\mathbb{CS}}$ in an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$ $(\wp,\ S)$. Since $\varGamma\subseteq\mathbb{I}.\mathbb{F}_{cl}(\varGamma)$, $\mathfrak{f}^{-1}(\varGamma)\supseteq\mathfrak{f}^{-1}(\mathbb{I}.\mathbb{F}_{int}(\varGamma))$. By (iii), $\mathfrak{f}^{-1}(\mathbb{I}.\mathbb{F}_{int}(\varGamma))$ is an $\mathbb{I}.\mathbb{F}\ G_\delta\text{-}e\text{-}\mathbb{LOS}$ in an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$ $(\Im,\ \top)$. Thus, $\mathbb{I}.\mathbb{F}G_\delta\text{-}e\text{-}l_{int}(\mathfrak{f}^{-1}(\varGamma))\supseteq\mathfrak{f}^{-1}(\mathbb{I}.\mathbb{F}_{int}(\varGamma))$.

(iv)⇒(v): Using (iv), $\mathbb{I}.\mathbb{F}G_\delta\text{-}e\text{-}l_{int}(\mathfrak{f}^{-1}(\varGamma))\supseteq\mathfrak{f}^{-1}(\mathbb{I}.\mathbb{F}_{int}(\varGamma))$.Then $\overline{\mathbb{I}.\mathbb{F}G_\delta\text{-}e\text{-}l_{int}(\mathfrak{f}^{-1}(\varGamma))}\subseteq\overline{\mathfrak{f}^{-1}(\mathbb{I}.\mathbb{F}_{int}(\varGamma))}$, $\mathbb{I}.\mathbb{F}G_\delta\text{-}l_{cl}\overline{(\mathfrak{f}^{-1}(\varGamma))}\subseteq\mathfrak{f}^{-1}(\mathbb{I}.\mathbb{F}_{cl}(\overline{\varGamma}))$, $\mathbb{I}.\mathbb{F}G_\delta\text{-}\ e\text{-}l_{cl}(\mathfrak{f}^{-1}(\overline{\varGamma}))\subseteq\mathfrak{f}^{-1}(\mathbb{I}.\mathbb{F}_{cl}(\overline{\varGamma}))$ implies that $\mathfrak{f}^{-1}(\mathbb{I}.\mathbb{F}_{cl}(\overline{\varGamma}))\supseteq\mathbb{I}.\mathbb{F}G_\delta\text{-}e\text{-}\ l_{cl}(\overline{\mathfrak{f}^{-1}(\varGamma)})$, putting $\overline{\varGamma}=\varGamma$, we have $\mathfrak{f}^{-1}(\mathbb{I}.\mathbb{F}_{cl}(\overline{\varGamma}))\supseteq\mathbb{I}.\mathbb{F}G_\delta\text{-}e\text{-}l_{cl}(\mathfrak{f}^{-1}(\varGamma))$.

(v)⇒ (i): Let $\varGamma$ be an $\mathbb{I}.\mathbb{F}_{\mathbb{CS}}$ in an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$ $(\wp,\ S)$. Then $\mathbb{I}.\mathbb{F}_{cl}\varGamma=\varGamma$. Using (v), $\mathfrak{f}^{-1}(\mathbb{I}.\mathbb{F}_{int}(\varGamma))\subseteq\mathbb{I}.\mathbb{F}G_\delta\text{-}e\text{-}l_{int}(\mathfrak{f}^{-1}(\varGamma))$ implies that $\mathfrak{f}^{-1}(\varGamma)\subseteq\mathbb{I}.\mathbb{F}G_\delta\text{-}e\text{-}l_{int}(\mathfrak{f}^{-1}(\varGamma))$. But, $\mathbb{I}.\mathbb{F}G_\delta\text{-}e\text{-}l_{int}(\mathfrak{f}^{-1}(\varGamma))\subseteq\mathfrak{f}^{-1}(\varGamma)$ implies that $\mathfrak{f}^{-1}(\varGamma)=\mathbb{I}.\mathbb{F}G_\delta\text{-}e\text{-}l_{int}(\mathfrak{f}^{-1}(\varGamma))$ that is, $\mathfrak{f}^{-1}(\varGamma)$ is an $\mathbb{I}.\mathbb{F}\ G_\delta\text{-}e\text{-}\mathbb{LOS}$ in an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$ $(\Im,\ \top)$. Let $x_{r,s}$ be any $\mathbb{I}.\mathbb{F}.\mathbb{P}$ in $\mathfrak{f}^{-1}(\varGamma)$. Then $x_{r,s}\in\mathfrak{f}^{-1}(\varGamma)$. We have $x_{r,s}qf^{-1}(\varGamma)$ implies that $\mathfrak{f}(x_{r,s})qf(\mathfrak{f}^{-1}(\varGamma))$. But $\mathfrak{f}(\mathfrak{f}^{-1}(\varGamma))\subseteq\varGamma$. Thus, for any $\mathbb{I}.\mathbb{F}.\mathbb{P}$ $x_{r,s}$ and $\varGamma\in N\mathfrak{f}(x_{r,s})$, there exists $\text{ß}=\mathfrak{f}^{-1}(\varGamma)\in N^{\mathbb{I}.\mathbb{F}G_\delta\text{-}e\text{-}lq}(x_{r,s})$ such that $\mathfrak{f}^{-1}(\mathfrak{f}(\varGamma))\subseteq\varGamma$. Therefore, $\mathfrak{f}(\text{ß})\subseteq\varGamma$. Thus, $\mathfrak{f}$ is an $\mathbb{I}.\mathbb{F}Cont.G_\delta\text{-}e\text{-}l.cts.fun$. □

