

ALGEBRAIC PROPERTIES OF KERNEL SYMMETRIC NEUTROSOPHIC FUZZY MATRICES

P. MURUGADAS ¹, T. SHYAMALADEVI ^{2,3,*}, M. ANANDHKUMAR ², §

ABSTRACT. We define secondary k-Kernel symmetric (KS) and provide numerical examples for neutrosophic fuzzy matrices (NFM). We discuss the relation between s-k- KS, s-KS, k- KS and KS NFM. We identify the necessary and sufficient conditions for a NFM to be a s-k- KS NFM. We demonstrate that k-symmetry implies k-KS and the converse is true. Also, we illustrate a graphical representation of KS adjacency and incidence NFM. Every adjacency NFM is symmetric, kernel symmetric but incidence matrix satisfies only kernel symmetric conditions. We establish the existence of multiple generalized inverses of NFM in F_n and establish the additional equivalent conditions for certain g-inverses of a s-k-KS NFM to be s-k-KS. Also, we characterize the generalized inverses belonging to the sets $\psi(1, 2)$, $\psi(1, 2, 3)$ and $\psi(1, 2, 4)$ of s-k- KS NFM ψ .

Keywords: Neutrosophic fuzzy matrices, s- Kernel symmetric, Adjacency Neutrosophic fuzzy matrices, Incidence Neutrosophic fuzzy matrices, Moore Penrose inverse.

AMS Subject Classification: 03E72, 15B15, 15B99.

1. INTRODUCTION

The study of fuzzy and neutrosophic fuzzy matrices has seen substantial advancements over the years, with contributions from various researchers exploring their theoretical foundations and applications. These matrices, which generalize classical and fuzzy matrix theories, offer a robust framework for handling uncertainty, vagueness, and imprecise information in mathematical models. This introduction provides an overview of significant works contributing to the development and understanding of fuzzy and neutrosophic fuzzy

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matrices. Anandhkumar et al. have made notable contributions in this field through multiple studies. Their works cover a broad spectrum of topics, including inverse properties of neutrosophic fuzzy matrices [1] reverse sharp and partial ordering operations [2] and interval-valued symmetric matrices [3]. They have also explored concepts such as pseudo-similarity [4], generalized symmetry [5], and k-idempotent properties [6], contributing to the structural understanding and applications of these matrices. Their recent advancements also include investigations into secondary k-column symmetry [7].

Earlier foundational studies laid the groundwork for these explorations. Lee [8] examined secondary symmetric matrices, introducing concepts pivotal for future developments. Dogra and Pal [9] have studied on Picture fuzzy matrix and its application. Hill and Waters [10] studied k-real and k-Hermitian matrices, while Jaya Shree [11,12] contributed to the theory of secondary k-kernel and k-range symmetric fuzzy matrices. These works have significantly influenced the subsequent research directions in neutrosophic fuzzy matrix theory. Recent developments in interval-valued and generalized matrices [13,14] have expanded the applicability of fuzzy matrix theory. The works of Kaliraja, Bhavani and Kim and Roush. Marimuthu and Chanthirababu [15] have studied On Schur Complement in k-Kernel Symmetric Neutrosophic and Intuitionistic Fuzzy Matrices.

Historical contributions by Meenakshi and collaborators [16–19] also play a vital role in the evolution of fuzzy matrix theory. Their research on secondary k-Hermitian matrices [16], s-k-EP matrices [17] and regular interval-valued fuzzy matrices [18] introduced essential concepts and methodologies that have been further explored and extended by contemporary researchers. Punithavalli [20] has discussed The Partial Orderings of m-Symmetric Fuzzy Matrices. Pal, M., Mondal [21–24] have studied Bipolar fuzzy matrices, Fuzzy matrices with fuzzy rows and columns, Interval-valued fuzzy matrices with interval-valued fuzzy rows and columns, Recent Developments of Fuzzy Matrix Theory and Applications. Shyamal and Pal [25] have focused on Interval valued Fuzzy matrices, Triangular Fuzzy Matrices, Two new operators on fuzzy matrices. The seminal work of Smarandache [21] on neutrosophic sets provided a generalization of intuitionistic fuzzy sets, which has been instrumental in the development of neutrosophic fuzzy matrices.

Together, these studies represent a cohesive and evolving body of knowledge in the domain of fuzzy and neutrosophic fuzzy matrices, addressing theoretical challenges and opening avenues for diverse applications. This comprehensive framework not only enhances mathematical understanding but also provides practical tools for decision-making and modeling under uncertainty. $\psi\{1\}$ denotes a regular fuzzy matrix and ψ is set of all g-inverses. A fuzzy matrix ψ is RS and KS is denoted by $R(\psi^T) = R(\psi)$ and $N(\psi^T) = N(\psi)$. It is identified that for complex matrices, the concepts of range and KS are equivalent. Additionally, this is not true for NFM. Only partial information taking into account truth membership and falsity-membership values can be handled by intuitionistic fuzzy sets. It does not deal with the ambiguous and contradictory data found in belief systems.

We introduce and extend the notion of RS NFM. In section 2, a kernel symmetric neutrosophic fuzzy matrix is described. In section 3, graphical representation of kernel symmetric adjacency and incidence NFM is given. In section 4, various generalized inverses of matrices in NFM are detailed. The comparable standards for various g-inverses of a s-k KS fuzzy matrix to be a s-k KS are established. The generalized inverses of a is $KS\psi$ corresponding to the sets $\psi\{1\}$, $\psi\{1, 2\}$, $\psi\{1, 2, 3\}$ and $\psi\{1, 2, 4\}$ are characterized.

1.1. Research gap. Meenakshi and Jayashri created the idea of range symmetric fuzzy matrices, while Meenakshi and Jayashri introduced the concept of k-KS matrices. In this study, we extend these ideas to Secondary k-KS NFM. This framework is essential to the structure of the hybrid real matrix and we apply it to fuzzy matrices, examining specific results in detail. Initially, we present alternative characterizations of Secondary k-KS NFM. Subsequently, we provide an example of a Secondary k-KS NFM. Also, we explore various g-inverses associated with regular matrices and establish a characterization of the set of all inverses using Secondary k-KS NFM.

1.2. Notations.

ψ^T = Transpose of the matrix ψ ,

ψ^\dagger = Moore-Penrose inverse of ψ ,

$R(\psi)$ = Row space of ψ ,

$N(\psi)$ = Null space of ψ ,

RS = Range symmetric ψ ,

KS = Kernel symmetric ψ ,

F_n = Square Neutrosophic Fuzzy Matrices.

2. KERNEL SYMMETRIC NEUTROSOPHIC FUZZY MATRICES

Definition 2.1. Let ψ be a NFM, if $N(\psi) = \{x\psi = (0, 0, 1) : x \in (NFM)_n\}$ then ψ is called as kernel or null space.

Definition 2.2. A NFM $\psi \in F_n$ is s-symmetric NFM $\Leftrightarrow \psi = V\psi^TV$.

