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ALGEBRAIC PROPERTIES OF KERNEL SYMMETRIC INTUITIONISTIC FUZZY MATRICES

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Abstract. The characterization of interval valued secondary k- kernel symmetric Intuitionistic fuzzy matrices have been examined in this study. It is discussed how interval valued s-k kernel symmetric, s- kernel symmetric, interval valued k- kernel symmetric, and interval valued kernel symmetric matrices relate to one another. We establish the necessary and sufficient criteria for interval valued s-k kernel symmetric Intuitionistic fuzzy matrices.

Keywords: Interval valued Intuitionistic Fuzzy matrix, kernel symmetric IV Intuitionistic fuzzy matrix, s-k- kernel symmetric interval valued Intuitionistic fuzzy matrix.

AMS Subject Classification: 68T27.

1. Introduction

Matrices are crucial in many fields of research in science and engineering. The traditional matrix theory is unable to address problems involving numerous kinds of uncertainties. Fuzzy matrices are used to solve certain kinds of issues. Many researchers have since completed numerous works. Only membership values are addressed by fuzzy matrices. These matrices cannot handle values that are not membership. Several properties on IFMs have been studied in Khan and Pal [5]. Atanassov [1,2] has discussed Intuitionistic Fuzzy Sets Theory and Applications, Intuitionistic fuzzy sets and Operations over interval-valued intuitionistic fuzzy sets. Hashimoto has studied Canonical form of a transitive matrix. Kim and Roush [4] have studied Generalized fuzzy matrices. Pal, Khanand and Shyamal have characterize Intuitionistic fuzzy matrices. Lee [6] has studied Secondary Skew Symmetric, Secondary Orthogonal Matrices. Hill and Waters [7] have analyze On k-Real

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and k-Hermitian matrices. Meenakshi [8] has studied Fuzzy Matrix: Theory and Applications. Meenakshi and Krishanmoorthy [9] have characterize On Secondary k-Hermitian matrices. Shyamal and Pal [10] Interval valued Fuzzy matrices. Meenakshi and Kalliraja [11] Regular Interval valued Fuzzy matrices. But, practically it is difficult to measure the membership or non-membership value as a point .So, we consider the membership value as an interval and also in the case of non-membership values, it is not selected as a point, it can be considered as an interval .Here, we introduce the Secondary k-Kernel Symmetric Intuitionistic Fuzzy Matrices and introduce some basic operators on IVIFMs.

The concept of interval-valued secondary k-range symmetric neutrosophic fuzzy matrices has been explored in recent works, including Anandhkumar et al. [13], where they investigate the properties of such matrices within neutrosophic fuzzy systems. Related studies by Punithavalli and Anandhkumar focus on reverse sharp and left-T right-T partial ordering in intuitionistic fuzzy matrices [14], contributing to the understanding of fuzzy matrix structures and their applications in decision-making models. Additionally, Anandhkumar et al. [15] examine reverse tilde (T) and minus partial ordering on intuitionistic fuzzy matrices, providing insights into their role in mathematical modeling.

Further, the study of kernel and k-kernel symmetric intuitionistic fuzzy matrices has been advanced by Punithavalli and Anandhkumar [16], deepening the understanding of symmetry properties in fuzzy matrix theory. The exploration of Schur complement in k-kernel symmetric block quadri-partitioned neutrosophic fuzzy matrices by Radhika et al. [18] offers valuable connections to advanced matrix operations. Additionally, the work of Radhika et al. [19] discusses interval-valued secondary k-range symmetric quadri-partitioned neutrosophic fuzzy matrices with a focus on decision-making, highlighting their practical implications in multi-criteria decision analysis. Another relevant study by Prathab et al. [20] provides insights into interval-valued secondary k-range symmetric fuzzy matrices with generalized inverses, expanding their theoretical framework.In this article we study characterization of interval valued secondary k- kernel symmetric Intuitionistic fuzzy matrices have been examined in this study. It is discussed how interval valued s-k kernel symmetric, s- kernel symmetric, interval valued k- kernel symmetric, and interval valued kernel symmetric matrices relate to one another. We establish the necessary and sufficient criteria for interval valued s-k kernel symmetric Intuitionistic fuzzy matrices.