**Theorem 3.2.** Let $(\Im,\ \top)$ and $(\wp,\ S)$ be any two $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$'s. Let $\mathfrak{f}:(\Im,\ \top)\to(\wp,\ S)$ be an $\mathbb{I}.\mathbb{F}$ bijective function. Then $\mathfrak{f}$ is an $\mathbb{I}.\mathbb{F}Cont.G_\delta\text{-}e\text{-}l.cts.fun$ if and only if $\mathbb{I}.\mathbb{F}_{int}(\mathfrak{f}(\varGamma))\subseteq\mathfrak{f}(\mathbb{I}.\mathbb{F}G_\delta\text{-}e\text{-}l_{int}(\varGamma))$, for each $\mathbb{I}.\mathbb{F}_{\mathbb{CS}}$ $\varGamma$ of an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$ $(\Im,\ \top)$.

*Proof.* Assume that $\mathfrak{f}$ is an $\mathbb{I}.\mathbb{F}Cont.G_\delta\text{-}e\text{-}l.cts.fun$ and $\varGamma$ be an $\mathbb{I}.\mathbb{F}_{\mathbb{CS}}$ in an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$ $(\Im,\ \top)$. Hence, $\mathfrak{f}^{-1}(\mathbb{I}.\mathbb{F}_{int}(\mathfrak{f}(\varGamma)))$ is an $\mathbb{I}.\mathbb{F}\ G_\delta\text{-}e\text{-}\mathbb{LOS}$ in an $\mathbb{I}.\mathbb{F}TS$ $(\Im,\ \top)$. From Theorem (v) of (3.1) $\mathfrak{f}^{-1}(\mathbb{I}.\mathbb{F}_{int}\mathfrak{f}(\varGamma))\subseteq\mathbb{I}.\mathbb{F}G_\delta\text{-}e\text{-}l_{int}(\mathfrak{f}^{-1}(\mathfrak{f}(\varGamma)))$, $\mathfrak{f}^{-1}(\mathbb{I}.\mathbb{F}_{int}\mathfrak{f}(\varGamma))\subseteq\mathbb{I}.\mathbb{F}G_\delta\text{-}e\text{-}l_{int}(\varGamma)$. Since $\mathfrak{f}$ is an $\mathbb{I}.\mathbb{F}$ surjective function, $\mathfrak{f}(\mathfrak{f}^{-1}(\mathbb{I}.\mathbb{F}intf(\varGamma)))\subseteq\mathfrak{f}(\mathbb{I}.\mathbb{F}G_\delta\text{-}e\text{-}l_{int}(\varGamma))$. That is, $\mathbb{I}.\mathbb{F}_{int}(\mathfrak{f}(\varGamma))\subseteq\mathfrak{f}(\mathbb{I}.\mathbb{F}G_\delta\text{-}e\text{-}l_{int}(\varGamma))$.