Example 2.1. Consider a NFM

$$\psi = \begin{bmatrix} \langle 0.5, 0.3, 0.2 \rangle & \langle 0, 0, 1 \rangle & \langle 0.6, 0.4, 0.3 \rangle \\ \langle 0, 0, 1 \rangle & \langle 0, 0, 1 \rangle & \langle 0, 0, 1 \rangle \\ \langle 0.6, 0.4, 0.3 \rangle & \langle 0, 0, 1 \rangle & \langle 0.3, 0.2, 0.4 \rangle \end{bmatrix},$$

$$V = \begin{bmatrix} \langle 0, 0, 0 \rangle & \langle 0, 0, 0 \rangle & \langle 1, 1, 0 \rangle \\ \langle 0, 0, 0 \rangle & \langle 1, 1, 0 \rangle & \langle 0, 0, 0 \rangle \\ \langle 1, 1, 0 \rangle & \langle 0, 0, 0 \rangle & \langle 0, 0, 0 \rangle \end{bmatrix}.$$

Definition 2.3. An adjacency matrix is a square matrix that serves as a representation for a finite graph. A NFM $\psi \in F_n$ is s- KS symmetric NFM $\Leftrightarrow N(\psi) = N(V\psi^TV)$.

Example 2.2. Consider a NFM

$$\psi = \begin{bmatrix} \langle 0.6, 0.4, 0.5 \rangle & \langle 0, 0, 1 \rangle & \langle 0.4, 0.2, 0.1 \rangle \\ \langle 0, 0, 1 \rangle & \langle 0, 0, 1 \rangle & \langle 0, 0, 1 \rangle \\ \langle 0.4, 0.2, 0.1 \rangle & \langle 0, 0, 1 \rangle & \langle 0.7, 0.7, 0.3 \rangle \end{bmatrix},$$

$$V = \begin{bmatrix} \langle 0, 0, 0 \rangle & \langle 0, 0, 0 \rangle & \langle 1, 1, 0 \rangle \\ \langle 0, 0, 0 \rangle & \langle 1, 1, 0 \rangle & \langle 0, 0, 0 \rangle \\ \langle 1, 1, 0 \rangle & \langle 0, 0, 0 \rangle & \langle 0, 0, 0 \rangle \end{bmatrix}.$$

Definition 2.4. A NFM $\psi \in F_n$ is s-k-symmetric NFM $\Leftrightarrow N(\psi) = N(KV\psi^TVK)$.

Example 2.3. Consider a NFM

$$\psi = \begin{bmatrix} \langle 0.6, 0.3, 0.4 \rangle & \langle 0.7, 0.3, 0.4 \rangle \\ \langle 0.7, 0.3, 0.4 \rangle & \langle 0.7, 0.3, 0.5 \rangle \end{bmatrix}, \quad V = \begin{bmatrix} \langle 0, 0, 0 \rangle & \langle 1, 1, 0 \rangle \\ \langle 1, 1, 0 \rangle & \langle 0, 0, 0 \rangle \end{bmatrix}.$$

Definition 2.5. A NFM ψ is kernel symmetric NFM $\Leftrightarrow N(\psi) = N(\psi^T)$.

Definition 2.6. A NFM ψ is column symmetric NFM $\Leftrightarrow C(\psi) = C(\psi^T)$.

Definition 2.7. A NFM ψ is range symmetric NFM $\Leftrightarrow R(\psi) = R(\psi^T)$.

Definition 2.8. The row space of a NFM is the set of all possible linear combinations of its rows, where the operations are defined according to the type of fuzzy logic or algebra being used. The rows of the matrix are treated as fuzzy vectors, and their combinations form a subspace within the fuzzy vector space. Key Characteristics: 1. Fuzzy Row Vectors: Each row in the fuzzy matrix represents a fuzzy vector. 2. Fuzzy Linear Combination: The operations used to combine rows depend on the fuzzy algebra, such as addition \oplus and scalar multiplication \odot defined for fuzzy sets or membership degrees. 3. Dimensionality: The dimension of the row space corresponds to the rank of the fuzzy matrix, which is the maximum number of linearly independent fuzzy rows.

Example Consider a 3×3 NFM

Example 2.4. Consider a NFM

$$\psi = \begin{bmatrix} \langle 0.2, 0.4, 0.8 \rangle & \langle 0.7, 0.3, 0.4 \rangle & \langle 0.3, 0.5, 0.6 \rangle \\ \langle 0.3, 0.4, 0.8 \rangle & \langle 0.5, 0.2, 0.1 \rangle & \langle 0.2, 0.2, 0.1 \rangle \\ \langle 0.1, 0.4, 0.4 \rangle & \langle 0.6, 0.4, 0.4 \rangle & \langle 0.2, 0.3, 0.5 \rangle \end{bmatrix},$$

$$\begin{aligned} R_1 &= [\langle 0.2, 0.4, 0.8 \rangle \langle 0.7, 0.3, 0.4 \rangle \langle 0.3, 0.5, 0.6 \rangle] \\ R_2 &= [\langle 0.3, 0.4, 0.8 \rangle \langle 0.5, 0.2, 0.1 \rangle \langle 0.2, 0.2, 0.1 \rangle] \\ R_3 &= [\langle 0.1, 0.4, 0.4 \rangle \langle 0.6, 0.4, 0.4 \rangle \langle 0.2, 0.3, 0.5 \rangle] \end{aligned}$$

Steps to Determine the Row Space: 1. Identify the rows 2. Apply fuzzy row reduction (if needed) to determine linearly independent rows. Use fuzzy linear combinations (e.g., $\alpha \odot R_1 \oplus \beta \odot R_2$) to describe all vectors in the row space.

Proposition 2.1. Let the function be defined as $V(y) = (y_{k[1]}, y_{k[2]}, y_{k[3]}, \dots, y_{k[n]}) \in F_{n \times 1}$ for $y = (y_1, y_2, \dots, y_n) \in F_{[1 \times n]}$, where V is permutation matrix and satisfies the following conditions, $VV^T = V^TV = \psi$ In then $V^T = V$ and $N(\psi) = N(V\psi), N(\psi) = N(\psi)$.

Remark 2.1. Every s - k -symmetric NFM is s - k -KS NFM since $\psi = KV\psi^TVK$ if ψ is s - k -symmetric NFM. Thus, $N(\psi) = N(KV\psi^TVK)$, signifying that is a NFM with s - k -KS.

