TWMS J. App. Eng. Math., V.15, No.9, 2025, pp. 2313-2330

A NOVEL APPROACH ON m-POLAR FUZZY SET WITH DOMBI POWER AGGREGATION OPERATORS TO SOLVE MULTI-ATTRIBUTE DECISION-MAKING PROBLEMS

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Abstract. This article introduces new aggregation operators by combining Dombi and power operators, resulting in m-polar fuzzy Dombi power averaging and geometric operators, with several key properties analyzed. A novel approach is then developed using these combined operators to assist in selecting the best company for stock market investments. Unlike existing research, which primarily utilizes homogeneous sub-characteristics for each attribute in the mPF environment, this study emphasizes the use of heterogeneous sub-characteristic collections in our application to address complex, uncertain decision-making challenges. Finally, the proposed approach is compared to various established operators and the MABAC method, with an analysis of its advantages and limitations.

Keyword: m-polar fuzzy set, Dombi and power aggregation operators, MABAC approach, Multi-attribute decision-making.

AMS Subject Classification: 90B50.

1. Introduction

1.1. **Research background and related works.** An m-polar fuzzy set extends fuzzy set theory to address complex scenarios with multiple dimensions of uncertainty and preference. Zadeh introduced FSs in 1965 [38] to handle vagueness by assigning a membership degree between 0 and 1 to each element. Atanassov developed IFSs in 1986 [7], adding a NMV alongside the MV, with their sum not exceeding 1. Later, in 1989, Atanassov and Gargov introduced IVIFS [32], where both MV and NMV are represented as intervals, addressing incomplete but not indeterminate information. To handle bipolar perspectives, Zhang developed the concept of bipolar fuzzy sets (BFSs) [9], where membership has two parts: a positive degree on [0, 1] and a negative degree on [-1, 0]. BFSs are widely used in decision-making applications, including medical sciences. Chen later expanded this idea, proposing mPF sets as a generalization of BFSs, designed to address even more nuanced, multi-dimensional uncertainty in decision-making. mPFSs provide a robust framework

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Manuscript received: August 05, 2024; accepted: December 02, 2024.

for situations where decisions are influenced by multiple criteria, each with unique uncertainties. This added flexibility makes them ideal for complex real-world applications, enhancing accuracy in modeling intricate decision-making processes.

AOs are mathematical tools that combine multiple inputs into a single output, widely applied in fields like data fusion, decision-making, and information retrieval. Commonly constructed using TN and TCN operators, AOs include those defined by Archimedean [26], Hamacher [29], Einstein [12], Bonferroni mean [8], Dombi [11], and Frank [33]. Yager introduced OWA operators in 1988 [36], which have since been applied to various set types, leading to advances in MADM. Notable works include Beliakov et al. [8], He et al. [13], and others.

Power averaging operators, introduced by Yager in 2001 [37], inspired further studies, including those by Wei [30], Wang [28], and Jana [17, 18, 19, 20]. Xu and Yager [35] later developed a version where weight vectors depend on input values. Dombi's versatile operations, proposed in 1982 [11], were later applied by Liu et al. [23] to solve MADM problems. Jana et al. [22] further used Dombi operations with picture fuzzy numbers for MADM. In 2024, Ruan and Chen [25] applied power Bonferroni operators to yield flexible MADM results. Recently, Jana et al. [15, 16] combined Dombi power aggregation for Pythagorean fuzzy sets, though no one has applied this approach in m-polar fuzzy environments. Waseem [29] developed mPF Hamacher AOs for MADM, and Akram et al. [1, 2, 3, 4, 5, 6] solved numerous MADM problems using mPFS-based approaches.

Despite progress, there remains a gap in research on a specific MADM method using varied operators for mPFs, especially in developing new mPF aggregation operators with the combined Dombi power operator. Additionally, in the existing applications, to aggregate the collections of mPFNs, homogeneous sub-characteristic collections have been utilized. There is a major problem whether the selection process contains so many attributes in which every attribute contains different numbers of sub-attributes. To address this gap, this study uses heterogeneous sub-characteristic collections to select best company in the stock market in which so many key words.

Notations and symbols In Table 1, some notations and abbreviation forms are given which are used in the entire paper.

TABLE 1. List of abbreviations.

Full name	Abbreviation	Full name	Abbreviation
Membership value	MV	Multi-attribute decision-making	MADM
Non membership value	NMV	Multi-criteria decision-making	MCDM
Fuzzy set	FS	t-norm and t-conorm	TN and TCN
Intuitionistic fuzzy set	IFS	m-polar fuzzy Dombi weighted avaraging	mPFDPWA
Interval valued intuitionistic fuzzy set	IVIFS	m-polar fuzzy Dombi weighted geometric	mPFDPWG
Bipolar fuzzy set	BFS	m-polar fuzzy Dombi power weighted averaging	mPFDPWA
m-polar fuzzy set	mPFS	m-polar fuzzy Dombi power weighted geometric	mPFDPWG
Aggregation operator	AO	m-polar fuzzy Einstein weighted averaging	mPFEWA
Ordered weighted aggregation	OWA	m-polar fuzzy Einstein weighted geometric	mPFEWG

1.2. **Motivation of the work.** To write this article, we are motivated from the following issues:

(1) The stock market is essential for India's economic growth, as it provides a platform for companies to raise capital for expansion, innovation, and job creation. Additionally, the stock market reflects economic stability and investor confidence, influencing economic policies that aim for sustainable growth. This, collectively,

strengthens India's economic development and global competitiveness. So, investigation in the stock market is very essential. To select the best company in the market, each company contain so many key words in that case FS, BPFS, and IFS can not handle, that is why we have used mPFS for this type of decision-making problems.

(2) Real-world decision problems often involve inter-dependencies among criteria. The combination of Dombi and power averaging operators can effectively capture these inter-dependencies, providing a more comprehensive aggregation mechanism that reflects the true nature of the decision context.

(3) The proposed combination of two operators overcomes the limitations of existing operators and make the optimal outcomes more accurate and definite.

1.3. **Novelty and contribution of the work.** The present study focuses to solve MADM problems by a novel process and combining Dombi and power aggregation operators on mPF information. Unlike previous research, which primarily used homogeneous sub-characteristics within each mPFN, our method incorporates heterogeneous sub-characteristics to aggregate mPF information. This innovation enhances the handling of complex, uncertain decision-making scenarios and our approach apart from existing methods.

The primary contributions of this article are as follows:

(1) In relation to some new mPF Dombi power operators, a novel MADM method is taken into consideration.
(2) A case study is performed for stock market and solved this problem to show applicability of the proposed method.
(3) At last, the validity and strengths of this combined operator are discussed through comparative analysis.

1.4. **Framework of this paper.** The study is organized as follows: First, we present essential definitions for mPFNs and their basic operations. In Section 3, we explore Dombi and power operators, as well as Dombi operations for mPFNs. Section 4 introduces the combination of Dombi and power aggregation operators, resulting in new operators like the mPFDPWA and mPFDPWG operators, along with their properties. Section 5 describes an algorithm for solving MADM problems using mPFNs and these operators. A very interesting application is provided in Section 6 to demonstrate feasibility. In Section 7, we validate the approach using MABAC, followed by a comparison and analysis of its advantages and limitations in Section 8. The paper concludes in Section 9.

2. Preliminaries

Some required definitions are introduced about mPF set on the finite non empty set ζ.

Definition 2.1. [9] *An mPFS χ on the reference set ζ expressed as $\chi : \zeta \to [0,1]^m$ defined by $\chi(z) = \langle\ \sigma_1, \sigma_2, \ldots, \sigma_m\ \rangle = \langle \sum \sigma_1 \rangle, z \in \zeta$ where, r-th component σ_r is defined by projection mapping $p_r \circ \chi(z) : [0,1]^m \to [0,1]$. Here, $\sum \sigma_1$ is not the sum of components, it represents all components $\sigma_1, \sigma_2, \ldots, \sigma_m$. Here all components of mPFS represent only membership values and m is any arbitrary positive integer.*

Definition 2.2. *Let $\chi = \langle \sum \sigma_1 \rangle = \langle \sigma_1, \sigma_2, \ldots, \sigma_m \rangle$ and $\psi = \langle \sum \rho_1 \rangle = \langle \rho_1, \rho_2, \ldots, \rho_m \rangle$ be two mPFNs. Then some operations on mPFNs are defined in the following:*

(1) $\chi \oplus \psi = \langle \sum (\sigma_1 + \rho_1 - \sigma_1 \rho_1) \rangle$;
(2) $\chi \otimes \psi = \langle \sum (\sigma_1 \rho_1) \rangle$;

(3) $\chi^c = \langle \sum(1-\sigma_1) \rangle$, χ^c is the complement of χ;
(4) $\delta\chi = \left\langle \sum(1-(1-\sigma_1)^\delta) \right\rangle$, $\delta > 0$;
(5) $(\chi)^\delta = \left\langle \sum(\sigma_1^\delta) \right\rangle$, $\delta > 0$;
(6) $\chi \subseteq \psi$ iff $\sigma_\gamma \leq \rho_\gamma$, for every $\gamma = 1, 2, \ldots, m$;
(7) $\chi \cap \psi = \langle \sum min\{\sigma_1, \rho_1\} \rangle$;
(8) $\chi \cup \psi = \langle \sum max\{\sigma_1, \rho_1\} \rangle$;.