1.1 NOTATIONS

P^T = Transpose of the matrix P,
$P^\dagger$ = Moore- Penrose inverse of P,
$R(P)$ = Row space of P,
$C(P)$ = Column space of P,
$N(P)$ = Null space of P,

1.2 The main contributions of our work

(i)Characterization of Interval-Valued Secondary k-Kernel Symmetric Intuitionistic Fuzzy Matrices: This study provides a comprehensive analysis of interval-valued secondary k-kernel symmetric intuitionistic fuzzy matrices. We explore their properties and relationships with other types of fuzzy matrices, particularly those related to kernel symmetry.

(ii)Relations Among Different Types of Matrices: We discuss and establish how interval-valued s-kernel symmetric, s-kernel symmetric, interval-valued k-kernel symmetric, and interval-valued kernel symmetric matrices are interrelated. This comparative analysis enhances the understanding of how these matrix types differ and connect in the context of intuitionistic fuzzy systems.

(iii)Necessary and Sufficient Criteria: The study provides the necessary and sufficient conditions for interval-valued s-kernel symmetric intuitionistic fuzzy matrices. This is a crucial step in developing a more profound theoretical understanding of these matrices and their application in various fuzzy systems and decision-making models.

1.3 Research gap

Punithavalli and Anandhkumar [20] studied Kernel and K-Kernel Symmetric Intuitionistic Fuzzy Matrices. I have applied the above concept to Interval-Valued Intuitionistic Fuzzy Matrices and have also detailed some of the results.

1.4 Novelty

The references provided showcase the evolving and diverse contributions to the theory and applications of intuitionistic fuzzy matrices, interval-valued fuzzy matrices, and their various extensions. Each of these works brings forward novel concepts, methodologies, and applications that push the boundaries of fuzzy mathematics and its interaction with real-world problems. Below is a summary of the novelties introduced by the key references:

Atanassov's Contributions (References 1, 2): Atanassov's foundational work on intuitionistic fuzzy sets (IFS) and interval-valued intuitionistic fuzzy sets (IVIFS) provides essential theoretical underpinnings for many later developments in fuzzy matrix theory. His exploration of operations on interval-valued intuitionistic fuzzy sets (IVIFS) in particular, helps set the stage for understanding more complex matrix structures and their algebraic properties.

Hashimoto (Reference 3): Hashimoto introduces a canonical form for transitive matrices, focusing on the role of fuzzy matrices in modeling relations that satisfy transitivity. This concept is significant as it allows the simplification of matrix representations in various fuzzy set applications, especially in decision-making and optimization problems.

Kim and Roush (Reference 4): The generalization of fuzzy matrices by Kim and Roush is notable as it broadens the scope of matrix theory in fuzzy set systems. Their work allows for more flexibility in modeling and solving problems involving uncertainty, providing a foundation for further research into generalized fuzzy matrix operations and their practical use.

Pal, Khan, and Shyamal (References 5, 10): The study of intuitionistic fuzzy matrices and interval-valued fuzzy matrices by Pal, Khan, and Shyamal introduces critical methodologies for handling uncertainty and vagueness in real-world systems. Their work in developing and characterizing these matrices offers robust theoretical tools for applications in areas such as decision-making, optimization, and systems analysis under fuzzy conditions.

Meenakshi and Jaya Shree (References 8,9,11): Meenakshi and Jaya Shree's exploration of k-kernel symmetric matrices, secondary k-Hermitian matrices, and k-range symmetric matrices adds an important dimension to the understanding of fuzzy matrix theory. Their study of k-kernel symmetric matrices establishes essential criteria for analyzing symmetric properties in fuzzy matrix systems, which is useful for optimization and classification tasks.

Anandhkumar et al. (References 15,17): Anandhkumar and collaborators contribute significantly to the theory of interval-valued fuzzy matrices and neutrosophic fuzzy matrices. They extend the concepts of symmetry, partial ordering, and decision-making using secondary k-range symmetric and other advanced matrix forms, which offer new computational methods and insights for dealing with imprecise data. Their work also includes applications to decision-making, optimization, and generalized inverses in matrix systems, making it highly relevant for practical problems in engineering, computer science, and applied mathematics.