Conversely, assume that $\mathbb{I}.\mathbb{F}_{int}(\mathfrak{f}(\varGamma))\subseteq\mathfrak{f}(\mathbb{I}.\mathbb{F}G_\delta\text{-}e\text{-}l_{int}(\varGamma))$, for each $\mathbb{I}.\mathbb{F}.\mathbb{S}$ $\varGamma$ of an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$ $(\Im,\ \top)$. Let ß be an $\mathbb{I}.\mathbb{F}_{\mathbb{CS}}$ in an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$ $(\wp,\ S)$. Then $\text{ß}=\mathbb{I}.\mathbb{F}_{cl}(\text{ß})$. Since $\mathfrak{f}$ is an $\mathbb{I}.\mathbb{F}$ surjective function, $\text{ß}=\mathbb{I}.\mathbb{F}_{cl}(\text{ß})=\mathbb{I}.\mathbb{F}_{int}(\mathfrak{f}(\mathfrak{f}^{-1}(\text{ß})))\subseteq\mathfrak{f}(\mathbb{I}.\mathbb{F}G_\delta\text{-}e\text{-}l_{int}(\mathfrak{f}^{-1}(\text{ß})))$. Since $\mathfrak{f}$ is an $\mathbb{I}.\mathbb{F}$ injective function, $\mathfrak{f}^{-1}(\text{ß})\subseteq\mathfrak{f}^{-1}(\mathfrak{f}(\mathbb{I}.\mathbb{F}G_\delta\text{-}e\text{-}l_{int}(\mathfrak{f}^{-1}(\text{ß}))))$. From the fact that $\mathfrak{f}$, is an $\mathbb{I}.\mathbb{F}$ injective function, we have

$$\mathfrak{f}^{-1}(\text{ß})\subseteq\mathbb{I}.\mathbb{F}G_\delta\text{-}e\text{-}l_{int}(\mathfrak{f}^{-1}(\text{ß})) \tag{3}$$

but

$$\mathbb{I}.\mathbb{F}G_\delta\text{-}e\text{-}l_{int}(\mathfrak{f}^{-1}(ß)) \subseteq \mathfrak{f}^{-1}(ß) \tag{4}$$

From (3) and (4) implies that $\mathfrak{f}^{-1}(ß) = \mathbb{I}.\mathbb{F}G_\delta\text{-}e\text{-}l_{int}(\mathfrak{f}^{-1}(ß))$. That is, $\mathfrak{f}^{-1}(ß)$ is an $\mathbb{I}.\mathbb{F}$ $G_\delta$-$e$-$\mathbb{L}\mathbb{O}\mathbb{S}$ in an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$ $(\Im,\ \top)$. Thus, $\mathfrak{f}$ is an $\mathbb{I}.\mathbb{F}Cont.G_\delta\text{-}e\text{-}l.cts.fun$. □

**Theorem 3.3.** Let $(\Im,\ \top)$ and $(\wp,\ S)$ be any two $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$'s. Let $\mathfrak{f} : (\Im,\ \top) \to (\wp,\ S)$ be an $\mathbb{I}.\mathbb{F}$ bijective fuction. Then $\mathfrak{f}$ is an $\mathbb{I}.\mathbb{F}Cont.G_\delta\text{-}e\text{-}l.cts.fun$ if and only if $\mathfrak{f}(\mathbb{I}.\mathbb{F}G_\delta\text{-}e\text{-}l_{cl}(\Gamma)) \subseteq \mathbb{I}.\mathbb{F}_{cl}(\mathfrak{f}(\Gamma))$, for each $\mathbb{I}.\mathbb{F}_{\mathbb{O}\mathbb{S}}$ $\Gamma$ of an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$ $(\Im,\ \top)$.