For ranking any two mPFNs, score and accuracy function are formulated as follow:

Definition 2.3. [4] *Let χ be an mPFN. Then score and accuracy function of an mPFN are respectively defined below:*

$$\Delta(\chi) = \frac{1}{m} \ (\sum_{\gamma=1}^{m} \sigma_\gamma), \ \ \Delta(\chi) \in [0, 1], \tag{1}$$

$$\Gamma(\chi) = \frac{1}{m} \ (\sum_{\gamma=1}^{m} (-1)^\gamma (\sigma_\gamma - 1), \ \ \Gamma(\chi) \in [-1, 1]. \tag{2}$$

Definition 2.4. *Any two mPFNs χ and ψ can be ordered from the following ordered relation that are defined based on the above score function.*
(1) $\chi > \psi$ *if* $\Delta(\chi) > \Delta(\psi)$*;*
(2) $\chi < \psi$ *if* $\Delta(\chi) < \Delta(\psi)$*;*
(3) *If* $\Delta(\chi) = \Delta(\psi)$ *, then*
- $\chi > \psi$ *if* $\Gamma(\chi) > \Gamma(\psi)$*;*
- $\chi < \psi$ *if* $\Gamma(\chi) > \Gamma(\psi)$*;*
- $\chi = \psi$ *if* $\Gamma(\chi) = \Gamma(\psi)$*.*

3. Dombi and power operators

The definitions of Dombi TN and Dombi TCN are provided below.

Definition 3.1. [17] *The Dombi TN and TCN between any two real numbers h and k are defined by*

$$D(h, k) = \frac{1}{1 + \{(\frac{1-h}{h})^\partial + (\frac{1-k}{k})^\partial\}^{\frac{1}{\partial}}}, \tag{3}$$

$$D^c(h, k) = 1 - \frac{1}{1 + \{(\frac{h}{1-h})^\partial + (\frac{k}{1-k})^\partial\}^{\frac{1}{\partial}}}, \tag{4}$$

where, $\partial \geq 1$ is any parameter and $h, k \in [0, 1]$.

Along with Dombi TN and TCN some essential Dombi operations on mPFNs are given below:

Definition 3.2. *Let $\chi = \langle \sum \sigma_1 \rangle$ and $\psi = \langle \sum \rho_1 \rangle$ be two mPFNs. Then their some operations including Dombi operator are defined as:*

(1) $\chi \bigoplus \psi = \left\langle \sum \left(1 - \frac{1}{1+\{(\frac{\sigma_1}{1-\sigma_1})^\partial + (\frac{\rho_1}{1-\rho_1})^\partial\}^{\frac{1}{\partial}}} \right) \right\rangle$;

(2) $\chi \bigotimes \psi = \left\langle \sum \left(\frac{1}{1+\{(\frac{1-\sigma_1}{\sigma_1})^\partial + (\frac{1-\rho_1}{\rho_1})^\partial\}^{\frac{1}{\partial}}} \right) \right\rangle$;

(3) $\hbar\chi = \left\langle \sum \left(1 - \frac{1}{1+\{\hbar(\frac{\sigma_1}{1-\sigma_1})^\partial\}^{\frac{1}{\partial}}} \right) \right\rangle$, $\hbar > 0$;

(4) $(\chi)^{\hbar} = \left\langle \sum \left(\frac{1}{1+\{\hbar(\frac{1-\sigma_1}{\sigma_1})^{\partial}\}^{\frac{1}{\partial}}} \right) \right\rangle$, $\hbar > 0$.

Definition 3.3. [17] *Consider $f_1, f_2, \ldots, f_b$ be any real numbers. Then power averaging(PA) operator is defined by $P_A : \mathbf{R^b} \to \mathbf{R}$ such that*

$$P_A(f_1, f_2, \ldots, f_b) = \frac{\sum_{p=1}^{b}(1 + S(f_p))f_p}{\sum_{p=1}^{b}(1 + S(f_p))}, \tag{5}$$

where, $S(f_p) = \sum_{p=1,p\neq q}^{b} Spt(f_p, f_q)$, and $Spt(f_p, f_q)$ represents the support between f_p and f_q and that obeys three conditions :

(1) $Spt(f_p, f_q) \in [0, 1]$,
(2) $Spt(f_p, f_q) = Spt(f_q, f_p)$,
(3) $Spt(f_p, f_q) \geq Spt(f_k, f_l)$ *if* $|f_p - f_q| < |f_k - f_l|$

i.e., third condition says that support between two real numbers is more strong when two values are more closer.

Definition 3.4. [17] *The power geometric operator on the real numbers $g_1, g_2, \ldots, g_b$ defined by the expression*

$$P_G(g_1, g_2, \ldots, g_b) = \prod_{t=1}^{b}(g_t)^{\frac{(1+S(g_t))}{\sum_{t=1}^{b}(1+S(g_t))}}, \tag{6}$$

where, $S(g_t)$ is previously defined.

4. Combination of Dombi and power aggregation operators

Combining the Dombi operator with the power operator, it is possible to leverage their respective strengths to model complex decision scenarios more effectively. Researchers have explored the synergy between two aggregation operators. Here, we apply on the set of mPFNs.

4.1. Arithmetic aggregation operators.

Definition 4.1. *Let $\chi_p = \langle \sigma_{1p}, \sigma_{2p}, \ldots, \sigma_{mp} \rangle = \langle \sum \sigma_{1p} \rangle$, $p = 1, 2, \ldots, v$, be the collection of mPFNs. Then mPF Dombi power averaging (mPFDPA) operator is a mapping defined by*

$$mPFDPA(\chi_1, \chi_2, \ldots, \chi_v) = \bigoplus_{p=1}^{v} \frac{(1 + S(\chi_p))\chi_p}{\sum_{p=1}^{v}(1 + S(\chi_p))}. \tag{7}$$

Based on the above combined operator some theories are deduced.

Theorem 4.1. *Let χ_{ϖ}, $\varpi = 1, 2, \ldots, v$ be the collections. Then using mPFDPA operator, the aggregated value of collections is also an mPFN.*
Mathematically, $mPFDPA(\chi_1, \chi_2, \ldots, \chi_v) = \bigoplus_{\varpi=1}^{v} \frac{(1+S(\chi_{\varpi}))\chi_{\varpi}}{\sum_{\varpi=1}^{v}(1+S(\chi_{\varpi}))}$

$$= \left\langle \left(1 - \frac{1}{1+\{\sum_{\varpi=1}^{v} \frac{(1+S(\chi_{\varpi}))}{\sum_{\varpi=1}^{v}(1+S(\chi_{\varpi}))}(\frac{\sigma_{1\varpi}}{1-\sigma_{1\varpi}})^{\partial}\}^{\frac{1}{\partial}}} \right), \ldots, \left(1 - \frac{1}{1+\{\sum_{\varpi=1}^{v} \frac{(1+S(\chi_{\varpi}))}{\sum_{\varpi=1}^{v}(1+S(\chi_{\varpi}))}(\frac{\sigma_{m\varpi}}{1-\sigma_{m\varpi}})^{\partial}\}^{\frac{1}{\partial}}} \right) \right\rangle$$

$$= \left\langle \sum \left(1 - \frac{1}{1 + \{\sum_{\varpi=1}^{v} \frac{(1+S(\chi_{\varpi}))}{\sum_{\varpi=1}^{v}(1+S(\chi_{\varpi}))}(\frac{\sigma_{1\varpi}}{1-\sigma_{1\varpi}})^{\partial}\}^{\frac{1}{\partial}}} \right) \right\rangle \tag{8}$$

Proof: This can be easily proved from the first and third operations of Definition 3.2.

Theorem 4.2. *Let χ_ϖ, $\varpi = 1, 2, \ldots, v$ be a collections of mPFNs. Then after use of mPFDPWA operator with weight vector $\vartheta = (\vartheta_1, \vartheta_2, \ldots, \vartheta_v)^T$ ($\vartheta_\varpi \in [0, 1]$) and ($\sum_{\varpi=1}^{v} \vartheta_\varpi = 1$) on mPFNs, the aggregated value is also a mPFN.*

$$mPFDPWA_\vartheta(\chi_1, \chi_2, \ldots, \chi_v) = \bigoplus_{\varpi=1}^{v} \frac{(\vartheta_\varpi(1 + S(\chi_\varpi))\chi_\varpi)}{\sum_{\varpi=1}^{v}(\vartheta_\varpi(1 + S(\chi_\varpi)))}$$

$$= \left\langle \sum \left(1 - \frac{1}{1 + \{\sum_{\varpi=1}^{v} \frac{(\vartheta_\varpi(1+S(\chi_\varpi)))}{\sum_{\varpi=1}^{v}(\vartheta_\varpi(1+S(\chi_\varpi)))} (\frac{\sigma_{1\varpi}}{1-\sigma_{1\varpi}})^\partial\}^{\frac{1}{\partial}}} \right) \right\rangle \quad (9)$$

where,

$$S(\chi_\varpi) = \sum_{\varpi=1, \varpi\neq\varsigma}^{v} \vartheta_\varpi Spt(\chi_\varpi, \chi_\varsigma). \quad (10)$$