Radhika et al. (References 22, 23, 24): Radhika and colleagues explore the role of Schur complements in k-kernel symmetric block quadri-partitioned matrices and their application in neutrosophic fuzzy matrices. These works contribute to the development of more advanced algebraic tools for matrix analysis, enabling more efficient handling of complex fuzzy systems and decision-making processes.

2. PRELIMINARIES AND DEFINITIONS

Definition 2.1. *Interval-valued intuitionistic fuzzy matrix (IVIFM): An interval valued intuitionistic fuzzy matrix (IVIFM) P of order* $m \times n$ *is defined as* $P = [X_{ij}, < p_{ij\mu}, p_{ijv} >]_{m\times n}$ *where* $p_{ij\mu}$ *and* p_{ijv} *are both the subsets of* $[0,1]$ *which are denoted by* $p_{ij\mu} = [p_{ij\mu L}, p_{ij\mu U}]$ *and* $p_{ijv} = [p_{ijvL}, p_{ijvU}]$ *which maintaining the condition* $0 \leq p_{ij\mu U} + p_{ij\mu U} \leq 1, 0 \leq p_{ij\mu L} + p_{ijvL} \leq 1, 0 \leq p_{\mu L} \leq p_{\mu U} \leq 1, 0 \leq p_{vL} \leq p_{vU} \leq 1$ *for* $i = 1, 2, , m$ *and* $j = 1, 2, , n$.

Example 2.1. *Let* $P = \begin{bmatrix} < [0.2, 0.2], [0.3, 0.3] > & < [0.2, 0.3], [0.3, 0.4] > \\ < [0.2, 0.3], [0.3, 0.4] > & < [0.2, 0.2], [0.3, 0.3] > \end{bmatrix}$

$P_L = \begin{bmatrix} < 0.2, 0.2 >, < 0.3, 0.3 > & \\ < [0.2, 0.2 >, < 0.3, 0.3 > & \end{bmatrix}$, $P_U = \begin{bmatrix} < 0.2, 0.3 >, < 0.3, 0.4 > & \\ < [0.3, 0.2 >, < 0.4, 0.3 > & \end{bmatrix}$

Definition 2.2. *If* $k(X) = (X_{k[1]}, X_{k[2]}, X_{k[3]}, \ldots, x_{k[n]} \in F_{n\times 1}$ *for* $X = X_1, X_2, .., X_n \in F_{[n\times 1]}$, *where K is involuntary, The corresponding permutation matrix is satisfied using the following.* $(P.2.1) KK^T = K^T K = I_n, K = K^T, K^2 = I$ *and* $R(x) = Kx$
By the definition of V,

$(P.2.2) V = V^T, VV^T = V^T V = In$ *and* $V^2 = I$

$(P.2.3) N([P_{\mu L}, P_{VL}]) = N([(P_{\mu L}, P_{VL}]V), N([(P_{\mu L}, P_{VL}]) = N([(P_{\mu L}, P_{VL}]K)$

$N([(P_{\mu U}, P_{VU}]) = N([(P_{\mu U}, P_{VU}]V), N([(P_{\mu U}, P_{VU}]) = N([(P_{\mu U}, P_{VU}]K)$

$(P.2.4) N([P_{\mu L}, P_{VL}]V)^T = N(V[P_{\mu L}, P_{VL}]^T), N(V[P_{\mu L}, P_{VL}])^T = N([P_{\mu L}, P_{VL}]^T V)$

$N([(P_{\mu U}, P_{VU}]V)^T = N(V[(P_{\mu U}, P_{VU}]^T), N(V[(P_{\mu U}, P_{VU}])^T = N([(P_{\mu U}, P_{VU}]^T V.$

Lemma 2.1. *For a matrix A belongs to F_n and a permutation fuzzy matrix P, $N(P) = N(Q)$ iff $N(APB^T) = N(AQB^T)$.*

Lemma 2.2. *For IV fuzzy matrix $P = KP^T K$ iff $KP = (KP)(KP)^T(KP)$, interval valued fuzzy matrix $\Leftrightarrow PK = (PK)(PK)^T(PK)$ IV fuzzy matrix.*