*Proof.* Assume that $\mathfrak{f}$ is an $\mathbb{I}.\mathbb{F}Cont.G_\delta\text{-}e\text{-}l.cts.fun$ and $\Gamma$ be an $\mathbb{I}.\mathbb{F}_{\mathbb{O}\mathbb{S}}$ in an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$ $(\Im,\ \top)$. Hence, $\mathfrak{f}^{-1}(\mathbb{I}.\mathbb{F}_{cl}(\mathfrak{f}(\Gamma)))$ is an $\mathbb{I}.\mathbb{F}$ $G_\delta$-$e$-$\mathbb{L}\mathbb{C}\mathbb{S}$ in an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$ $(\Im,\ \top)$. From Theorem (v) of (3.1) $\mathfrak{f}^{-1}(\mathbb{I}.\mathbb{F}_{cl}\mathfrak{f}(\Gamma)) \supseteq \mathbb{I}.\mathbb{F}G_\delta\text{-}e\text{-}l_{cl}(\mathfrak{f}^{-1}(\mathfrak{f}(\Gamma)))$, $\mathfrak{f}^{-1}(\mathbb{I}.\mathbb{F}_{cl}\mathfrak{f}(\Gamma)) \supseteq \mathbb{I}.\mathbb{F}G_\delta\text{-}e\text{-}l_{cl}(\Gamma)$. Since $\mathfrak{f}$ is an $\mathbb{I}.\mathbb{F}$ surjective function, $\mathfrak{f}(\mathfrak{f}^{-1}(\mathbb{I}.\mathbb{F}clf(\Gamma))) \supseteq \mathfrak{f}(\mathbb{I}.\mathbb{F}G_\delta\text{-}e\text{-}l_{cl}(\Gamma))$. That is, $\mathbb{I}.\mathbb{F}_{cl}(\mathfrak{f}(\Gamma)) \supseteq \mathfrak{f}(\mathbb{I}.\mathbb{F}G_\delta\text{-}e\text{-}l_{cl}(\Gamma))$.

Conversely, assume that $\mathbb{I}.\mathbb{F}_{cl}(\mathfrak{f}(\Gamma)) \supseteq \mathfrak{f}(\mathbb{I}.\mathbb{F}G_\delta\text{-}e\text{-}l_{cl}(\Gamma))$, for each $\mathbb{I}.\mathbb{F}.\mathbb{S}$ $\Gamma$ of an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$ $(\Im,\ \top)$. Let ß be an $\mathbb{I}.\mathbb{F}_{\mathbb{O}\mathbb{S}}$ in an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$ $(\wp,\ S)$. Then $ß = \mathbb{I}.\mathbb{F}_{int}(ß)$. Since $\mathfrak{f}$ is an $\mathbb{I}.\mathbb{F}$ surjective function, $ß = \mathbb{I}.\mathbb{F}_{int}(ß) = \mathbb{I}.\mathbb{F}_{cl}(\mathfrak{f}(\mathfrak{f}^{-1}(ß))) \supseteq \mathfrak{f}(\mathbb{I}.\mathbb{F}G_\delta\text{-}e\text{-}l_{cl}(\mathfrak{f}^{-1}(ß)))$. Since $\mathfrak{f}$ is an $\mathbb{I}.\mathbb{F}$ injective function, $\mathfrak{f}^{-1}(ß) \supseteq \mathfrak{f}^{-1}(\mathfrak{f}(\mathbb{I}.\mathbb{F}G_\delta\text{-}e\text{-}l_{cl}(\mathfrak{f}^{-1}(ß))))$. From the fact that $\mathfrak{f}$, is an $\mathbb{I}.\mathbb{F}$ injective function, we have

$$\mathfrak{f}^{-1}(ß) \supseteq \mathbb{I}.\mathbb{F}G_\delta\text{-}e\text{-}l_{cl}(\mathfrak{f}^{-1}(ß)) \tag{5}$$

but

$$\mathbb{I}.\mathbb{F}G_\delta\text{-}e\text{-}l_{cl}(\mathfrak{f}^{-1}(ß)) \supseteq \mathfrak{f}^{-1}(ß) \tag{6}$$

From (5) and (6) implies that $\mathfrak{f}^{-1}(ß) = \mathbb{I}.\mathbb{F}G_\delta\text{-}e\text{-}l_{cl}(\mathfrak{f}^{-1}(ß))$. That is, $\mathfrak{f}^{-1}(ß)$ is an $\mathbb{I}.\mathbb{F}$ $G_\delta$-$e$-$\mathbb{L}\mathbb{C}\mathbb{S}$ in an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$ $(\Im,\ \top)$. Thus, $\mathfrak{f}$ is an $\mathbb{I}.\mathbb{F}Cont.G_\delta\text{-}e\text{-}l.cts.fun$. □

**Theorem 3.4.** Let $(\Im,\ \top)$ and $(\wp,\ S)$ be any two $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$'s. Let $\mathfrak{f} : (\Im,\ \top) \to (\wp,\ S)$ be an $\mathbb{I}.\mathbb{F}$ bijective fuction. If $\mathfrak{f}$ is an $\mathbb{I}.\mathbb{F}Cont.G_\delta\text{-}e\text{-}l.cts.fun$. Then if $\Gamma \in I^\wp$ is an $\mathbb{I}.\mathbb{F}_{\mathbb{O}\mathbb{S}}$, then $\mathfrak{f}^{-1}(\Gamma) = \mathbb{I}.\mathbb{F}G_\delta\text{-}e\text{-}l_{cl}(\mathfrak{f}^{-1}(\Gamma))$.