Proof: Use mathematical induction technique to prove this theorem.
First consider $v = 2$, then we get,
$mPFDPWA_\vartheta(\chi_1, \chi_2) = \chi_1 \bigoplus \chi_2 = \langle \sum \sigma_{11} \rangle \bigoplus \langle \sum \sigma_{12} \rangle$
$= \langle \sigma_{11}, \sigma_{21}, \ldots, \sigma_{m1} \rangle \bigoplus \langle \sigma_{12}, \sigma_{22}, \ldots, \sigma_{m2} \rangle$

$$= \left\langle \sum \left(1 - \frac{1}{1+\{\frac{(\vartheta_1(1+S(\chi_1)))}{\sum_{\varpi=1}^{2}(\vartheta_\varpi(1+S(\chi_\varpi)))}(\frac{\sigma_{11}}{1-\sigma_{11}})^\partial + \frac{(\vartheta_2(1+S(\chi_2)))}{\sum_{\varpi=1}^{2}(\vartheta_\varpi(1+S(\chi_p)))}(\frac{\sigma_{12}}{1-\sigma_{12}})^\partial\}^{\frac{1}{\partial}}} \right) \right\rangle \text{ (by equation (8))}$$

$$= \left\langle \sum \left(1 - \frac{1}{1+\{\sum_{\varpi=1}^{2} \frac{(\vartheta_\varpi(1+S(\chi_\varpi)))}{\sum_{\varpi=1}^{2}(\vartheta_\varpi(1+S(\chi_\varpi)))}(\frac{\sigma_{1\varpi}}{1-\sigma_{1\varpi}})^\partial\}^{\frac{1}{\partial}}} \right) \right\rangle$$

So, equation (9) is valid for $v = 2$.
Next assume, the equation (9) is valid for $v = d$, i.e.,

$$mPFDPWA_\vartheta(\chi_1, \chi_2, \ldots, \chi_d) = \bigoplus_{\varpi=1}^{d} \frac{(\vartheta_\varpi(1 + S(\chi_\varpi))\chi_\varpi)}{\sum_{\varpi=1}^{d}(\vartheta_\varpi(1 + S(\chi_\varpi)))}$$

$$= \left\langle \sum \left(1 - \frac{1}{1 + \{\sum_{\varpi=1}^{d} \frac{(\vartheta_\varpi(1+S(\chi_\varpi)))}{\sum_{\varpi=1}^{d}(\vartheta_\varpi(1+S(\chi_\varpi)))} (\frac{\sigma_{1\varpi}}{1-\sigma_{1\varpi}})^\partial\}^{\frac{1}{\partial}}} \right) \right\rangle \quad (11)$$

Next, for $v = d + 1$,
$mPFDPWA_\vartheta(\chi_1, \ldots, \chi_d, \chi_{d+1}) = \bigoplus_{\varpi=1}^{d} \frac{(\vartheta_\varpi(1+S(\chi_\varpi))\chi_\varpi)}{\sum_{\varpi=1}^{d}(\vartheta_\varpi(1+S(\chi_\varpi)))} \bigoplus \left(\frac{\vartheta_{d+1}(1+S(\chi_{d+1}))\chi_{d+1}}{(\vartheta_{d+1}(1+S(\chi_{d+1})))} \right)$

$$= \left\langle \sum \left(1 - \frac{1}{1+\{\sum_{\varpi=1}^{d} \frac{(\vartheta_\varpi(1+S(\chi_\varpi)))}{\sum_{\varpi=1}^{d}(\vartheta_\varpi(1+S(\chi_\varpi)))}(\frac{\sigma_{1\varpi}}{1-\sigma_{1\varpi}})^\partial\}^{\frac{1}{\partial}}} \right) \right\rangle \bigoplus$$

$$\left\langle \sum \left(1 - \frac{1}{1 + \{\frac{(\vartheta_{d+1}(1+S(\chi_{d+1})))}{(\vartheta_{d+1}(1+S(\chi_{d+1})))}(\frac{\sigma_{1(d+1)}}{1-\sigma_{1(d+1)}})^\partial\}^{\frac{1}{\partial}}} \right) \right\rangle$$

$$= \left\langle \sum \left(1 - \frac{1}{1 + \{\sum_{\varpi=1}^{d+1} \frac{(\vartheta_\varpi(1+S(\chi_\varpi)))}{\sum_{\varpi=1}^{d+1}(\vartheta_\varpi(1+S(\chi_\varpi)))} (\frac{\sigma_{1\varpi}}{1-\sigma_{1\varpi}})^\partial\}^{\frac{1}{\partial}}} \right) \right\rangle \quad (12)$$

This shows that the equation (9) is valid for $v = d + 1$.
Therefore, by mathematical induction process the equation (9) is true for all natural number v.

Theorem 4.3. *Let χ_ϖ, $\varpi = 1, 2, \ldots, v$ be a collections of mPFNs. If every mPFN is same to a mPFN χ then $mPFDPWA_\vartheta(\chi_1, \chi_2, \ldots, \chi_v) = \chi$.*

Proof: Here, $\chi_\varpi = \chi = \langle \sum \sigma_1, \sigma_2, \ldots, \sigma_m \rangle$, for every $\varpi = 1, 2, \ldots, v$.
Now from the equation (9) we get,
$mPFDPWA_\vartheta(\chi_1, \chi_2, \ldots, \chi_v) = \bigoplus_{\varpi=1}^{v} \frac{(\vartheta_\varpi(1+S(\chi_\varpi))\chi_\varpi)}{\sum_{\varpi=1}^{v}(\vartheta_\varpi(1+S(\chi_\varpi)))}$
$= \left\langle \sum \left(1 - \frac{1}{1+\{\sum_{\varpi=1}^{v} \frac{(\vartheta_\varpi(1+S(\chi_\varpi)))}{\sum_{\varpi=1}^{v}(\vartheta_\varpi(1+S(\chi_\varpi)))}(\frac{\sigma_{1\varpi}}{1-\sigma_{1\varpi}})^\partial\}^{\frac{1}{\partial}}} \right) \right\rangle$
$= \left\langle \sum \left(1 - \frac{1}{1+\{\sum_{\varpi=1}^{v} \frac{(\vartheta_\varpi(1+S(\chi_\varpi)))}{\sum_{\varpi=1}^{v}(\vartheta_\varpi(1+S(\chi_\varpi)))}(\frac{\sigma_1}{1-\sigma_1})^\partial\}^{\frac{1}{\partial}}} \right) \right\rangle$
$= \left\langle \sum \left(1 - \frac{1}{1+\{(\frac{\sigma_1}{1-\sigma_1})^\partial\}^{\frac{1}{\partial}}} \right) \right\rangle$
$= \langle \sum \sigma_1 \rangle = \chi.$

Theorem 4.4. *Let χ_ϖ, $\varpi = 1, 2, \ldots, v$, be the collections. Let $A = \langle \sum \sigma_1^A \rangle = min(\chi_1, \chi_2, \ldots, \chi_v)$, and $B = \langle \sum \sigma_1^B \rangle = max(\chi_1, \chi_2, \ldots, \chi_v)$ then $A \leq mPFDPWA_\vartheta(\chi_1, \chi_2, \ldots, \chi_v) \leq B$.*

Proof: Consider $\sigma_1^A = min_\varpi\{\sigma_{1\varpi}\}$, and $\sigma_1^B = max_\varpi\{\sigma_{1\varpi}\}$, $\varpi = 1, 2, \ldots, v$.
Then we have,

$$\begin{aligned}
&1 - \frac{1}{1 + \{\sum_{\varpi=1}^{v} \frac{(\vartheta_\varpi(1+S(\chi_\varpi)))}{\sum_{\varpi=1}^{v}(\vartheta_\varpi(1+S(\chi_\varpi)))}(\frac{\sigma_1^A}{1-\sigma_1^A})^\partial\}^{\frac{1}{\partial}}} \\
&\leq 1 - \frac{1}{1 + \{\sum_{\varpi=1}^{v} \frac{(\vartheta_\varpi(1+S(\chi_\varpi)))}{\sum_{\varpi=1}^{v}(\vartheta_\varpi(1+S(\chi_\varpi)))}(\frac{\sigma_{1\varpi}}{1-\sigma_{1\varpi}})^\partial\}^{\frac{1}{\partial}}} \\
&\leq 1 - \frac{1}{1 + \{\sum_{\varpi=1}^{v} \frac{(\vartheta_\varpi(1+S(\chi_\varpi)))}{\sum_{\varpi=1}^{v}(\vartheta_\varpi(1+S(\chi_\varpi)))}(\frac{\sigma_1^B}{1-\sigma_1^B})^\partial\}^{\frac{1}{\partial}}},
\end{aligned}$$

Similar inequalities hold for m components of mPFN and this implies that

$$A \leq mPFDPWA_\vartheta(\chi_1, \chi_2, \ldots, \chi_v) \leq B.$$

Theorem 4.5. *Consider the two collections of mPFNs be $\chi_\varpi = \langle \sum \sigma_{1\varpi} \rangle$, and $\chi'_\varpi = \langle \sum \sigma'_{1\varpi} \rangle$, $\varpi = 1, 2, \ldots, v$. If $\chi_\varpi \leq \chi'_\varpi, \forall \varpi$, then*

$$mPFDPWA_\vartheta(\chi_1, \chi_2, \ldots, \chi_v) \leq mPFDPWA_\vartheta(\chi'_1, \chi'_2, \ldots, \chi'_v). \tag{13}$$

Definition 4.2. *The mPF Dombi power ordered weighted avaraging(mPFDPOWA) operator for v number of mPFNs $\chi_\varpi = \langle \sum \sigma_{1\varpi} \rangle$, $\varpi = 1, 2, \ldots, v$, is defined by*

$$mPFDPOWA_\vartheta(\chi_1, \chi_2, \ldots, \chi_v) = \bigoplus_{\varpi=1}^{v} \frac{(\vartheta_\varpi(1 + S(\chi_{\varrho(\varpi)}))\chi_{\varrho(\varpi)})}{\sum_{\varpi=1}^{v}(\vartheta_\varpi(1 + S(\chi_{\varrho(\varpi)})))}$$

$$= \left\langle \sum \left(1 - \frac{1}{1 + \{\sum_{\varpi=1}^{v} \frac{(\vartheta_\varpi(1+S(\chi_{\varrho(\varpi)})))}{\sum_{\varpi=1}^{v}(\vartheta_\varpi(1+S(\chi_{\varrho(\varpi)})))}(\frac{\sigma_{1\varrho(\varpi)}}{1-\sigma_{1\varrho(\varpi)}})^\partial\}^{\frac{1}{\partial}}} \right) \right\rangle \tag{14}$$

where $(\varrho(1), \varrho(2), \ldots, \varrho(v))$ is a permutation of $(\varpi = 1, 2, \ldots, v)$, such that $\chi_{\varrho(\varpi-1)} \geq \chi_{\varrho(\varpi)}$, $\forall \varpi = 1, 2, \ldots, v$.