3. Interval valued Secondary k - ks Intuitionistic Fuzzy Matrix

Definition 3.1. *For an Intuitionistic fuzzy matrix $P =< [P_{\mu L}, P_{\mu U}], [P_{vL}, P_{vU}] > \in IVIFM_{nn}$ is an IV s - symmetric fuzzy matrix iff $[P_{\mu L}, P_{vL}] = V[P_{\mu L}, P_{vL}]^T V$ and $[P_{\mu U}, P_{vU}] = V[P_{\mu U}, P_{vU}]^T V$.*

Definition 3.2. *For an Intuitionistic fuzzy matrix $P =< [P_{\mu L}, P_{\mu U}], [P_{vL}, P_{vU}] > \in IVIFM_{nn}$ is an IV s - ks fuzzy matrix iff $N([P_{\mu L}, P_{vL}]) = N(V[P_{\mu L}, P_{vL}]^T V)$, $N([P_{\mu U}, P_{vU}]) = N([P_{\mu U}, P_{vU}]^T V)$.*

Definition 3.3. *For an Intuitionistic fuzzy matrix $P =< [P_{\mu L}, P_{\mu U}], [P_{vL}, P_{vU}] > \in IVIFM_{nn}$ is an IV s - k - ks fuzzy matrix iff $N([P_{\mu L}, P_{vL}]) = N(KV[P_{\mu L}, P_{vL}]^T VK, [P_{\mu U}, P_{vU}] = N(KV[P_{\mu U}, P_{vU}]^T VK)$.*

Lemma 3.1. *For an Intuitionistic fuzzy matrix $P =< [P_{\mu L}, P_{\mu U}], [P_{vL}, P_{vU}] > \in IVIM_{nn}$ is interval valued s-kernel symmetric Intuitionistic fuzzy matrix $\Leftrightarrow VP =< [P_{\mu L}, P_{vL}], V[P_{\mu U}, P_{vU}] >$ interval valued kernel symmetric Intuitionistic fuzzy matrix $\Leftrightarrow PV =< [P_{\mu L}, P_{vL}]V, [P_{\mu U}, P_{vU}] >$ is interval valued kernel symmetric Intuitionistic fuzzy matrix.*

Proof. *An Intuitionistic fuzzy matrix is s-ks fuzzy matrix*
$P =< [P_{\mu L}, P_{\mu U}], [P_{vL}, P_{vU}] > \in IVIM_{nn}$ is s - ks fuzzy matrix.
$\Leftrightarrow N([P_{\mu L}, P_{vL}]) = N(V[P_{\mu L}, P_{vL}]^T V) \quad [Definition 3.2]$
$\Leftrightarrow N([P_{\mu L}, P_{vL}]V) = N([P_{\mu L}, P_{vL}]V)$
$\Leftrightarrow N([P_{\mu L}, P_{vL}]$ *is ks.* *[By P.2.2]*
$\Leftrightarrow N(V[P_{\mu L}, P_{vL}]VV^T) = N(VV[P_{\mu L}, P_{vL}]^T V)$
$\Leftrightarrow N(V[P_{\mu L}, P_{vL}]) = N(V[P_{\mu L}, P_{vL}]^T)$
$\Leftrightarrow V[P_{\mu L}, P_{vL}]$ *is kernal symmetric.*
Similar manner
$\Leftrightarrow N([P_{\mu U}, P_{vU}]) = N(V[P_{\mu V}, P_{vU}]^T V)$
$\Leftrightarrow N([P_{\mu L}, P_{vL}]) = N(V[P_{\mu L}, P_{vL}]^T V)$ *is ks.*
$\Leftrightarrow N(V[P_{\mu U}, P_{vU}]) = N(VV[P_{\mu U}, P_{vU}]^T V)$
$\Leftrightarrow N(V[P_{\mu U}, P_{vU}]) = N(V[P_{\mu U}, P_{vU}]^T)$
$\Leftrightarrow V[P_{\mu U}, P_{vU}] iskernalsymmetric$ *is kernel symmetric.*
Therefore, $VP =< V[P_{\mu L}, P_{vU}], V[P_{\mu U}, P_{vU}] >$ is an internal valued symmetric.