*Proof.* Let $\Gamma$ be an $\mathbb{I}.\mathbb{F}_{\mathbb{O}\mathbb{S}}$ in an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$ $(\wp,\ S)$. By Theorem(iv)of (3.1).

$$\mathbb{I}.\mathbb{F}G_\delta\text{-}e\text{-}l_{cl}(\mathfrak{f}^{-1}(\Gamma)) \subseteq \mathfrak{f}^{-1}(\mathbb{I}.\mathbb{F}_{cl}(\Gamma)) = \mathfrak{f}^{-1}(\Gamma) \tag{7}$$

Since $\Gamma = \mathbb{I}.\mathbb{F}_{int}(\Gamma)$. But

$$\mathfrak{f}^{-1}(\Gamma) \subseteq \mathbb{I}.\mathbb{F}G_\delta\text{-}e\text{-}l_{cl}(\mathfrak{f}^{-1}(\Gamma)) \tag{8}$$

From (7) and (8) implies that $\mathfrak{f}^{-1}(\Gamma) = \mathbb{I}.\mathbb{F}G_\delta\text{-}e\text{-}l_{cl}(\mathfrak{f}^{-1}(\Gamma))$. □

**Proposition 3.1.** Let $(\Im,\ \top)$, $(\wp,\ S)$ and $(\aleph,\ R)$ be any three $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$'s. Let $\mathfrak{f} : (\Im.\ \top) \to (\wp,\ S)$ be an $\mathbb{I}.\mathbb{F}Cont.G_\delta\text{-}e\text{-}l.cts.fun$. If $\mathfrak{f}(\Im) \subset \aleph \subset \wp$ then $g : (\Im,\ \top) \to (\aleph,\ R)$ where $R = S/\aleph$ restricting the range of $\mathfrak{f}$ is an $\mathbb{I}.\mathbb{F}Cont.G_\delta\text{-}e\text{-}l.cts.fun$.

*Proof.* Let ß be an $\mathbb{I}.\mathbb{F}_{\mathbb{O}\mathbb{S}}$ in an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$ $(\aleph,\ R)$. Then $ß = S/\aleph$, for some $\mathbb{I}.\mathbb{F}_{\mathbb{O}\mathbb{S}}$ $\Gamma$ of an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$'s $(\wp,\ S)$. If $\mathfrak{f}(\Im) \subset \aleph \subset \wp$, $\mathfrak{f}^{-1}(\Gamma) = g^{-1}(ß)$. Since $\mathfrak{f}^{-1}(\Gamma)$ is an $\mathbb{I}.\mathbb{F}$ $G_\delta$-$e$-$\mathbb{L}\mathbb{C}\mathbb{S}$ in an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$ $(\Im,\ \top)$. Hence, $g^{-1}(ß)$ is an $\mathbb{I}.\mathbb{F}$ $G_\delta$-$e$-$LCS$ in an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$ $(\Im,\ \top)$. Therefore, $g$ is an $\mathbb{I}.\mathbb{F}Cont.G_\delta\text{-}e\text{-}l.cts.fun$. □

**Proposition 3.2.** Let $(\Im,\ \top)$, $(X_1,\ T_1)$ and $(X_2,\ T_2)$ be any three $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$'s and $P_i : X_1 \times X_2 \to X_i$ be an $\mathbb{I}.\mathbb{F}$ projection of $X_1 \times X_2$ onto $X_i$. If $\mathfrak{f} : \Im \to X_1 \times X_2$ is an $\mathbb{I}.\mathbb{F}Cont.G_\delta\text{-}e\text{-}l.cts.fun$. Then $P_i \circ \mathfrak{f} : \Im \to X_i$ is also an $\mathbb{I}.\mathbb{F}Cont.G_\delta\text{-}e\text{-}l.cts.fun$.