By the help of the above mPFDPOWA operator we can easily prove the subsequent theorems.

Theorem 4.6. *If the collections of mPFNs χ_{ϖ}, $\varpi = 1, 2, \ldots, v$ are identical to a mPFN χ then $mPFDPOWA_{\vartheta}(\chi_1, \chi_2, \ldots, \chi_v) = \chi$.*

Theorem 4.7. *Let χ_{ϖ}, $\varpi = 1, 2, \ldots, v$, be the collections. If $A = min_{\varpi}(\chi_{\varpi})$, and $B = max_{\varpi}(\chi_{\varpi})$ then $A \leq mPFDPOWA_{\vartheta}(\chi_1, \chi_2, \ldots, \chi_v) \leq B$.*

Theorem 4.8. *If the two collections of mPFNs be ordered i.e. if $\chi_{\varpi} \leq \chi'_{\varpi}, \forall \varpi = 1, 2, \ldots, v$, then their ordered aggregation operator could be ordered i.e.,*

$$mPFDPOWA_{\vartheta}(\chi_1, \chi_2, \ldots, \chi_v) \leq mPFDPOWA_{\vartheta}(\chi'_1, \chi'_2, \ldots, \chi'_v). \tag{15}$$

Theorem 4.9. *Consider two collections χ_{ϖ_p}, and $\chi_{\varsigma_p}, \forall p = 1, 2, \ldots, v$. If $\varsigma_p, \forall p$ is any permutation of ϖ_p then,*

$$mPFDPOWA_{\vartheta}(\chi_{\varpi_1}, \chi_{\varpi_2}, \ldots, \chi_{\varpi_v}) = mPFDPOWA_{\vartheta}(\chi_{\varsigma_1}, \chi_{\varsigma_2}, \ldots, \chi_{\varsigma_v}). \tag{16}$$

4.2. **Geometric aggregation operators.**

Definition 4.3. *Let $\chi_{\varpi} = \langle \sum \sigma_{1\varpi} \rangle$, $\varpi = 1, 2, \ldots, v$, be the collection of mPFNs. Then mPFDPG operator is a mapping defined by*

$$mPFDPG(\chi_1, \chi_2, \ldots, \chi_v) = \bigotimes_{\varpi=1}^{v} (\chi_{\varpi})^{\frac{(1+S(\chi_{\varpi}))}{\sum_{\varpi=1}^{v}(1+S(\chi_{\varpi}))}}, \tag{17}$$

where, $S(\chi_{\varpi}) = \sum_{\varpi=1, \varpi \neq \varsigma}^{v} Spt(\chi_{\varpi}, \chi_{\varsigma})$, and $Spt(\chi_{\varpi}, \chi_{\varsigma})$ represents the support of χ_{ϖ} along with χ_{ς}.

Now we develop some theories on the basis of the above combined geometric Dombi power AO and we can easily proof as the above theorems for mPFDPWA operator.

Theorem 4.10. *Let χ_{ϖ}, $\varpi = 1, 2, \ldots, v$ be the collections. Then using mPFDPWG operator, the aggregated value of collections is also an mPFN.*
Mathematically, $mPFDPWG_{\vartheta}(\chi_1, \chi_2, \ldots, \chi_v) = \bigotimes_{\varpi=1}^{v} (\chi_{\varpi})^{\frac{(1+S(\chi_{\varpi}))}{\sum_{\varpi=1}^{v}(1+S(\chi_{\varpi}))}}$

$$= \left\langle \sum \left(\frac{1}{1 + \{\sum_{\varpi=1}^{v} \frac{(1+S(\chi_{\varpi}))}{\sum_{\varpi=1}^{v}(1+S(\chi_{\varpi}))} (\frac{1-\sigma_{1\varpi}}{\sigma_{1\varpi}})^{\vartheta}\}^{\frac{1}{\vartheta}}} \right) \right\rangle \tag{18}$$

Theorem 4.11. *If all the collections of mPFNs χ_{ϖ}, $\varpi = 1, 2, \ldots, v$ are similar to another mPFN χ then $mPFDPWG_{\vartheta}(\chi_1, \chi_2, \ldots, \chi_v) = \chi$.*

Theorem 4.12. *Let χ_{ϖ}, $\varpi = 1, 2, \ldots, v$, be the collections. If $C = min_{\varpi}(\chi_{\varpi})$, and $D = max_{\varpi}(\chi_{\varpi})$, we have, $C \leq mPFDPWG_{\vartheta}(\chi_1, \chi_2, \ldots, \chi_v) \leq D$.*

Theorem 4.13. *If the two collections of mPFNs be ordered i.e. if $\chi_{\varpi} \leq \chi'_{\varpi}, \forall \varpi = 1, 2, \ldots, v$, then their ordered aggregation operator could be ordered i.e.,*

$$mPFDPWG_{\vartheta}(\chi_1, \chi_2, \ldots, \chi_v) \leq mPFDPWG_{\vartheta}(\chi'_1, \chi'_2, \ldots, \chi'_v). \tag{19}$$

5. Novel approach for MADM using mPFS

Here to solve the MADM problem we provide a novel approach according the above proposed combined Dombi power aggregation operator using mPF information and described a very nice numerical explanation concerning best company selection for buying and selling shares of publicly listed companies at the stock market.

Let $\mathbf{L} = (L_1, L_2, \ldots, L_u)$ represent a finite set of alternative and $\beta = (\beta_1, \beta_2, \ldots, \beta_v)$ denote a finite set of attributes. For an mPFS, there are one or more than one attribute and every attribute contains some sub-criteria or sub-attributes, here for novelty we have utilized heterogeneous sub-attributes collection that means here the number of sub-attributes in each attribute may not be equal. Also each sub-attribute contains the weight, let $\Omega = (\Omega_1, \Omega_2, \ldots, \Omega_m)$ be the weight for the sub-attributes in each attribute such that every $\Omega_x \in [0, 1]$ and $\sum_{x=1}^{m} \Omega_x = 1$, the weight vector Ω is distinct for each main attribute. Also $\vartheta = (\vartheta_1, \vartheta_2, \ldots, \vartheta_v)$ is the weight vector for the main attribute satisfies every $\vartheta_y \in [0, 1]$ and $\sum_{y=1}^{v} \vartheta_y = 1$. Let $\tilde{M} = (a_{ij})_{u\times v}$ be the mPF decision matrix considered by the decision-makers, where $a_{ij} = \langle \sigma_{ij1}, \sigma_{ij2}, \ldots, \sigma_{ijm} \rangle$ is an mPFN at which the alternate L_i satisfy the attribute β_j, $i = 1, 2 \ldots, u$ and $j = 1, 2, \ldots, v$.

Algorithm.
Step 1. Identify the alternatives and criteria and construct an mPF decision-matrix $\tilde{M} = (a_{ij})_{u\times v}$ in such a way that for benefit type sub-attribute consider the points for the alternatives at increasing order, i.e. the best alternate gets highest point and for cost type attribute consider the points for alternatives at decreasing order, i.e. the best alternate gets lowest point, so that there are no need to normalize the decision-matrix, which is displayed in Table 1.
Step 2. Consider every mPFN a_{ij} is the collection of 1-polar fuzzy number(1PFN) then proceed to compute normalized Hamming distance $d(\sigma_{ijx}, \sigma_{ijy})$ for every a_{ij}, where

$$d(\sigma_{ijx}, \sigma_{ijy}) = |\sigma_{ijx} - \sigma_{ijy}|,\ x, y = 1, \ldots, m,\ x \neq y; \tag{20}$$

then compute the weighted support $S(\sigma_{ijx})$ for every component of the element a_{ij},

$$S(\sigma_{ijx}) = \sum_{y=1, y\neq x}^{m} \Omega_x Spt(\sigma_{ijx}, \sigma_{ijy}), \forall x = 1, 2, \ldots, m; \tag{21}$$

$$Spt(\sigma_{ijx}, \sigma_{ijy}) = 1 - d(\sigma_{ijx}, \sigma_{ijy}); \tag{22}$$

where $Spt(\sigma_{ijx}, \sigma_{ijy})$ represents the support of σ_{ijx} along with σ_{ijy} described in the above Definition 3.3.
Step 3. Assuming every mPFN a_{ij} is the collection of 1PFN, utilize the equation (9) i.e. use mPFDPWA operator for every mPFN a_{ij} to reduce into a FN and hence create 1-PF decision-matrix $\tilde{B} = (b_{ij})_{u\times v}$ i.e. for every $i = 1, \ldots, u$, $j = 1, \ldots, v$ we have,

$$b_{ij} = mPFDPWA(\sigma_{ij1}, \sigma_{ij2}, \ldots, \sigma_{ijm}) = \bigoplus_{x=1}^{m} \frac{(\Omega_x(1 + S(\sigma_{ijx}))\sigma_{ijx})}{\sum_{x=1}^{m}(\Omega_x(1 + S(\sigma_{ijx})))}$$

$$= \left\langle 1 - \frac{1}{1 + \{\sum_{x=1}^{m} \frac{(\Omega_x(1+S(\sigma_{ijx})))}{\sum_{x=1}^{m}(\Omega_x(1+S(\sigma_{ijx})))}(\frac{\sigma_{ijx}}{1-\sigma_{ijx}})^{\partial}\}^{\frac{1}{\partial}}} \right\rangle, \tag{23}$$

or, use mPFDPWG operator in that case,

$$b_{ij} = mPFDPWG(\sigma_{ij1}, \sigma_{ij2}, \ldots, \sigma_{ijm}) = \bigotimes_{x=1}^{m} \frac{(\Omega_x(1 + S(\sigma_{ijx}))\sigma_{ijx})}{\sum_{x=1}^{m}(\Omega_x(1 + S(\sigma_{ijx})))}$$

$$= \left\langle \frac{1}{1 + \{\sum_{x=1}^{m} \frac{(\Omega_x(1+S(\sigma_{ijx})))}{\sum_{x=1}^{m}(\Omega_x(1+S(\sigma_{ijx})))}(\frac{1-\sigma_{ijx}}{\sigma_{ijx}})^{\partial}\}^{\frac{1}{\partial}}} \right\rangle. \tag{24}$$