Example 2.5. Consider a NFM

$$V = \begin{bmatrix} \langle 0, 0, 0 \rangle & \langle 1, 1, 0 \rangle \\ \langle 1, 1, 0 \rangle & \langle 0, 0, 0 \rangle \end{bmatrix}, \psi = \begin{bmatrix} \langle 0.9, 0.3, 0.4 \rangle & \langle 0.2, 0.3, 0.4 \rangle \\ \langle 0.2, 0.3, 0.4 \rangle & \langle 0.7, 0.3, 0.5 \rangle \end{bmatrix},$$

$$K = \begin{bmatrix} \langle 1, 1, 0 \rangle & \langle 0, 0, 0 \rangle \\ \langle 0, 0, 0 \rangle & \langle 1, 1, 0 \rangle \end{bmatrix}.$$

$$\begin{aligned} KV\psi^TVK &= \begin{bmatrix} \langle 1, 1, 0 \rangle & \langle 0, 0, 0 \rangle \\ \langle 0, 0, 0 \rangle & \langle 1, 1, 0 \rangle \end{bmatrix} \begin{bmatrix} \langle 0, 0, 0 \rangle & \langle 1, 1, 0 \rangle \\ \langle 1, 1, 0 \rangle & \langle 0, 0, 0 \rangle \end{bmatrix} \\ &= \begin{bmatrix} \langle 0.9, 0.3, 0.4 \rangle & \langle 0.2, 0.3, 0.4 \rangle \\ \langle 0.2, 0.3, 0.4 \rangle & \langle 0.7, 0.3, 0.5 \rangle \end{bmatrix} \begin{bmatrix} \langle 0, 0, 0 \rangle & \langle 1, 1, 0 \rangle \\ \langle 1, 1, 0 \rangle & \langle 0, 0, 0 \rangle \end{bmatrix} \begin{bmatrix} \langle 1, 1, 0 \rangle & \langle 0, 0, 0 \rangle \\ \langle 0, 0, 0 \rangle & \langle 1, 1, 0 \rangle \end{bmatrix} \\ KV\psi^TVK &= \begin{bmatrix} \langle 0.9, 0.3, 0.4 \rangle & \langle 0.2, 0.3, 0.4 \rangle \\ \langle 0.2, 0.3, 0.4 \rangle & \langle 0.7, 0.3, 0.5 \rangle \end{bmatrix} = \psi. \end{aligned}$$

ψ is symmetric, s - k -symmetric which implies s - k -KS NFM.

Example 2.6. Consider a NFM

$$\begin{aligned} K &= \begin{bmatrix} \langle 0, 0, 0 \rangle & \langle 1, 1, 0 \rangle & \langle 0, 0, 0 \rangle \\ \langle 1, 1, 0 \rangle & \langle 0, 0, 0 \rangle & \langle 0, 0, 0 \rangle \\ \langle 0, 0, 0 \rangle & \langle 0, 0, 0 \rangle & \langle 1, 1, 0 \rangle \end{bmatrix}, V = \begin{bmatrix} \langle 0, 0, 0 \rangle & \langle 0, 0, 0 \rangle & \langle 1, 1, 0 \rangle \\ \langle 0, 0, 0 \rangle & \langle 1, 1, 0 \rangle & \langle 0, 0, 0 \rangle \\ \langle 1, 1, 0 \rangle & \langle 0, 0, 0 \rangle & \langle 0, 0, 0 \rangle \end{bmatrix} \\ \psi &= \begin{bmatrix} \langle 0, 0, 0 \rangle & \langle 0, 0, 0 \rangle & \langle 1, 1, 0 \rangle \\ \langle 0.5, 0.3, 0.4 \rangle & \langle 1, 1, 0 \rangle & \langle 0, 0, 0 \rangle \\ \langle 0.4, 0.2, 0.6 \rangle & \langle 0.5, 0.3, 0.4 \rangle & \langle 0, 0, 0 \rangle \end{bmatrix} \end{aligned}$$

$$\begin{aligned}
KV &= \begin{bmatrix} \langle 0, 0, 0 \rangle & \langle 1, 1, 0 \rangle & \langle 0, 0, 0 \rangle \\ \langle 1, 1, 0 \rangle & \langle 0, 0, 0 \rangle & \langle 0, 0, 0 \rangle \\ \langle 0, 0, 0 \rangle & \langle 0, 0, 0 \rangle & \langle 1, 1, 0 \rangle \end{bmatrix} \begin{bmatrix} \langle 0, 0, 0 \rangle & \langle 0, 0, 0 \rangle & \langle 1, 1, 0 \rangle \\ \langle 0, 0, 0 \rangle & \langle 1, 1, 0 \rangle & \langle 0, 0, 0 \rangle \\ \langle 1, 1, 0 \rangle & \langle 0, 0, 0 \rangle & \langle 0, 0, 0 \rangle \end{bmatrix} \\
KV &= \begin{bmatrix} \langle 0, 1, 0 \rangle & \langle 0, 1, 0 \rangle & \langle 0, 1, 0 \rangle \\ \langle 0, 1, 0 \rangle & \langle 0, 1, 0 \rangle & \langle 1, 1, 0 \rangle \\ \langle 1, 1, 0 \rangle & \langle 0, 1, 0 \rangle & \langle 0, 1, 0 \rangle \end{bmatrix} \\
VK &= \begin{bmatrix} \langle 0, 0, 0 \rangle & \langle 0, 0, 0 \rangle & \langle 1, 1, 0 \rangle \\ \langle 0, 0, 0 \rangle & \langle 1, 1, 0 \rangle & \langle 0, 0, 0 \rangle \\ \langle 1, 1, 0 \rangle & \langle 0, 0, 0 \rangle & \langle 0, 0, 0 \rangle \end{bmatrix} \begin{bmatrix} \langle 0, 0, 0 \rangle & \langle 1, 1, 0 \rangle & \langle 0, 0, 0 \rangle \\ \langle 1, 1, 0 \rangle & \langle 0, 0, 0 \rangle & \langle 0, 0, 0 \rangle \\ \langle 0, 0, 0 \rangle & \langle 0, 0, 0 \rangle & \langle 1, 1, 0 \rangle \end{bmatrix} \\
VK &= \begin{bmatrix} \langle 0, 1, 0 \rangle & \langle 0, 1, 0 \rangle & \langle 1, 1, 0 \rangle \\ \langle 1, 1, 0 \rangle & \langle 0, 1, 0 \rangle & \langle 0, 1, 0 \rangle \\ \langle 0, 1, 0 \rangle & \langle 1, 1, 0 \rangle & \langle 0, 1, 0 \rangle \end{bmatrix} \\
\psi^T VK &= \begin{bmatrix} \langle 0.5, 0.8, 0.4 \rangle & \langle 0.4, 0.8, 0.6 \rangle & \langle 0, 0, 0.4 \rangle \\ \langle 0, 0.7, 0 \rangle & \langle 0.5, 0.7, 0 \rangle & \langle 0, 0.7, 0 \rangle \\ \langle 0, 0, 0 \rangle & \langle 0, 0, 0 \rangle & \langle 1, 0, 0 \rangle \end{bmatrix} \\
KV\psi^T VK &= \begin{bmatrix} \langle 0, 1, 0 \rangle & \langle 0, 1, 0 \rangle & \langle 0, 1, 0 \rangle \\ \langle 0, 1, 0 \rangle & \langle 0, 1, 0 \rangle & \langle 1, 1, 0 \rangle \\ \langle 1, 1, 0 \rangle & \langle 0, 1, 0 \rangle & \langle 0, 1, 0 \rangle \end{bmatrix} \\
&\begin{bmatrix} \langle 0.5, 0.8, 0.4 \rangle & \langle 0.4, 0.8, 0.6 \rangle & \langle 0, 0, 0.4 \rangle \\ \langle 0, 0.7, 0 \rangle & \langle 0.5, 0.7, 0 \rangle & \langle 0, 0.7, 0 \rangle \\ \langle 0, 0, 0 \rangle & \langle 0, 0, 0 \rangle & \langle 1, 0, 0 \rangle \end{bmatrix} \\
&\begin{bmatrix} \langle 0, 1, 0 \rangle & \langle 0, 1, 0 \rangle & \langle 1, 1, 0 \rangle \\ \langle 1, 1, 0 \rangle & \langle 0, 1, 0 \rangle & \langle 0, 1, 0 \rangle \\ \langle 0, 1, 0 \rangle & \langle 1, 1, 0 \rangle & \langle 0, 1, 0 \rangle \end{bmatrix} \\
KV\psi^T VK &= \begin{bmatrix} \langle 0, 0, 0 \rangle & \langle 0, 0.2, 0 \rangle & \langle 0, 0, 0 \rangle \\ \langle 0, 0, 0 \rangle & \langle 0, 0, 0 \rangle & \langle 1, 0, 0 \rangle \\ \langle 0.5, 0, 0 \rangle & \langle 0.4, 0, 0 \rangle & \langle 0, 0, 0 \rangle \end{bmatrix} \neq \psi. \\
\psi &\neq KV\psi^T VK \text{ is not } s\text{-}k\text{-symmetric iff not is-} k\text{-KS}.
\end{aligned}$$