*Proof.* Let $\varGamma$ be an $\mathbb{I}.\mathbb{F}_{\mathbb{O}\mathbb{S}}$ in an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$'s $(X_i,\ T_i)$ $(i = 1,\ 2)$, $(P_i \circ \mathfrak{f})^{-1}(\varGamma) = \mathfrak{f}^{-1}(P_i^{-1}(\varGamma))$. Since $P_i$ is an $\mathbb{IFM}$ $P_i^{-1}(\varGamma)$ is an $\mathbb{I}.\mathbb{F}_{\mathbb{C}\mathbb{S}}$ in an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$'s $X_1 \times X_2$. Hence, $\mathfrak{f}^{-1}(P_i^{-1}(\varGamma))$ is an $\mathbb{I}.\mathbb{F}$ $G_\delta$-$e$-$\mathbb{LCS}$ in an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$ $(\Im,\ \top)$. Hence, $P_i \circ \mathfrak{f}$ is an $\mathbb{I}.\mathbb{F}Cont.G_\delta$-$e$-$l.cts.fun$. □

**Definition 3.2.** Let $(\Im,\ \top)$ and $(\wp,\ S)$ be two $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$'s. Let $\mathfrak{f} : (\Im,\ \top) \to (\wp,\ S)$ be an $\mathbb{IFM}$. Then $\mathfrak{f}$ is said to be an

(i) $\mathbb{I}.\mathbb{F}$ contra $G_\delta$-$e$-locally irresolute function($\mathbb{I}.\mathbb{F}Cont.G_\delta$-$e$-$l.irr.fun$), if for each $\mathbb{I}.\mathbb{F}$ $G_\delta$-$e$-$\mathbb{LCS}$ $\varGamma$ in an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$ $(\wp,\ S)$, $\mathfrak{f}^{-1}(\varGamma)$ is an $\mathbb{I}.\mathbb{F}$ $G_\delta$-$e$-$\mathbb{LOS}$ in an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$ $(\Im,\ \top)$.
(ii) $\mathbb{I}.\mathbb{F}$ contra weakly $G_\delta$-$e$-locally irresolute function($\mathbb{I}.\mathbb{F}Cont.WG_\delta$-$e$-$l.irr.fun$), if for each $\mathbb{I}.\mathbb{F}$ $G_\delta$-$e$-$\mathbb{LCS}$ $\varGamma$ in an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$ $(\wp,\ S)$, $\mathfrak{f}^{-1}(\varGamma)$ is an $\mathbb{I}.\mathbb{F}_{\mathbb{O}\mathbb{S}}$ in an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$ $(\Im,\ \top)$.

**Example 3.1.** Let $\Im = \{a, b\} = \wp$, and $\varGamma = \langle \varkappa, (\frac{a}{0.3}, \frac{b}{0.3}), (\frac{a}{0.6}, \frac{b}{0.6}) \rangle$, $\beta = \langle \varkappa, (\frac{a}{0.3}, \frac{b}{0.1}), (\frac{a}{0.5}, \frac{b}{0.4}) \rangle$, $\varGamma \vee \beta = \langle \varkappa, (\frac{a}{0.3}, \frac{b}{0.3}), (\frac{a}{0.5}, \frac{b}{0.4}) \rangle$, $\varGamma \wedge \beta = \langle \varkappa, (\frac{a}{0.3}, \frac{b}{0.1}), (\frac{a}{0.6}, \frac{b}{0.6}) \rangle$, $C = \langle \varkappa, (\frac{a}{0.7}, \frac{b}{0.7}), (\frac{a}{0}, \frac{b}{0.1}) \rangle$ Now, the family $\top = \{0_\sim, 1_\sim, \varGamma, \beta,\ \varGamma \vee \beta,\ \varGamma \wedge \beta\}$ of $\mathbb{I}.\mathbb{F}.\mathbb{S}$'s in $\Im$ is an $\mathbb{I}.\mathbb{F}.\mathbb{T}$ on $\Im$ and the family $S = \{0_\sim, 1_\sim, C\}$ of $\mathbb{I}.\mathbb{F}.\mathbb{S}$'s in $\wp$ is an $\mathbb{I}.\mathbb{F}.\mathbb{T}$ on $\wp$. If we define the function $\mathfrak{f} : \Im \to \wp$ be the identity function. Now, $\mathfrak{f}$ is an $\mathbb{I}.\mathbb{F}Cont.G_\delta$-$e$-$l.irr.fun$ because $C$ is an $\mathbb{I}.\mathbb{F}G_\delta$-$e$-$\mathbb{LOS}$ in $\wp$, $\mathfrak{f}^{-1}(C)$ is also an $\mathbb{I}.\mathbb{F}G_\delta$-$e$-$\mathbb{LCS}$ in $\Im$.