Step 4. Again use mPFDPWA operator on the 1-PF decision matrix $\tilde{B}$ for each alternate L_i with the weight vector ϑ we have, for $i = 1, \ldots, v$,

$$\Gamma_i = mPFDPWA(b_{i1}, b_{i2}, \ldots, b_{iv}) = \left\langle 1 - \frac{1}{1 + \{\sum_{x=1}^{v} \frac{(\vartheta_x(1+S(b_{ix})))}{\sum_{x=1}^{v}(\vartheta_x(1+S(b_{ix})))} (\frac{b_{ix}}{1-b_{ix}})^{\partial}\}^{\frac{1}{\partial}}} \right\rangle \tag{25}$$

For mPFDPWG operator we have,

$$\Gamma_i = mPFDPWG(b_{i1}, b_{i2}, \ldots, b_{iv}) = \left\langle \frac{1}{1 + \{\sum_{x=1}^{v} \frac{(\vartheta_x(1+S(b_{ix})))}{\sum_{x=1}^{v}(\vartheta_x(1+S(b_{ix})))} (\frac{1-b_{ix}}{1-b_{ix}})^{\partial}\}^{\frac{1}{\partial}}} \right\rangle \tag{26}$$

where,

$$S(b_{ix}) = \sum_{y=1, y \neq x}^{v} \vartheta_x Spt(b_{ix}, b_{iy}), \forall x = 1, 2, \ldots, v; \tag{27}$$

$Spt(b_{ix}, b_{iy})$ represents the support of b_{ix} along with b_{iy} described in the above Definition 3.3.

Step 5. According to the values of 1PFN Γ_i, rank all the corresponding alternative L_i, $i = 1, \ldots, u$, therefore the choice having the highest value is selected as the best performing alternative.

Step 6.

End.

The time complexity of the above algorithm is calculated below:

Step 1 takes (uv) times. The eq. (20), (21) and (22) need $O(uvm)$ times. Similarly, step 3 takes (uvm) times. But, eq. (25) and (26) can be computed only using (v^2) times. The last step 5 can be computed in (u).

Finally, the time complexity of the entire algorithm is $O(uvm + v^2)$.

Flowchart of the above algorithm is shown below.

6. Numerical application

The stock market, also known as the share market, is vital to the global economy. The stock market facilitates the exchange of ownership in companies through shares. Investors participate in this market to grow their wealth, hedge against inflation, and fund corporate expansion. Understanding the stock market is essential for both individual investors and policymakers. It serves as a platform for buying and selling shares of publicly listed companies. Companies go public by issuing shares through an initial public offering(IPO). Stock exchanges like the New York stock Exchange(NYSE) and NASDAQ offer a platform for trading shares. Buyers and sellers interact through brokers or electronic trading systems.

Table 2. Fuzzy decision matrix with heterogeneous sub-characteristics.

	β_1	β_2	β_3	β_4	β_5
L_1	$\langle 0.7, 0.62, 0.5.0.57\rangle$	$\langle 0.9, 0.6, 0.8, 0.7, 0.52\rangle$	$\langle 0.8, 0.8, 0.6, 0.5\rangle$	$\langle 0.9, 0.3, 0.9\rangle$	$\langle 0.04, 0.03, 0.01, 0.02, 0.01\rangle$
L_2	$\langle 0.4, 0.56, 0.6, 0.73\rangle$	$\langle 0.5, 0.62, 0.2, 0.5, 0.4\rangle$	$\langle 0.5, 0.6, 0.3, 0.53\rangle$	$\langle 0.8, 0.3, 0.5\rangle$	$\langle 0.03, 0.02, 0.01, 0.01, 0.01\rangle$
L_3	$\langle 0.3, 0.66, 0.4, 0.63\rangle$	$\langle 0.8, 0.69, 0.6, 0.6, 0.51\rangle$	$\langle 0.8, 0.6, 0.7, 0.66\rangle$	$\langle 0.78, 0.35, 0.1\rangle$	$\langle 0.02, 0.01, 0.01, 0.01, 0.01\rangle$
L_4	$\langle 0.6, 0.42, 0.5, 0.9\rangle$	$\langle 0.6, 0.3, 0.2, 0.55, 0.45\rangle$	$\langle 0.3, 0.4, 0.1, 0.4\rangle$	$\langle 0.7, 0.4, 0.1\rangle$	$\langle 0.02, 0.01, 0.01, 0.01, 0.01\rangle$
L_5	$\langle 0.3, 0.68, 0.55, 0.74\rangle$	$\langle 0.8, 0.68, 0.3, 0.61, 0.5\rangle$	$\langle 0.7, 0.62, 0.62, 0.6\rangle$	$\langle 0.67, 0.2, 0.5\rangle$	$\langle 0.02, 0.01, 0.01, 0.01, 0.01\rangle$
L_6	$\langle 0.3, 0.72, 0.45, 0.62\rangle$	$\langle 0.6, 0.65, 0.4, 0.53, 0.53\rangle$	$\langle 0.6, 0.58, 0.61, 0.5\rangle$	$\langle 0.3, 0.2, 0.56\rangle$	$\langle 0.02, 0.01, 0.01, 0.01, 0.01\rangle$
L_7	$\langle 0.25, 0.6, 0.3, 0.73\rangle$	$\langle 0.6, 0.7, 0.35, 0.5, 0.45\rangle$	$\langle 0.61, 0.6, 0.6, 0.5\rangle$	$\langle 0.2, 0.2, 0.55\rangle$	$\langle 0.02, 0.01, 0.01, 0.01, 0.01\rangle$

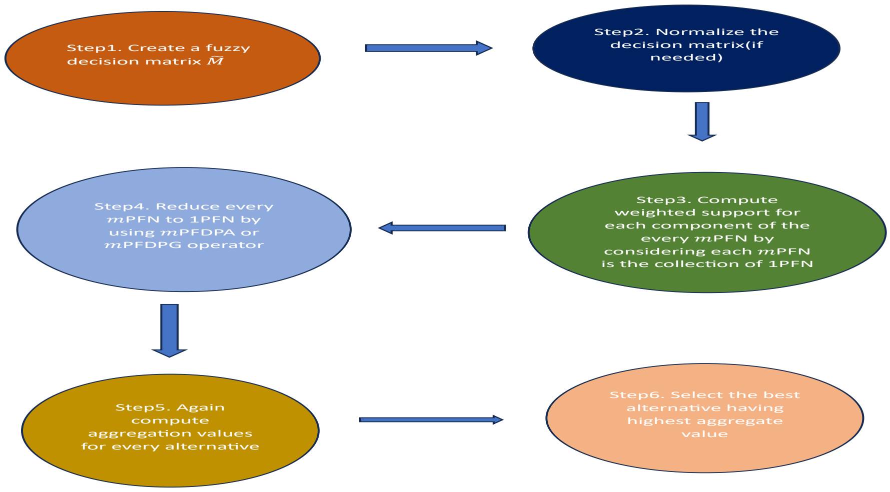
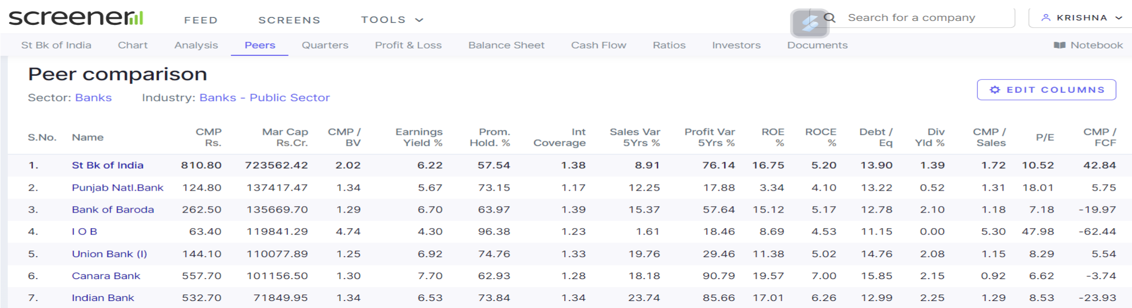