2.1. Graphical Representation of kernel symmetric Adjacency NFM.

Definition 2.9. Adjacency NFM : An adjacency Neutrosophic Fuzzy Matrix is a square matrix that serves as a representation for a finite graph. The matrix's elements convey information regarding whether pairs of vertices within the graph are connected or not. In the specific scenario of a finite simple graph, the adjacency matrix can be described as a binary matrix, often denoted as a $(1, 1, 0)$ and $(0, 0, 1)$ -matrix, where the diagonal elements are uniformly set to $(0, 0, 1)$. If $G(V, E)$ denote a simple graph with n vertices. The adjacency matrix $A = [a_{ij}]$ is a symmetric matrix defined

$$A = [a_{ij}] = \begin{cases} (1, 1, 0) & \text{when } v_i \text{ is adjacent to } v_j \\ (0, 0, 1) & \text{otherwise} \end{cases} \quad \text{denoted by } A(G) \text{ or } A_G.$$

Example 2.7. Consider an adjacency NFM and a corresponding graph

$$A = \begin{bmatrix} v1 & v3 & v4 & v2 & v5 \\ v1 & \langle 0, 0, 1 \rangle & \langle 1, 1, 0 \rangle & \langle 1, 1, 0 \rangle & \langle 0, 0, 1 \rangle & \langle 0, 0, 1 \rangle \\ v3 & \langle 1, 1, 0 \rangle & \langle 0, 0, 1 \rangle & \langle 0, 0, 1 \rangle & \langle 0, 0, 1 \rangle & \langle 1, 1, 1 \rangle \\ v4 & \langle 1, 1, 0 \rangle & \langle 0, 0, 1 \rangle & \langle 0, 0, 1 \rangle & \langle 1, 1, 0 \rangle & \langle 0, 0, 1 \rangle \\ v2 & \langle 0, 0, 1 \rangle & \langle 0, 0, 1 \rangle & \langle 1, 1, 0 \rangle & \langle 0, 0, 1 \rangle & \langle 1, 1, 0 \rangle \\ v5 & \langle 0, 0, 1 \rangle & \langle 1, 1, 0 \rangle & \langle 0, 0, 1 \rangle & \langle 1, 1, 0 \rangle & \langle 0, 0, 1 \rangle \end{bmatrix}$$

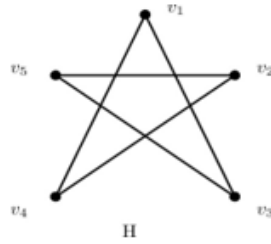


FIGURE 1

Definition 2.10. Incidence NFM: If $G(V, E)$ represent a simple graph with n vertices. Let $V = V_1, V_2, \dots, V_n$ and $E = e_1, e_2, \dots, e_m$. Then, the incidence NFM $I = [m_{ij}]$ is a matrix defined by

$$A = [a_j] = \begin{cases} (1, 1, 0) & \text{when } v \text{ is incidence to } e_j \\ (0, 0, 1) & \text{otherwise} \end{cases} \quad \text{denoted by } A(G) \text{ or } A_G.$$

Example 2.8. Consider an incidence NFM and a corresponding graph. The incidence NFM is

$$A = \begin{bmatrix} \langle 1, 1, 0 \rangle & \langle 1, 1, 0 \rangle & \langle 0, 0, 0 \rangle & \langle 0, 0, 1 \rangle & \langle 0, 0, 1 \rangle & \langle 0, 0, 1 \rangle & \langle 1, 1, 0 \rangle \\ \langle 1, 1, 0 \rangle & \langle 1, 1, 0 \rangle & \langle 0, 0, 1 \rangle & \langle 1, 1, 0 \rangle & \langle 0, 0, 1 \rangle & \langle 1, 1, 0 \rangle & \langle 0, 0, 1 \rangle \\ \langle 0, 0, 1 \rangle & \langle 1, 1, 0 \rangle & \langle 1, 1, 0 \rangle & \langle 1, 0, 1 \rangle & \langle 0, 0, 1 \rangle & \langle 0, 0, 1 \rangle & \langle 0, 0, 1 \rangle \\ \langle 0, 0, 1 \rangle & \langle 0, 0, 1 \rangle & \langle 1, 1, 0 \rangle & \langle 1, 1, 0 \rangle & \langle 1, 1, 0 \rangle & \langle 0, 0, 1 \rangle & \langle 0, 0, 1 \rangle \\ \langle 0, 0, 1 \rangle & \langle 0, 0, 1 \rangle & \langle 0, 0, 1 \rangle & \langle 0, 0, 1 \rangle & \langle 1, 1, 0 \rangle & \langle 1, 1, 0 \rangle & \langle 1, 1, 0 \rangle \end{bmatrix}$$