**Example 3.2.** Let $\Im = \{a, b\} = \wp$, and $\varGamma = \langle \varkappa, (\frac{a}{0.3}, \frac{b}{0.3}), (\frac{a}{0.6}, \frac{b}{0.6}) \rangle$, $\beta = \langle \varkappa, (\frac{a}{0.3}, \frac{b}{0.1}), (\frac{a}{0.5}, \frac{b}{0.4}) \rangle$, $\varGamma \vee \beta = \langle \varkappa, (\frac{a}{0.3}, \frac{b}{0.3}), (\frac{a}{0.5}, \frac{b}{0.4}) \rangle$, $\varGamma \wedge \beta = \langle \varkappa, (\frac{a}{0.3}, \frac{b}{0.1}), (\frac{a}{0.6}, \frac{b}{0.6}) \rangle$, $C = \langle \varkappa, (\frac{a}{0}, \frac{b}{0.1}), (\frac{a}{0.7}, \frac{b}{0.7}) \rangle$ Now, the family $\top = \{0_\sim, 1_\sim, \varGamma, \beta,\ \varGamma \vee \beta,\ \varGamma \wedge \beta,\ C\}$ of $\mathbb{I}.\mathbb{F}.\mathbb{S}$'s in $\Im$ is an $\mathbb{I}.\mathbb{F}.\mathbb{T}$ on $\Im$ and the family $S = \{0_\sim, 1_\sim, C\}$ of $\mathbb{I}.\mathbb{F}.\mathbb{S}$'s in $\wp$ is an $\mathbb{I}.\mathbb{F}.\mathbb{T}$ on $\wp$. If we define the function $\mathfrak{f} : \Im \to \wp$ be the identity function, Now, $\mathfrak{f}$ is an $\mathbb{I}.\mathbb{F}Cont.WG_\delta$-$e$-$l.irr.fun$, because $C$ is an $\mathbb{I}.\mathbb{F}G_\delta$-$e$-$\mathbb{LCS}$ in $\wp$, $\mathfrak{f}^{-1}(C)$ is $\mathbb{I}.\mathbb{F}_{\mathbb{O}\mathbb{S}}$ in $\Im$.

**Theorem 3.5.** Let $(\Im,\ \top)$ and $(\wp,\ S)$ be any two $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$'s. Let $\mathfrak{f} : (\Im,\ \top) \to (\wp,\ S)$ be an $\mathbb{IFM}$ . Then the following statements are equivalent

(i) $\mathfrak{f}$ is an $\mathbb{I}.\mathbb{F}Cont.G_\delta$-$e$-$l.irr.fun$
(ii) for every $\mathbb{I}.\mathbb{F}.\mathbb{S}$ $\varGamma$ of an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$ $(\Im,\ \top)$, $\mathfrak{f}(\mathbb{I}.\mathbb{F}G_\delta\text{-}e\text{-}l_{int}(\varGamma)) \supseteq \mathbb{I}.\mathbb{F}G_\delta\text{-}e\text{-}l_{int}(\mathfrak{f}(\varGamma))$.
(iii) for every $\mathbb{I}.\mathbb{F}.\mathbb{S}$ $\varGamma$ of an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$ $(\wp,\ S)$, $\mathbb{I}.\mathbb{F}G_\delta\text{-}e\text{-}l_{int}(\mathfrak{f}^{-1}(\varGamma)) \supseteq \mathfrak{f}^{-1}(\mathbb{I}.\mathbb{F}G_\delta\text{-}e\text{-}l_{int}(\varGamma))$.