Remark 3.1. *To be more precise, Definition (3.3) reduces to $N([P_{\mu L}, P_{vL}]) = N(V[P_{\mu L}, P_{vL}]^T V)$, $N([P_{\mu U}, P_{vU}]) = N(V[P_{\mu U}, P_{vU}]^T V)$, meaning that the appropriate Intuitionistic fuzzy permutation matrix K is an interval valued s-kernel symmetric Intuitionistic fuzzy matrix when $k(i) = i for i = 1, 2, \ldots, n$.*

Remark 3.2. *For $k(i) = n - i + 1$, the analogous permutation Intuitionistic fuzzy matrix K can be reduced to V.* $N([P_{\mu L}, P_{vL}]) = N([P_{\mu L}, P_{vL}]^T)$, $N([P_{\mu U}, P_{vU}]) = N(V[P_{\mu U}, P_{vU}]^T V)$, *means that is an IV kernel symmetric in* $P =< [P_{\mu L}, P_{vL}], [P_{\mu U}, P_{vU}] >$ *Definition (3.3).*

Remark 3.3. *If A is interval valued s-k-symmetric, then* $[P_{\mu L}, P_{vL}] = KV[P_{\mu L}, P_{vL}]^T VK$, *and* $AU = KVA_U^T VK$, *indicating that it is interval valued (IV) s-k-ks IFM, then* $N([P_{\mu L}, P_{vL}]) = N(KV[P_{\mu L}, P_{vL}]^T VK), N([P_{\mu}v, P_{vU}]) = N(KV[P_{\mu v}, P_{vU}]^T VK)$. *We note that s-k-symmetric Intuitionistic fuzzy matrix is s-k-ks Intuitionistic fuzzy matrix. The opposite isn't always true, though. The example that follows illustrates this V.*

Example 3.1. *Consider IVIFM*

$$K = \begin{bmatrix} <1,0> & <0,1> \\ <0,1> & <1,0> \end{bmatrix}, V = \begin{bmatrix} <0,1> & <1,0> \\ <1,0> & <0,1> \end{bmatrix}$$

$P{=}[P_{\mu L, P_\mu U}], [P_{vL, P_v U}] >\in IVIFM_{nn}$

$$P = \begin{bmatrix} <[0.2,0.2],[0.3,0.3]> & <[0.2,0.3],[0.3,0.4]> \\ <[0.2,0.3],[0.3,0.4]> & <[0.2,0.2],[0.3,03.]> \end{bmatrix},$$

is an IV symmetric, IV s-k symmetric and hence therefore IV s-k kernal symmetric. Hence

$$P_L = \begin{bmatrix} <0.2,0.2> & <0.3,0.3> \\ <0.2,0.2> & <0.3,0.3> \end{bmatrix},$$

$$P_U = \begin{bmatrix} <0.2,0.3> & <0.3,0.4> \\ <0.3,0.2> & <0.4,0.3> \end{bmatrix},$$

$$KV = \begin{bmatrix} <1,0> & <0,1> \\ <0,1> & <1,0> \end{bmatrix} \begin{bmatrix} <0,1> & <1,0> \\ <1,0> & <0,1> \end{bmatrix} = \begin{bmatrix} <0,1> & <1,0> \\ <1,0> & <0,1> \end{bmatrix}$$

$$VK = \begin{bmatrix} <0,1> & <1,0> \\ <1,0> & <0,1> \end{bmatrix} \begin{bmatrix} <1,0> & <0,1> \\ <0,1> & <1,0> \end{bmatrix} = \begin{bmatrix} <0,1> & <1,0> \\ <1,0> & <0,1> \end{bmatrix}$$

$$KVP_L^T VK = \begin{bmatrix} <0,1> & <1,0> \\ <1,0> & <0,1> \end{bmatrix} \begin{bmatrix} <0.2,0.2> & <0.2,0.3> \\ <0.2,0.3> & <0.2,0.2> \end{bmatrix} \begin{bmatrix} <0,1> & <1,0> \\ <1,0> & <0,1> \end{bmatrix}$$

$= P_L$

$$KVP_L^T VK = \begin{bmatrix} <0,1> & <1,0> \\ <1,0> & <0,1> \end{bmatrix} \begin{bmatrix} <0.3,0.3> & <0.3,0.4> \\ <0.3,0.4> & <0.3,0.3> \end{bmatrix} \begin{bmatrix} <0,1> & <1,0> \\ <1,0> & <0,1> \end{bmatrix}$$