Corresponding Graph

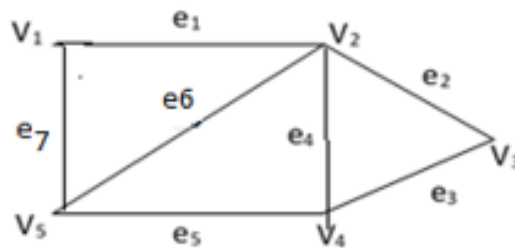


FIGURE 2

2.2. Relation between isomorphism, non-isomorphism and KS.

Graph A

Consider the graph G and name as follows

Let us consider adjacency matrix of the given graph is

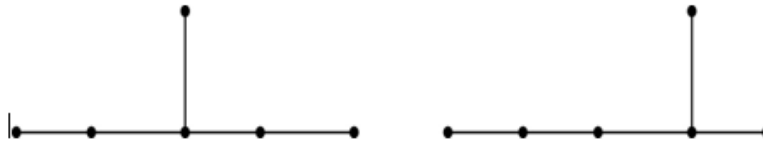


FIGURE 3

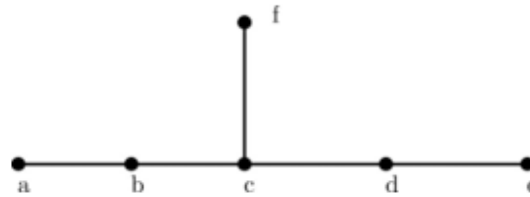


FIGURE 4

$$A = \begin{bmatrix} \langle 0, 0, 1 \rangle & \langle 0, 0, 1 \rangle & \langle 0, 0, 1 \rangle & \langle 0, 0, 1 \rangle & \langle 0, 0, 1 \rangle & \langle 0, 0, 1 \rangle \\ \langle 1, 1, 0 \rangle & \langle 0, 0, 1 \rangle & \langle 1, 1, 0 \rangle & \langle 0, 0, 1 \rangle & \langle 0, 0, 1 \rangle & \langle 1, 1, 0 \rangle \\ \langle 0, 0, 1 \rangle & \langle 1, 1, 0 \rangle & \langle 0, 0, 1 \rangle & \langle 1, 1, 0 \rangle & \langle 0, 0, 1 \rangle & \langle 1, 1, 0 \rangle \\ \langle 0, 0, 1 \rangle & \langle 0, 0, 1 \rangle & \langle 1, 1, 0 \rangle & \langle 0, 0, 1 \rangle & \langle 1, 1, 0 \rangle & \langle 0, 0, 1 \rangle \\ \langle 0, 0, 1 \rangle & \langle 0, 0, 1 \rangle & \langle 0, 0, 1 \rangle & \langle 1, 1, 0 \rangle & \langle 0, 0, 1 \rangle & \langle 0, 0, 1 \rangle \\ \langle 0, 0, 1 \rangle & \langle 0, 0, 1 \rangle & \langle 1, 1, 0 \rangle & \langle 0, 0, 1 \rangle & \langle 0, 0, 1 \rangle & \langle 0, 0, 1 \rangle \end{bmatrix}$$

Consider the graph H and name as follows

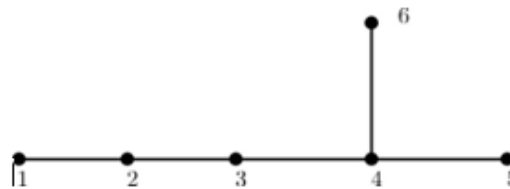


FIGURE 5

Consider the graph H and name as follows

Let us consider adjacency matrix of the given graph is

$$A = \begin{bmatrix} \langle 0, 0, 1 \rangle & \langle 0, 0, 1 \rangle & \langle 0, 0, 1 \rangle & \langle 0, 0, 1 \rangle & \langle 0, 0, 1 \rangle & \langle 0, 0, 1 \rangle \\ \langle 1, 1, 0 \rangle & \langle 0, 0, 1 \rangle & \langle 1, 1, 0 \rangle & \langle 0, 0, 1 \rangle & \langle 0, 0, 1 \rangle & \langle 1, 1, 0 \rangle \\ \langle 0, 0, 1 \rangle & \langle 1, 1, 0 \rangle & \langle 0, 0, 1 \rangle & \langle 1, 1, 0 \rangle & \langle 0, 0, 1 \rangle & \langle 1, 1, 0 \rangle \\ \langle 0, 0, 1 \rangle & \langle 0, 0, 1 \rangle & \langle 1, 1, 0 \rangle & \langle 0, 0, 1 \rangle & \langle 1, 1, 0 \rangle & \langle 0, 0, 1 \rangle \\ \langle 0, 0, 1 \rangle & \langle 0, 0, 1 \rangle & \langle 0, 0, 1 \rangle & \langle 1, 1, 0 \rangle & \langle 0, 0, 1 \rangle & \langle 0, 0, 1 \rangle \\ \langle 0, 0, 1 \rangle & \langle 0, 0, 1 \rangle & \langle 1, 1, 0 \rangle & \langle 0, 0, 1 \rangle & \langle 0, 0, 1 \rangle & \langle 0, 0, 1 \rangle \end{bmatrix}$$

The two graphs have the same number of vertices, same number of edges and same degree sequence. Though both the graphs have 3 pendent vertices, 2 vertices of degree 2 and 1 vertex of degree 3, the incidence relation of 3 pendent vertices are not preserved because in graph G 2 pendent vertices are attached to vertices of degree 2 and 1 pendent vertex is attached to vertex of degree 3 but in graph H only 1 pendent vertex is attached to

vertex of degree 2 and 2 pendent vertices are attached to vertex of degree 3. Therefore, the isomorphism between the two graphs cannot be established.

Thus, the given two graphs are non-isomorphic.

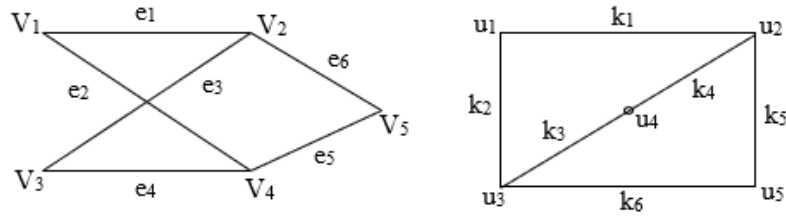


FIGURE 6

Let us consider adjacency matrix of the given graph is

$$G = \begin{bmatrix} \langle 0, 0, 1 \rangle & \langle 1, 1, 0 \rangle & \langle 0, 0, 1 \rangle & \langle 1, 1, 0 \rangle & \langle 0, 0, 1 \rangle \\ \langle 1, 1, 0 \rangle & \langle 0, 0, 1 \rangle & \langle 1, 1, 0 \rangle & \langle 0, 0, 1 \rangle & \langle 1, 1, 0 \rangle \\ \langle 0, 0, 1 \rangle & \langle 1, 1, 0 \rangle & \langle 0, 0, 1 \rangle & \langle 1, 1, 0 \rangle & \langle 0, 0, 1 \rangle \\ \langle 1, 1, 0 \rangle & \langle 0, 0, 1 \rangle & \langle 1, 1, 0 \rangle & \langle 0, 0, 1 \rangle & \langle 1, 1, 0 \rangle \\ \langle 0, 0, 1 \rangle & \langle 1, 1, 0 \rangle & \langle 0, 0, 1 \rangle & \langle 1, 1, 0 \rangle & \langle 0, 0, 1 \rangle \end{bmatrix}$$

$$H = \begin{bmatrix} \langle 0, 0, 1 \rangle & \langle 1, 1, 0 \rangle & \langle 0, 0, 1 \rangle & \langle 1, 1, 0 \rangle & \langle 0, 0, 1 \rangle \\ \langle 1, 1, 0 \rangle & \langle 0, 0, 1 \rangle & \langle 1, 1, 0 \rangle & \langle 0, 0, 1 \rangle & \langle 1, 1, 0 \rangle \\ \langle 0, 0, 1 \rangle & \langle 1, 1, 0 \rangle & \langle 0, 0, 1 \rangle & \langle 1, 1, 0 \rangle & \langle 0, 0, 1 \rangle \\ \langle 1, 1, 0 \rangle & \langle 0, 0, 1 \rangle & \langle 1, 1, 0 \rangle & \langle 0, 0, 1 \rangle & \langle 1, 1, 0 \rangle \\ \langle 0, 0, 1 \rangle & \langle 1, 1, 0 \rangle & \langle 0, 0, 1 \rangle & \langle 1, 1, 0 \rangle & \langle 0, 0, 1 \rangle \end{bmatrix}$$

There is a 1-1 correspondence between the vertices and edges. Therefore, the two graphs G and H are isomorphic.