FIGURE 1. Flowchart of the approach



Peer comparison

Sector: Banks Industry: Banks - Public Sector

S.No.	Name	CMP Rs.	Mar Cap Rs.Cr.	CMP / BV	Earnings Yield %	Prom. Hold. %	Int Coverage	Sales Var 5Yrs %	Profit Var 5Yrs %	ROE %	ROCE %	Debt / Eq	Div Yld %	CMP / Sales	P/E	CMP / FCF
1.	St Bk of India	810.80	723562.42	2.02	6.22	57.54	1.38	8.91	76.14	16.75	5.20	13.90	1.39	1.72	10.52	42.84
2.	Punjab Natl.Bank	124.80	137417.47	1.34	5.67	73.15	1.17	12.25	17.88	3.34	4.10	13.22	0.52	1.31	18.01	5.75
3.	Bank of Baroda	262.50	135669.70	1.29	6.70	63.97	1.39	15.37	57.64	15.12	5.17	12.78	2.10	1.18	7.18	-19.97
4.	I O B	63.40	119841.29	4.74	4.30	96.38	1.23	1.61	18.46	8.69	4.53	11.15	0.00	5.30	47.98	-62.44
5.	Union Bank (I)	144.10	110077.89	1.25	6.92	74.76	1.33	19.76	29.46	11.38	5.02	14.76	2.08	1.15	8.29	5.54
6.	Canara Bank	557.70	101156.50	1.30	7.70	62.93	1.28	18.18	90.79	19.57	7.00	15.85	2.15	0.92	6.62	-3.74
7.	Indian Bank	532.70	71849.95	1.34	6.53	73.84	1.34	23.74	85.66	17.01	6.26	12.99	2.25	1.29	8.53	-23.93

FIGURE 2. Several data for seven Indian Banks

When selecting a share of a company to invest in the stock market, especially aiming for a company considered one of the best, there are some criteria you should think. Here are some key factors to watch:

Revenue and Earnings growth: Look for consistent growth in earnings and revenue over time. These are the benefit factors for a company.

Profitability: Check the company's profit margins and return on equity (ROE), return on capital employed(ROCE) to assess how efficiently it generates profits. So, these are the benefit factors.

Debt levels: Ensure the company has manageable levels of debt. A high debt-to-equity ratio can be risky. So this is the cost factor for a company.

Cash flow: Evaluate the operating cash flow of the company to see if it consistently generates enough cash to support its operations. So, the factor price to free cash flow(CMP/FCF) is the benefit factor.

Price to earnings ratio (P/E): Compare P/E ratio of a company to its historical average and industry peers to evaluate if it's undervalued or overvalued. This is the cost factor.
Price-to-book value ratio (CMP/BV): Consider the CMP/BV ratio to evaluate if the stock price is reasonable relative to the book value of a company. This is the cost factor.
Dividend history: If you are interested in dividends, examine the company's dividend yield, payout ratio, and potential for future dividend growth. These are the benefit factors.
Understand risks: Identify and understand credit risk, market risk, regulatory risk, liquidity risk, geopolitical risks associated with the company's business model, industry, or economic factors.
However, to choose the best one for buying shares in seven companies, we consider seven companies $\{L_1 =$ St. Bank of India(SBI), $L_2 =$Punjab Nati. Bank(PNB), $L_3 =$ Bank of Baroda(BOB), $L_4 =$ Union Bank(UB), $L_5 =$ Indian Overseas Bank(IOB), $L_6 =$ Canara Bank(CB), $L_7 =$ Indian Bank(IB)$\}$ as the set of alternatives. Our choice is according to the some criteria, the criteria are the attributes under consideration. Here for calculation we chose five attributes as $\{\beta_1 =$ Company fundamentals, $\beta_2 =$ Macroeconomic factors, $\beta_3 =$ Valuation metrics, $\beta_4 =$ Financial health, $\beta_5 =$ Risk$\}$. Now to formulate fuzzy decision-matrix $\tilde{M} = (a_{ij})_{7\times 5} = (\langle \sigma_{ij1}, \sigma_{ij2}, \dots, \sigma_{ijm}\rangle)_{7\times 5}$, in which every every elements of the 1st, 2nd,3rd, 4-th and 5-th columns are respectively 4PFN, 5PFN, 4PFN, 3PFN and 5PFN. For this case we consider as follows:

- The "Company fundamental" includes four sub-criteria as revenue growth, earnings yield ratio, net profit, promoter holding and suppose $\Omega = \{\Omega_1 = 0.25, \Omega_2 = 0.25, \Omega_3 = 0.25, \Omega_4 = 0.25\}$ be the weight vector for these corresponding sub-criteria.
- The "Macroeconomic factors" includes five sub-criteria as interest coverage ratio, sales growth 5 years, profit growth 5 years, return on equity(ROE), return on capital employed(ROCE) and in this case chose corresponding weight $\Omega = \{\Omega_1 = 0.2, \Omega_2 = 0.2, \Omega_3 = 0.2, \Omega_4 = 0.2, \Omega_5 = 0.2\}$.
- The "Valuation metrics" includes four sub-criteria as price to earning(P/E) ratio, price to book value(P/B) ratio, dividend yield, price to sales(CMP/Sales) ratio and corresponding weight vector is $\Omega = \{\Omega_1 = 0.2, \Omega_2 = 0.2, \Omega_3 = 0.4, \Omega_4 = 0.2\}$.
- The "Financial health" includes three sub-criteria as market capitalization, debt to equity, price to free cash flow(CMP/FCF) and the corresponding weight vector is $\Omega = \{\Omega_1 = 0.4, \Omega_2 = 0.2, \Omega_3 = 0.4\}$.
- The "Risk" factor includes five sub-criteria as credit risk, market risk, regulatory risk, liquidity risk, geopolitical risk and in this factor $\Omega = \{\Omega_1 = 0.2, \Omega_2 = 0.2, \Omega_3 = 0.2, \Omega_4 = 0.2, \Omega_5 = 0.2\}$ is the corresponding weight.

Now we follow the above algorithm.
Step 1. The decision-matrix $\tilde{A}$ (shown in Table 2) is formulated from the Picture 1(picture is collected from website) according to the above process that described in step 1 of algorithm so that there are no need to normalize the decision matrix.
Step 2. Using the equations (21), (22), (23) we compute the weighted support for every component of the element a_{ij} of the decision matrix.
For the element $a_{11} = \langle \sigma_{111}, \sigma_{112}, \sigma_{113}, \sigma_{114}\rangle$, we already chose the weight $\Omega = \{\Omega_1 = 0.25, \Omega_2 = 0.25, \Omega_3 = 0.25, \Omega_4 = 0.25\}$ and this weight is chosen for computing weighted support for all elements of the 1st column.

TABLE 3. $\tilde{B}$(Aggregated values for $\partial = 2$ using mPFDPWA operator)

	β_1	β_2	β_3	β_4	β_5
L_1	$\langle 0.6220\rangle$	$\langle 0.8202\rangle$	$\langle 0.7244\rangle$	$\langle 0.8921\rangle$	$\langle 0.0251\rangle$
L_2	$\langle 0.6297\rangle$	$\langle 0.5046\rangle$	$\langle 0.4872\rangle$	$\langle 0.7235\rangle$	$\langle 0.0180\rangle$
L_3	$\langle 0.5751\rangle$	$\langle 0.6957\rangle$	$\langle 0.7168\rangle$	$\langle 0.6920\rangle$	$\langle 0.0127\rangle$
L_4	$\langle 0.8168\rangle$	$\langle 0.4916\rangle$	$\langle 0.3098\rangle$	$\langle 0.6033\rangle$	$\langle 0.0127\rangle$
L_5	$\langle 0.6555\rangle$	$\langle 0.6867\rangle$	$\langle 0.6366\rangle$	$\langle 0.5929\rangle$	$\langle 0.0127\rangle$
L_6	$\langle 0.6139\rangle$	$\langle 0.5688\rangle$	$\langle 0.5899\rangle$	$\langle 0.4625\rangle$	$\langle 0.0127\rangle$
L_7	$\langle 0.6088\rangle$	$\langle 0.5803\rangle$	$\langle 0.5900\rangle$	$\langle 0.4422\rangle$	$\langle 0.0127\rangle$

TABLE 4. Aggregated values for $\partial = 2$ using mPFDPWG operator

	β_1	β_2	β_3	β_4	β_5
L_1	$\langle 0.5783\rangle$	$\langle 0.6424\rangle$	$\langle 0.6111\rangle$	$\langle 0.5183\rangle$	$\langle 0.0143\rangle$
L_2	$\langle 0.5200\rangle$	$\langle 0.3344\rangle$	$\langle 0.3707\rangle$	$\langle 0.4598\rangle$	$\langle 0.0122\rangle$
L_3	$\langle 0.4110\rangle$	$\langle 0.6079\rangle$	$\langle 0.6795\rangle$	$\langle 0.1454\rangle$	$\langle 0.0108\rangle$
L_4	$\langle 0.5197\rangle$	$\langle 0.3152\rangle$	$\langle 0.1389\rangle$	$\langle 0.1471\rangle$	$\langle 0.0108\rangle$
L_5	$\langle 0.4458\rangle$	$\langle 0.4603\rangle$	$\langle 0.6267\rangle$	$\langle 0.3599\rangle$	$\langle 0.0108\rangle$
L_6	$\langle 0.4245\rangle$	$\langle 0.5127\rangle$	$\langle 0.5771\rangle$	$\langle 0.3030\rangle$	$\langle 0.0108\rangle$
L_7	$\langle 0.3398\rangle$	$\langle 0.4664\rangle$	$\langle 0.5780\rangle$	$\langle 0.2408\rangle$	$\langle 0.0108\rangle$

For example, $S(\sigma_{111}) = \Omega_1(Spt(\sigma_{111}, \sigma_{112}) + Spt(\sigma_{111}, \sigma_{113}) + Spt(\sigma_{111}, \sigma_{114})) = 0.6475$, $S(\sigma_{112}) = \Omega_2(Spt(\sigma_{112}, \sigma_{111}) + Spt(\sigma_{112}, \sigma_{113}) + Spt(\sigma_{112}, \sigma_{114})) = 0.6875$, $S(\sigma_{113}) = 0.6525$, $S(\sigma_{114}) = 0.6875$; Similarly all other weighted supports are computed.
Observe that, the weight vectors for the 2nd, 3rd, 4th, and 5th columns are respectively $\Omega = \{\Omega_1 = 0.2, \Omega_2 = 0.2, \Omega_3 = 0.2, \Omega_4 = 0.2, \Omega_5 = 0.2\}$, $\Omega = \{\Omega_1 = 0.2, \Omega_2 = 0.2, \Omega_3 = 0.4, \Omega_4 = 0.2\}$, $\Omega = \{\Omega_1 = 0.4, \Omega_2 = 0.2, \Omega_3 = 0.4\}$, $\Omega = \{\Omega_1 = 0.2, \Omega_2 = 0.2, \Omega_3 = 0.2, \Omega_4 = 0.2, \Omega_5 = 0.2\}$.