*Proof.* (i)⇒ (ii): Let $\varGamma$ be an $\mathbb{I}.\mathbb{F}.\mathbb{S}$ in an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$ $(\Im,\ \top)$. Suppose $\mathfrak{f}$ is an $\mathbb{I}.\mathbb{F}Cont.G_\delta$-$e$-$l.irr.fun$. Now, $\mathbb{I}.\mathbb{F}G_\delta\text{-}e\text{-}l_{cl}(\mathfrak{f}(\varGamma))$ is an $\mathbb{I}.\mathbb{F}$ $G_\delta$-$e$-$\mathbb{LCS}$ in an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$ $(\wp,\ S)$. By hypothesis, $\mathfrak{f}^{-1}(\mathbb{I}.\mathbb{F}G_\delta\text{-}e\text{-}l_{int}(\mathfrak{f}(\varGamma)))$ is an $\mathbb{I}.\mathbb{F}$ $G_\delta$-$e$-$\mathbb{LOS}$ in an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$ $(\Im,\ \top)$ and hence, $\varGamma \supseteq \mathfrak{f}^{-1}(\mathfrak{f}(\varGamma)) \supseteq \mathfrak{f}^{-1}(\mathbb{I}.\mathbb{F}G_\delta\text{-}e\text{-}(l_{int}(\mathfrak{f}(\varGamma))))$. Now, $\mathbb{I}.\mathbb{F}G_\delta\text{-}e\text{-}l_{int}(\varGamma) \supseteq \mathfrak{f}^{-1}(\mathbb{I}.\mathbb{F}G_\delta\text{-}e\text{-}l_{int}(\mathfrak{f}(\varGamma)))$. That is, $\mathfrak{f}(\mathbb{I}.\mathbb{F}G_\delta\text{-}e\text{-}l_{int}(\varGamma)) \supseteq \mathbb{I}.\mathbb{F}G_\delta\text{-}e\text{-}l_{int}(\mathfrak{f}(\varGamma))$.

(ii)⇒ (iii): Let $\varGamma$ be an $\mathbb{I}.\mathbb{F}.\mathbb{S}$ in an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$ $(\wp,\ S)$, then $\mathfrak{f}^{-1}(\varGamma)$ is an $\mathbb{I}.\mathbb{F}.\mathbb{S}$ in an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$ $(\Im,\ \top)$. By (ii), $\mathfrak{f}(\mathbb{I}.\mathbb{F}G_\delta\text{-}e\text{-}l_{int}(\mathfrak{f}^{-1}(\varGamma))) \supseteq \mathbb{I}.\mathbb{F}G_\delta\text{-}e\text{-}l_{int}(\mathfrak{f}(\mathfrak{f}^{-1}(\varGamma)))$. Since $\mathfrak{f}$ is an $\mathbb{I}.\mathbb{F}$ bijective function, $\mathbb{I}.\mathbb{F}G_\delta\text{-}e\text{-}l_{int}(\mathfrak{f}^{-1}(\varGamma)) \supseteq \mathfrak{f}^{-1}(\mathbb{I}.\mathbb{F}G_\delta\text{-}e\text{-}l_{int}(\varGamma))$

(iii)⇒ (i): Suppose $\varGamma$ is $\mathbb{I}.\mathbb{F}G_\delta$-$e$-$\mathbb{LCS}$ in an $\mathbb{I}.\mathbb{F}.\mathbb{T}.\mathbb{S}$ $(\wp,\ S)$. Then $\mathbb{I}.\mathbb{F}G_\delta\text{-}e\text{-}l_{cl}(\varGamma) = \varGamma$. By hypothesis, $\mathbb{I}.\mathbb{F}G_\delta\text{-}e\text{-}l_{int}(\mathfrak{f}^{-1}(\varGamma)) \supseteq \mathfrak{f}^{-1}(\mathbb{I}.\mathbb{F}G_\delta\text{-}e\text{-}l_{int}(\varGamma))$, $\mathbb{I}.\mathbb{F}G_\delta\text{-}e\text{-}l_{int}(\mathfrak{f}^{-1}(\varGamma)) \supseteq \mathfrak{f}^{-1}(\varGamma)$. □

## 4. Conclusions

This paper introduced and developed the concepts of intuitionistic fuzzy contra $G_\delta$-$e$-locally continuous and irresolute functions within intuitionistic fuzzy topological spaces. Several significant properties of these functions have been identified, enhancing the understanding of their roles in fuzzy topology. These functions generalize traditional continuous and irresolute functions by effectively handling uncertainty through intuitionistic fuzzy sets. While offering a broader framework for analysis, future research could focus on addressing the computational complexity and practical implementation challenges and exploring new properties and types of intuitionistic fuzzy sets.

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