The given two graphs have same number of vertices, edges and degree sequence and also the adjacency matrices are equal. Therefore the given Graph is isomorphic and also KS NFM.

Every isomorphic and non-isomorphic graph is KS adjacency NFM but converse need not be true.

2.3. Theorems and Results.

Theorem 2.1. The subsequent conditions are equivalent for $\psi \in F_n$

- (i) $N(\psi) = N(\psi^T)$.
- (ii) $\psi^T = \psi H = K\psi$ for several NFM H, K and $\rho(\psi) = r$.

Theorem 2.2. The subsequent conditions are equivalent for $\psi \in F_n$

- (i) $N(\psi) = N(KV\psi^TVK)$
- (ii) $N(KV\psi) = N((KV\psi)^T)$
- (iii) $N(\psi KV) = N((\psi KV)^T)$
- (iv) $N(V\psi) = N(K(V\psi)^TK)$
- (v) $N(\psi K) = N(V(\psi K)^TV)$
- (vi) $N(\psi^T) = N(KV(\psi)VK)$
- (vii) $N(\psi) = N(\psi^TVK)$
- (viii) $N(\psi^T) = N(\psi KV)$
- (ix) $\psi = VK\psi^TVKH_1$ for $H_1 \in F_n$
- (x) $\psi = H_1KV\psi^TVK$ for $H_1 \in F_n$

(xi) $\psi^T = KV\psi VKH$ for $H \in F_n$

(xii) $\psi^T = HKV\psi KV$ for $H \in F_n$.

Proof: (i) \Leftrightarrow (ii) \Leftrightarrow (iv)

ψ is $s - k - KS$

$\Leftrightarrow N(\psi) = N(KV\psi^T VK)$

$\Leftrightarrow N(KV\psi) = N((KV\psi)^T)$

[By Definition : 2.1]

$\Leftrightarrow KV \psi$ is KS

$\Leftrightarrow VP$ is $k - KS$

Hence, (i) \Leftrightarrow (ii) \Leftrightarrow (iv) hold.

(i) \Leftrightarrow (iii) \Leftrightarrow (v)

λ is $s - k - KS \Leftrightarrow N(\psi) = N(KV\psi^T VK)$

[By Definition 2.4]

$\Leftrightarrow N(KV\psi) = N((KV\psi)^T)$

[By Definition: 2.5]

$\Leftrightarrow N(VK(KV\psi)(VK)^T) = N((VK)\psi^T VK(VK)^T)$

$\Leftrightarrow N(KV) = N((KV)^T)$

$\Leftrightarrow \psi KV$ is KS

$\Leftrightarrow \psi K$ is $s - KS$

Hence, (i) \Leftrightarrow (iii) \Leftrightarrow (v) hold.

(ii) \Leftrightarrow (vii)

$KV\psi$ is $KS \Leftrightarrow N(KV\psi) = N((KV\psi)^T)$

$\Leftrightarrow N(\psi) = N((KV\psi)^T)$

[By Proposition 2.1]

$\Leftrightarrow N(\lambda) = N(\lambda^T VK)$

Hence, (ii) \Leftrightarrow (vii) hold.

(iii) iff (viii) :

ψVK is $KS \Leftrightarrow N(\psi VK) = N((\psi VK)^T)$

$\Leftrightarrow N(\psi VK) = N(\psi^T)$

[By Proposition 2.1]

Hence, (iii) \Leftrightarrow (viii) hold.

(i) iff (vi)

ψ is $s - k - KS \Leftrightarrow N(\psi) = N(KV\psi^T VK)$

$\Leftrightarrow N(KV\psi) = N((KV\psi)^T)$

[By Proposition 2.1]

$\Leftrightarrow (KV\psi)^T$ is KS

$\Leftrightarrow \psi^T VK$ is KS

$\Leftrightarrow \psi^T$ is $s - k - KS$

Hence, (i) \Leftrightarrow (vi) hold.

(i) iff (xi) iff (x)

ψ is $s - k - KS \Leftrightarrow N(\lambda) = N(KV\lambda^T VK)$

$\Leftrightarrow N(\psi^T) = N(KVVK)$

$\Leftrightarrow \psi^T = KV\psi VKH$

[By Theorem 2.1]

$\psi = H_1 KV\psi^T VK$ for $H_1 \in F_n$

Hence, (i) \Leftrightarrow (xi) iff \Leftrightarrow (x) hold.

(ii) \Leftrightarrow (xii) iff \Leftrightarrow (ix)

KVP is $RS \Leftrightarrow V\psi$ is $k - KS$

$\Leftrightarrow N(V\psi) = N(K(V\psi)^T K)$

$\Leftrightarrow (\psi) = N(\psi^T VK)$

[By Proposition 2.1]

$\Leftrightarrow N(\psi^T) = N(KV\psi)$

$\Leftrightarrow \psi^T = HKV\psi$ for $H \in F_n$

[By Theorem 2.1]

$\Leftrightarrow \psi^T = HKV\psi KV$

$\Leftrightarrow \psi = VK\psi^T VKH_1$ for $H_1 \in F_n$

Hence, (ii) \Leftrightarrow (xii) iff \Leftrightarrow (ix) hold.

Corollary 2.1. *The subsequent conditions are equivalent for $\lambda \in F_n$*

- (i) $N(\psi) = N(V\psi^T V)$
- (ii) $N(V\psi) = N(V\psi)^T$
- (iii) $N(\psi V) = N(\psi V)^T$
- (iv) ψ is $s - KS$
- (v) $N(\psi^T) = N(V\psi)$
- (vi) $N(\psi) = N(\psi^T V)$
- (vii) $N(\psi^T) = N(\psi V)$
- (viii) $N(KV\psi) = N((V\psi)^T)$
- (ix) $\psi = V\psi^T V H_1$ for $H_1 \in F_n$
- (x) $\psi = H_1 V\psi^T V$ for $H_1 \in F_n$
- (xi) $\psi^T = V\psi V H$ for $H \in F_n$
- (xii) $\psi^T = H V\psi V$ for $H \in F$.

Theorem 2.3. *For then any pair of the following statements indicate the other one*

- (i) $N(\psi) = N(K\psi^T K)$
- (ii) $N(\psi) = N(VK\psi^T \psi KV)$
- (iii) $N(\psi^T) = N((VK\psi)^T)$

Proof: (i) and (ii) iff (iii)

$$\psi \text{ is } s - k - KS \Rightarrow R(\psi) = R(\psi^T V K)$$

[By Theorem 2.2]

$$\Rightarrow N(K\psi K) = N(K\psi^T K)$$

$$\text{Hence (i) and (ii)} \Rightarrow N(\psi^T) = N((V\psi K)^T)$$

Hence, (iii) hold.