Step 3. Using the equation (23) i.e. using mPFDPWA operator we have formulated 1PF decision-matrix $\tilde{B} = (b_{ij})_{7\times 5}$, $i = 1, 2, \dots, 7$, $j = 1, 2, \dots, 5$, that is presented in the Table 3.
Using the equation (24) i.e. using mPFDPWG operator also we get 1PF decision matrix $\tilde{B}$ that is displayed in the Table 4.

Step 4. Again use mPFDPWA operator on the 1PF decision matrix $\tilde{B}$ (given in Table 3) for every alternative L_i and using the equations (25), (27) we get, $\alpha(L_1) = \langle 0.8295\rangle$, $\alpha(L_2) = \langle 0.6256\rangle$, $\alpha(L_3) = \langle 0.6801\rangle$, $\alpha(L_4) = \langle 0.6981\rangle$, $\alpha(L_5) = \langle 0.6455\rangle$, $\alpha(L_6) = \langle 0.5679\rangle$, $\alpha(L_7) = \langle 0.5673\rangle$. Here, $\alpha(L_1)$ having the highest value.
For the geometric operator using the equations (26), (27) and from Table 4 we get, $\alpha(L_1) = \langle 0.0847\rangle$, $\alpha(L_2) = \langle 0.0721\rangle$, $\alpha(L_3) = \langle 0.0612\rangle$, $\alpha(L_4) = \langle 0.0608\rangle$, $\alpha(L_5) = \langle 0.0641\rangle$, $\alpha(L_6) = \langle 0.0639\rangle$, $\alpha(L_7) = \langle 0.0632\rangle$. In this case, $\alpha(L_1)$ having the highest value.
Step 5. Based on the values of α we order all the alternatives L_i, $i = 1, 2, \dots, 7$ as $L_1 > L_4 > L_3 > L_5 > L_2 > L_6 > L_7$ for mPFDPWA operator and $L_1 > L_2 > L_5 > L_6 > L_7 > L_3 > L_4$ for mPFDPWG operator.
In both cases the best alternative for this approach is L_1.

Next we compare the above result with MABAC approach.

TABLE 5. The values of ξ_{ij}

	β_1	β_2	β_3	β_4	β_5
L_1	$\langle .2468, .2528, .2476, .2528\rangle$	$\langle .1932, .2005, .2005, .2029, .2029\rangle$	$\langle .1773, .1773, .4728, .1726\rangle$	$\langle .4216, .1567, .4216\rangle$	$\langle .1994, .20, .20, .2003, .2003\rangle$
L_2	$\langle .2432, .2556, .2556, .2456\rangle$	$\langle .2051, .1964, .1882, .2051, .2051\rangle$	$\langle .1828, .1795, .4548, .1828\rangle$	$\langle .3989, .1698, .4313\rangle$	$\langle .1993, .20, .2002, .2002, .2002\rangle$
L_3	$\langle .2447, .2502, .2526, .2526\rangle$	$\langle .1925, .2003, .2024, .2024, .2024\rangle$	$\langle .1718, .1736, .4782, .1764\rangle$	$\langle .3969, .1850, .4180\rangle$	$\langle .1995, .2001, .2001, .2001, .2001\rangle$
L_4	$\langle .2576, .2512, .2576, .2335\rangle$	$\langle .1990, .2015, .1941, .2027, .2027\rangle$	$\langle .1827, .1827, .4519, .1827\rangle$	$\langle .4091, .1818, .4091\rangle$	$\langle .1995, .2001, .2001, .2001, .2001\rangle$
L_5	$\langle .2362, .2562, .2562, .2514\rangle$	$\langle .1953, .2042, .1884, .2060, .2060\rangle$	$\langle .1716, .1752, .4789, .1743\rangle$	$\langle .4086, .1648, .4226\rangle$	$\langle .1995, .2001, .2001, .2001, .2001\rangle$
L_6	$\langle .2430, .2470, .2550, .2550\rangle$	$\langle .2015, .1980, .1940, .2032, .2032\rangle$	$\langle .1755, .1755, .4771, .1719\rangle$	$\langle .4288, .1693, .4019\rangle$	$\langle .1995, .2001, .2001, .2001, .2001\rangle$
L_7	$\langle .2496, .2537, .2537, .2431\rangle$	$\langle .2017, .1945, .1957, .2041, .2041\rangle$	$\langle .1748, .1753, .4790, .1708\rangle$	$\langle .4317, .1729, .3953\rangle$	$\langle .1995, .2001, .2001, .2001, .2001\rangle$

7. JUSTIFICATION BY MABAC APPROACH

Now, we justify the previous result of the mPFDPWA operator using MABAC approach. For this, we apply MABAC approach on the same problem, i.e. the data described in the Table 2. The following steps are as follows:

TABLE 6. The values of ξ_{ij}

	β_1	β_2	β_3	β_4	β_5
L_1	$\langle .2571, .2170, .1577, .1921\rangle$	$\langle .3592, .1678, .2758, .2167, .1383\rangle$	$\langle .2482, .2482, .3516, .1127\rangle$	$\langle .6212, .0544, .6212\rangle$	$\langle .0081, .0061, .002, .004, .002\rangle$
L_2	$\langle .1168, .1892, .2088, .2750\rangle$	$\langle .1325, .1731, .0411, .1325, .0995\rangle$	$\langle .1190, .1517, .1497, .1289\rangle$	$\langle .4738, .0588, .2584\rangle$	$\langle .0061, .004, .002, .002, .002\rangle$
L_3	$\langle .0836, .2365, .1210, .2221\rangle$	$\langle .2664, .2091, .1693, .1693, .1344\rangle$	$\langle .2416, .1471, .4377, .1732\rangle$	$\langle .4518, .0766, .0431\rangle$	$\langle .004, .002, .002, .002, .002\rangle$
L_4	$\langle .2103, .1279, .1635, .4159\rangle$	$\langle .1667, .0693, .0424, .1494, .1141\rangle$	$\langle .0631, .0891, .0465, .0891\rangle$	$\langle .3889, .0887, .0422\rangle$	$\langle .004, .002, .002, .002, .002\rangle$
L_5	$\langle .0808, .2531, .1850, .2873\rangle$	$\langle .2698, .2076, .0650, .1763, .1330\rangle$	$\langle .1867, .1559, .3709, .1476\rangle$	$\langle .3643, .0361, .2560\rangle$	$\langle .004, .002, .002, .002, .002\rangle$
L_6	$\langle .0830, .2698, .1414, .2186\rangle$	$\langle .1686, .1877, .0944, .1422, .1422\rangle$	$\langle .1485, .1412, .3619, .1123\rangle$	$\langle .1418, .0371, .2810\rangle$	$\langle .004, .002, .002, .002, .002\rangle$
L_7	$\langle .0693, .2074, .0865, .2726\rangle$	$\langle .1687, .2087, .0808, .1319, .1148\rangle$	$\langle .1518, .1484, .3553, .1117\rangle$	$\langle .0918, .0378, .2707\rangle$	$\langle .004, .002, .002, .002, .002\rangle$

Step 1. We use the decision-matrix shown in Table 2.
Step 2. In this step normalization is required, for this case no need to normalize the decision-matrix.
Step 3. Next, the weighted normalization matrix $\Omega\tilde{M}$ (shown in Table 6) is formulated. For the normalized matrix $\tilde{M} = (a_{ij})_{7\times 5}$, $a_{ij} = \langle \sum \sigma_{ij1}\rangle$, $i = 1, 2, \ldots, 7$, $j = 1, 2, \ldots, 5$. We compute normalized fuzzy weighted matrix $\Omega\tilde{M} = (\Omega a_{ij})_{7\times 5}$ (shown in Table 6) by using the equation: $\Omega\tilde{M} = \Omega \oplus \tilde{M} = \left\langle \sum \sigma'_{ij1} \right\rangle = \left\langle \sum (1 - (1 - \sigma_{ij1}))^{\xi_{ij1}} \right\rangle$, where $\xi_{ijx} = \frac{\Omega_x(1+S(\sigma_{ijx}))}{\sum_{x=1}^{m}(\Omega_x(1+S(\sigma_{ijx})))}$, $\xi = (\xi_{ij})_{7\times 5} = (\langle \sum \xi_{ij1}\rangle)_{7\times 5}$ is shown in Table 5.
Notice that, the respectively weights $\Omega = \{\Omega_1 = 0.25, \Omega_2 = 0.25, \Omega_3 = 0.25, \Omega_4 = 0.25\}$ $\Omega = \{\Omega_1 = 0.2, \Omega_2 = 0.2, \Omega_3 = 0.2, \Omega_4 = 0.2, \Omega_5 = 0.2\}$, $\Omega = \{\Omega_1 = 0.2, \Omega_2 = 0.2, \Omega_3 = 0.4, \Omega_4 = 0.2\}$, $\Omega = \{\Omega_1 = 0.4, \Omega_2 = 0.2, \Omega_3 = 0.4\}$, $\Omega = \{\Omega_1 = 0.2, \Omega_2 = 0.2, \Omega_3 = 0.2, \Omega_4 = 0.2, \Omega_5 = 0.2\}$ for the 1st, 2nd, 3rd, 4th, and 5th columns have been utilized.
Step 4. In this step, we formulate the border approximation areas (BAA) matrix $N = [n_1\ n_2\ n_3\ n_4\ n_5]_{1\times 5}$, where $n_j = \left(\prod_{i=1}^{7} \Omega a_{ij}\right)^{1/7} = \langle \sigma''_1, \sigma''_2, \ldots, \sigma''_m\rangle$, $j = 1, 2, \ldots, 5$, *i.e.*, $n_1 = \left\langle \left(\prod_{i=1}^{7} \sigma_{i11}^{1/7}, \ldots, \prod_{i=1}^{7} \sigma_{i14}\right)^{1/7} \right\rangle = \langle 0.1137, 0.2093, 0.1469, 0.2615\rangle$, similarly $n_2 = \langle 0.2069, 0.1656, 0.0878, 0.1575, 0.1243\rangle$, $n_3 = \langle 0.1522, 0.1487, 0.2437, 0.1225\rangle$, $n_4 = \langle 0.3049, 0.0525, 0.1781\rangle$, $n_5 = \langle 0.0047, 0.0026, 0.0020, 0.0022, 0.0020\rangle$.
Step 5. Next the distance matrix $T = (t_{ij})_{7\times 5}$ is evaluated (shown in Table 7). The distance t_{ij} between BAA matrix and every alternative is calculated by the following condition:

$$t_{ij} = \begin{cases} d(\Omega a_{ij}, n_j) & \text{if } \Omega a_{ij} > n_j; \\ 0 & \text{if } \Omega a_{ij} = n_j; \\ -d(\Omega a_{ij}, n_j) & \text{if } \Omega a_{ij} < n_j. \end{cases} \tag{28}$$

where $d(\Omega a_{ij}, n_j)$ is the mean distance between Ωa_{ij} and n_j, i.e.,

$$d(\Omega a_{ij}, n_j) = \frac{1}{m}\left[|\sigma'_{ij1} - \sigma''_1| + \ldots + |\sigma'_{ijm} - \sigma''_m|\right]. \tag{29}$$

Step 6. Next, for each alternative the sums of the distances $V_i = \sum_{j=1}^{5} t_{ij}$, $i = 1, 2, \ldots, 7$ are evaluated. So, from Table 7 we get,
$V_1 = 0.5358$, $V_2 = 0.1502$, $V_3 = 0.1134$, $V_4 = -0.1769$, $V_5 = 0.1526$, $V_6 = -0.2128$, $V_7 = -0.2307$. From the magnificent evaluation of V_i we get the order list of alternatives as $L_1 > L_5 > L_2 > L_3 > L_4 > L_6 > L_7$. L_1 is the most desirable option.

8. Comparative analysis with merits and demerits

In this article, we compare our proposed approach, which utilizes Dombi and power aggregation operators, with existing four recent operators such as mPFDWA, mPFDWG[4],

TABLE 7. Distance between every alternative and BAA

	β_1	β_2	β_3	β_4	β_5
L_1	0.0578	0.0831	0.0783	0.1805	0.1361
L_2	0.0246	−0.0357	−0.0341	0.1318	0.0636
L_3	−0.0306	−0.0413	0.0839	0.1246	−0.0232
L_4	0.0872	−0.0400	−0.0948	−0.1038	−0.0255
L_5	0.0351	0.0310	0.0485	0.0957	−0.0577
L_6	−0.0349	−0.0200	0.0349	−0.1299	−0.0629
L_7	−0.0294	−0.0247	0.0308	−0.1464	−0.0610

TABLE 8. Final aggregated values of different operators and sum of distances in MABAC method

Different AO and method	L_1	L_2	L_3	L_4	L_5	L_6	L_7	Ranking orders
mPFDWA[4]	0.8277	0.6268	0.6799	0.7084	0.6437	0.5647	0.5657	$L_1 > L_4 > L_3 > L_5 > L_2 > L_7 > L_6$
mPFDWG[4]	0.0675	0.0579	0.0511	0.0505	0.05164	0.05160	0.0515	$L_1 > L_2 > L_5 > L_6 > L_7 > L_3 > L_4$
mPFEWA[14]	0.7114	0.5151	0.4907	0.4457	0.5719	0.5030	0.4796	$L_1 > L_5 > L_2 > L_6 > L_3 > L_7 > L_4$
mPFEWG[14]	0.6044	0.4372	0.4562	0.3202	0.4886	0.4333	0.3983	$L_1 > L_5 > L_3 > L_2 > L_6 > L_7 > L_4$
Proposed mPFDPWA	0.8295	0.6256	0.6801	0.6981	0.6455	0.5679	0.5673	$L_1 > L_4 > L_3 > L_5 > L_2 > L_6 > L_7$
Proposed mPFDPWA	0.0847	0.0721	0.0612	0.0608	0.0641	0.0639	0.0632	$L_1 > L_2 > L_5 > L_6 > L_7 > L_3 > L_4$
MABAC method[17]	0.5358	0.1502	0.1134	−0.1769	0.1526	−0.2128	−0.2307	$L_1 > L_5 > L_2 > L_3 > L_4 > L_6 > L_7$

mPFEWA[14] and mPFEWG[14] operators, as well as the MABAC approach. The final aggregation values and ranking orders of different approaches are given in Table 8. Here we observe that, for mPFDWA and mPFDPWA operators the ranking orders almost same; and mPFDWG and mPFDPWG operators show exactly same ranking orders in this application but aggregated values in mPFDPWG operator are greater than mPFDWG operator; for mPFEWA and mPFEWG operators, the ranking orders almost same though values are different. However, from all AOs and MABAC approach, observe that L_1 having highest value, therefore L_1 is the most desirable company to invest money although their ranking orders are slightly different in different approaches. Thus proposed operators implement a new flexible and reliable measure for decision-makers to control mPF information in MADM problems. For buying or selling the shares of that company we must watch the market price. So, our proposed approach is stable and effective. The proposed approach on mPF environment in this article is fully different from the the existing methods.

Several merits of the proposed approach are given below:

(1) In the existing approach, all researchers have used homogeneous sub-characteristics in every attribute in mPF environment, but in our novel approach heterogeneous sub-characteristics could be used. This is very essential and beneficial for multi-polar environment.
(2) The operational parameter ∂ in the combined operations for this article offers significant flexibility advantages for aggregation.
(3) Our developed aggregation process is more effective and stable with multi-polar information.

Despite some advantages in our approach, there are some limitations which are listed below:

(1) If two alternatives have equal final aggregate values then we cannot choose the best alternative between them.

(2) Final ranking orders of the alternatives may vary by removing or adding more attributes with their weights to the existing decision matrix.
(3) By using different operators and different methods we get different values, ranking orders may vary that shown in a comparative study.
(4) There is another major limitation is the calculation process is very lengthy.

9. Conclusions

By extending the fuzzy set theory to accommodate multiple membership functions, mPFSs offer enhanced flexibility and accuracy in modeling real-world problems. In this article, we have developed a novel process for solving the MADM problem. From the motivation of Dombi power operations we proposed some arithmetic and geometric aggregation operators as mPF Dombi power average and mPF Dombi power geometric aggregation operators. Additionally, we have proposed some of their properties including commutativity, idempotency, boundedness and monotonicity properties. However, our developed operators are stable and effective for dealing with mPF environments. In real world, most of all get confused which company is the best for buying the shares in the stock market. Understanding the stock market requires continuous learning and adaptability. Investors must stay informed, diversify their portfolios and make informed decisions based on research and analysis. our method offers significant benefits for selection the best company to invest money in the stock market. The comparison analysis highlights the usefulness of the decision-making process by evaluating the proposed operators against existing ones. Finally, the merits and demerits of our proposed approach are discussed.

Future scopes

In the future, our research will focus on developing new combined aggregation operators of Hamachar norm, Heronian mean, Bonferroni mean, and Maclaurin mean weighted aggregation operators etc. to effectively aggregate multi polar uncertain information. In future work, we can apply these operators in other domains, such as mPF soft set, mPF neutrosophic set, mPF cubic set etc. We will also explore the application of our framework in various fields, including image clustering, pattern recognition, fault analysis, medical diagnosis etc. Furthermore, we intend to investigate statistical methods such as time series analysis, regression analysis, and forecasting to complement our framework. mPF information may be processed further using a variety of hybrid theories, including hesitant FSs, COPRAS, EDAS, ELECTRE-I and ELECTRE-II method etc.

Declarations

Conflict of interest: The authors declare that they have no conflicts of interest.
Ethics approval: This article does not include any studies involving human participants or animals conducted by the authors.
Consent to participate: Not applicable.
Consent for publication/Informed consent: Not applicable.

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