(i) and (iii) iff (ii)

$$\psi \text{ is } k - KS \Rightarrow N(\psi) = N(K\psi^T K)$$

$$\Rightarrow N(K\psi K) = N(\psi^T)$$

$$\text{Hence (i) and (iii)} \Rightarrow N(K\psi K) = N((V\psi K)^T)$$

$$\Rightarrow N(\psi) = N(\psi^T V K)$$

$$\Rightarrow N(\psi) = N((KV\psi)^T)$$

$$\Rightarrow \psi \text{ is } s - k - KS$$

[By Theorem 2.2]

Therefore, (ii) hold.

(ii) and (iii) \Leftrightarrow (i)

$$\psi \text{ is } s - k - KS \Rightarrow N(\psi) = N(\psi^T V K)$$

$$\Rightarrow N(K\psi K) = N(K\psi^T V)$$

[By Definition: 4.5]

$$\text{Hence (ii) and (iii)} \Rightarrow N(K\psi K) = N(\psi^T)$$

$$\Rightarrow N(\psi) = N(K\psi^T K)$$

$$\Rightarrow \psi \text{ is } k - KS$$

Therefore, (i) hold.

Hence the Theorem.

3. S- K-KERNEL SYMMETRIC REGULAR NFM

We show the existence of several generalized inverses of NFM in F_n and determine the conditions for different g-inverses of a s-k-KS NFM to be s-k KS NFM. Generalized inverses belonging to the sets $\psi\{1, 2\}$, $\psi\{1, 2, 3\}$ and $\psi\{1, 2, 4\}$ of s-k-KS NFM are characterized.

Theorem 3.1. *Let $(\psi_T, \psi_I, \psi_F) \in NFM_n$, $Z \in NFM_n\{1, 2\}$ and $(\psi_T, \psi_I, \psi_F)Z$, $Z(\psi_T, \psi_I, \psi_F)$ are s-k-KS NFM. Then (ψ_T, ψ_I, ψ_F) is s - k - KS NFM $\Leftrightarrow Z$ is s - k - KS NFM.*

Proof: Since $N(KV\lambda) = N(KV(\psi_T, \psi_I, \psi_F)Z(\psi_T, \psi_I, \psi_F)) \subseteq N(Z(\psi_T, \psi_I, \psi_F))$
 $= N(ZVV(\psi_T, \psi_I, \psi_F)) = N(ZVKKV(\psi_T, \psi_I, \psi_F)) \subseteq N(KV(\psi_T, \psi_I, \psi_F))$

$$\begin{aligned}
& \text{Hence, } N(KV(\psi_T, \psi_I, \psi_F)) = N(Z(\psi_T, \psi_I, \psi_F)) \\
& = N(KV(Z(\psi_T, \psi_I, \psi_F))^T VK) \quad [Z \text{ is } s-k-KSNFM] \\
& = N((\psi_T, \psi_I, \psi_F)^T Z^T VK) \\
& = N(Z^T VK) \\
& = N((KVZ)^T) \\
& N((KV(\psi_T, \psi_I, \psi_F))^T) = N((\psi_T, \psi_I, \psi_F)^T VK) \\
& = N(Z^T(\psi_T, \psi_I, \psi_F)^T VK) \\
& = N((KV(\psi_T, \psi_I, \psi_F)Z)^T) \\
& = N(KV\lambda Z) \quad [V \text{ is } s-k-KS] \\
& = N(KVZ) \\
& KVZ \text{ is } KS \Leftrightarrow N(KV(\psi_T, \psi_I, \psi_F)) = N((KV\psi)^T) \\
& \Leftrightarrow N((KVZ)^T) = N(KVZ) \\
& \Leftrightarrow KVZ \text{ is } KS \\
& \Leftrightarrow Z \text{ is } s-k-KS.
\end{aligned}$$

Theorem 3.2. Let $Z \in \{1, 2, 3\}$, $N(KV) = N((KVZ)^T)$. Then (ψ_T, ψ_I, ψ_F) is $s-k-KS$ NFM $\Rightarrow Z$ is $s-k-KS$ NFM.

Proof: Since $Z \in (\psi_T, \psi_I, \psi_F)\{1, 2, 3\}$,

$$\begin{aligned}
& \text{Hence } (\psi_T, \psi_I, \psi_F)Z(\psi_T, \psi_I, \psi_F) = (\psi_T, \psi_I, \psi_F), Z(\psi_T, \psi_I, \psi_F)Z = Z, ((\psi_T, \psi_I, \psi_F)Z)^T \\
& = (\psi_T, \psi_I, \psi_F)Z \\
& N((KV(\psi_T, \psi_I, \psi_F))^T) = N(Z^T(\psi_T, \psi_I, \psi_F)^T VK) \quad [By \text{ using } \psi Z \psi = \psi] \\
& = N(KV(\lambda Z)^T) = N((\lambda Z)^T) = N(Z) \quad [(\psi Z)^T = \psi Z] \\
& = N(Z) \quad [By \text{ using } Z = Z\lambda Z] \\
& = N(KVZ) \quad KV(\psi_T, \psi_I, \psi_F) \text{ is } KSNFM \Leftrightarrow N(KV\lambda) = N((KV\lambda)^T) \\
& \Leftrightarrow N((KVZ)^T) = N(KVZ) \\
& \Leftrightarrow KVZ \text{ is } KS \\
& \Leftrightarrow Z \text{ is } s-k-KS.
\end{aligned}$$

Theorem 3.3. Let $(\psi_T, \psi_I, \psi_F) \in NFM_n$, $Z \in (\psi_T, \psi_I, \psi_F)\{1, 2, 4\}$, $N((KVP)^T) = N(KVZ)$. Then Z is $s-k-KS$ NFM $\Leftrightarrow Z$ is $s-k-KS$ NFM.

Proof: Since $Z \Leftrightarrow (\psi_T, \psi_I, \psi_F)\{1, 2, 4\}$, we have $(\psi_T, \psi_I, \psi_F)Z(\psi_T, \psi_I, \psi_F)$

$$\begin{aligned}
& = (\psi_T, \psi_I, \psi_F), Z(\psi_T, \psi_I, \psi_F)Z = Z, (Z(\psi_T, \psi_I, \psi_F))^T = Z(\psi_T, \psi_I, \psi_F) \\
& N(KV) = N((\psi_T, \psi_I, \psi_F)) \\
& = N(Z(\psi_T, \psi_I, \psi_F))[Z(\psi_T, \psi_I, \psi_F)Z = Z, (\psi_T, \psi_I, \psi_F)Z\psi] \\
& = N((Z(\psi_T, \psi_I, \psi_F))^T)[(Z(\psi_T, \psi_I, \psi_F))^T = Z(\psi_T, \psi_I, \psi_F)] \\
& = N((\psi_T, \psi_I, \psi_F)Z^T) \\
& = N(Z^T) \\
& = N((KVZ)^T) \\
& KV(\psi_T, \psi_I, \psi_F) \text{ is } KSNFM \Leftrightarrow N(KV\lambda) = N((KV\lambda)^T) \\
& \Leftrightarrow N((KVZ)^T) = N(KVZ) \\
& \Leftrightarrow KVZ \text{ is } KSNFM \\
& \Leftrightarrow Z \text{ is } s-k-KS \text{ NFM.}
\end{aligned}$$

In particular for $K = I$, the above Theorems reduces to equivalent conditions for various g -inverses of a s -kernel symmetric NFM to be secondary kernel symmetric NFM.

Corollary 3.1. Let (ψ_T, ψ_I, ψ_F) belongs to n , Z belongs $(\psi_T, \psi_I, \psi_F)(1, 2)$ and $(\psi_T, \psi_I, \psi_F)Z$, $Z(\psi_T, \psi_I, \psi_F)$ are KS NFM. Then (ψ_T, ψ_I, ψ_F) is $s-KS$ NFM $\Leftrightarrow Z$ is $s-KS$ NFM.

Proof: Since $N(V\lambda) = N(V(\psi_T, \psi_I, \psi_F)Z(\psi_T, \psi_I, \psi_F)) \subseteq N(Z(\psi_T, \psi_I, \psi_F))$

$$= N(ZVV(\psi_T, \psi_I, \psi_F)) = N(ZVV(\psi_T, \psi_I, \psi_F)) \subseteq N(V(\psi_T, \psi_I, \psi_F))$$

$$\text{Hence, } N(V(\psi_T, \psi_I, \psi_F)) = N(Z(\psi_T, \psi_I, \psi_F))$$

$$\begin{aligned}
&= N(V(Z(\psi_T, \psi_I, \psi_F))^T V) && [Z \text{ is } s - KSNFM] \\
&= N((\psi_T, \psi_I, \psi_F)^T Z^T V) \\
&= N(Z^T V) \\
&= N((VZ)^T) \\
&N((V(\psi_T, \psi_I, \psi_F))^T) = N((\psi_T, \psi_I, \psi_F)^T V) \\
&= N(Z^T (\psi_T, \psi_I, \psi_F)^T V) \\
&= N((V(\psi_T, \psi_I, \psi_F) Z)^T) \\
&= N(V\lambda Z) && [V \text{ is } s - KS] \\
&= N(VZ) \\
&VZ \text{ is } KS \Leftrightarrow N(V(\psi_T, \psi_I, \psi_F)) = N((V\psi)^T) \\
&\Leftrightarrow N((VZ)^T) = N(VZ) \\
&\Leftrightarrow VZ \text{ is } KS \\
&\Leftrightarrow Z \text{ is } s - KS.
\end{aligned}$$

Corollary 3.2. Let (ψ_T, ψ_I, ψ_F) belongs to F_n , Z belongs to $(\psi_T, \psi_I, \psi_F)(1, 2, 3)$, $N(KV(\psi_T, \psi_I, \psi_F)) = N((VX)^T)$. Then $is - KS NFM \Leftrightarrow Z \text{ is } s - KS NFM$.

Proof: Since $Z \in (\psi_T, \psi_I, \psi_F)\{1, 2, 3\}$,

Hence $(\psi_T, \psi_I, \psi_F) Z (\psi_T, \psi_I, \psi_F) = (\psi_T, \psi_I, \psi_F)$, $Z (\psi_T, \psi_I, \psi_F) Z = Z$, $((\psi_T, \psi_I, \psi_F) Z)^T = (\psi_T, \psi_I, \psi_F) Z$

$$\begin{aligned}
&N((V(\psi_T, \psi_I, \psi_F))^T) = N(Z^T (\psi_T, \psi_I, \psi_F)^T V) && [By \text{ using } \psi Z \psi = \psi] \\
&= N(V(\lambda Z)^T) = N((\lambda Z)^T) = N(Z) && [(\psi Z)^T = \psi Z] \\
&= N(Z) && [By \text{ using } Z = Z\lambda Z] \\
&= N(VZ) V(\psi_T, \psi_I, \psi_F) \text{ is } KSNFM \Leftrightarrow N(V\lambda) = N((V\lambda)^T) \\
&\Leftrightarrow N((VZ)^T) = N(VZ) \\
&\Leftrightarrow VZ \text{ is } KS \\
&\Leftrightarrow Z \text{ is } s - KS.
\end{aligned}$$

Corollary 3.3. Let ψ belongs to F_n , Z belongs to $(\psi_T, \psi_I, \psi_F)(1, 2, 4)$, $N((V\psi)^T) = N(VZ)$. Then (ψ_T, ψ_I, ψ_F) is $s - KS NFM \Leftrightarrow Z \text{ is } s - KS NFM$.

Proof: Since $Z \Leftrightarrow (\psi_T, \psi_I, \psi_F)\{1, 2, 4\}$, we have $(\psi_T, \psi_I, \psi_F) Z (\psi_T, \psi_I, \psi_F)$

$= (\psi_T, \psi_I, \psi_F)$, $Z (\psi_T, \psi_I, \psi_F) Z = Z$, $(Z(\psi_T, \psi_I, \psi_F))^T = Z(\psi_T, \psi_I, \psi_F)$

$$\begin{aligned}
&N(KV) = N((\psi_T, \psi_I, \psi_F)) \\
&= N(Z(\psi_T, \psi_I, \psi_F)) [Z(\psi_T, \psi_I, \psi_F) Z = Z, (\psi_T, \psi_I, \psi_F) Z \psi] \\
&= N((Z(\psi_T, \psi_I, \psi_F))^T) [(Z(\psi_T, \psi_I, \psi_F))^T = Z(\psi_T, \psi_I, \psi_F)] \\
&= N((\psi_T, \psi_I, \psi_F) Z^T) \\
&= N(Z^T) \\
&= N((VZ)^T) \\
&V(\psi_T, \psi_I, \psi_F) \text{ is } SNFM \Leftrightarrow N(V\lambda) = N((V\lambda)^T) \\
&\Leftrightarrow N((VZ)^T) = N(VZ) \\
&\Leftrightarrow VZ \text{ is } KSNFM \\
&\Leftrightarrow Z \text{ is } s - KS NFM.
\end{aligned}$$

4. CONCLUSION

We show that k-symmetry implies k-KS, though the reverse is universally applicable. We elucidated the equivalent criteria for the g-inverses of s-KS NFM to retain their s-KS. Our work has provided a characterization of the generalized inverses of s-KS NFM ψ for specific sets $\psi\{1, 2\}$, $\psi\{1, 2, 3\}$, and $\psi\{1, 2, 4\}$. We show that s-k- KS, s- KS, k- KS, and KS NFM relate to each other. For a NFM to be a s-k- KS NFM, necessary and

sufficient requirements are identified. It is demonstrated that k -symmetry implies k -KS and the reverse is necessarily true. We also describe Graphical Representation of Range symmetric, Column symmetric and kernel symmetric Adjacency and Incidence NFM is characterized. Every Adjacency NFM is symmetric, range symmetric, column symmetric and kernel symmetric but Incidence matrix satisfies only kernel symmetric conditions. Every range symmetric Adjacency NFM is kernel symmetric Adjacency NFM but kernel symmetric Adjacency NFM need not be range symmetric NFM. In the future, we will prove additional properties relating to g -inverses of Secondary k -Kernel Symmetric Neutrosophic Fuzzy Matrices with Generalized Inverses.

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