$= P_L$

$N(P_L) = N(KVP_L^T VK) =< 0,0 >$

$P = [P_L, P_U]$ *is an IV s-k kernal symmetric.*

Example 3.2. *For k = (1,2)*

$$K = \begin{bmatrix} <1,0> & <0,1> \\ <0,1> & <1,0> \end{bmatrix}, V = \begin{bmatrix} <0,1> & <1,0> \\ <1,0> & <0,1> \end{bmatrix}$$

$P =< [P_{\mu L}, P_{vU}], [P_{vL}, P_{vU}] >\in IVIFM_{nn}$

$$P = \begin{bmatrix} <[0,0.2],[0,1]> & <[0.2,0.4],[0.2,0.3]> \\ <[0.2,0.4],[0.2,0.3]> & <[0.2,0.2]>, <[0.3,0.4]> \end{bmatrix}$$

$$P_U = \begin{bmatrix} <0,1> & <0.2,0.3> \\ <0.2,0.3> & <0.3,0.4> \end{bmatrix}$$

$$KVP_U^T VK = \begin{bmatrix} <1,0> & <0,1> \\ <0,1> & <1,0> \end{bmatrix} \begin{bmatrix} <0,1> & <1,0> \\ <1,0> & <0,1> \end{bmatrix} \begin{bmatrix} <0,1> & <0.2,0.3> \\ <0.2,0.3> & <0.3,0.4> \end{bmatrix}$$

$$\begin{bmatrix} <0,1> & <1,0> \\ <1,0> & <0,1> \end{bmatrix} \begin{bmatrix} <1,0> & <0,1> \\ <0,1> & <1,0> \end{bmatrix}$$

KVP_L^TVK *not equal to* P^T
Here $P = KP^T - UK$
Therefore P is symmetric IFM, s- k-symmetric and IFM but not s- k- symmetric IFM.

Theorem 3.1. *The following conditions are equivalent* $P \in IVIF_n$.
(1)$P = [P_{\mu L}, P_{vL}], [P_{\mu U}, P_{vU}]$*is an IV s-k kS.*
(2)$KVP =< [KV[P_{\mu L}, P_{vL}], KV[P_{\mu U}, P_{vU}] >$*is an IV kernal symmetric.*
(3)$PKV =< [P_{\mu L}, P_{vL}]KV, [P_{\mu U}, P_{vU}] >$*is an IV kernal symmetric.*
(4)$VP =< [V[P_{\mu L}, P_{vL}], V[P_{\mu U}, P_{vU}] >$*is an IV kernal symmetric.*
(5)$PK =< [[P_{\mu L}, P_{vL}]K, [P_{\mu U}, P_{vU}]K] >$*is an IV kernal symmetric.*
(6)P^T *is an IV s-k symmetric*
(7)$N([P_{\mu L}, P_{vL}]) = N([P_{\mu L}, P_{vL}])^T VK, N([P_{\mu U}, P_{vU}]) = N([P_{\mu U}, P_{vU}]^T VK)$.
(8)$N([P_{\mu L}, P_{vL}]^T) = N([P_{\mu L}, P_{vL}])VK, N([P_{\mu U}, P_{vU}]^T)N([P_{\mu U}, P_{vU}]VK)$.
(9)$N(KV[P_{\mu L}, P_{vL}]) = N(KV[P_{\mu L}, P_{vL}]^T)^T, N(KV[P_{\mu U}, P_{vU}]) = N(KV[P_{\mu U}, P_{vU}]^T)$.
(10) $PVK =< [P_{\mu L}, P_{vL}]VK, [P_{\mu U}, P_{vU}] > VK$*is an IV kernal symmetric.*
(11)$PV =< [P_{\mu L}, P_{vL}]V, [P_{\mu U}, P_{vU}] > V$*is an IV kernal symmetric.*
(12)$VKP =< VK[P_{\mu L}, P_{vL}], VK[P_{\mu U}, P_{vU}] >$*is an IV kernal symmetric.*
(xii)$KP =< K[P_{\mu L}, P_{vL}], K[P_{\mu U}, P_{vU}] >$*is an IV kernal symmetric.*

Proof: *(1) iff (2) iff (4)*
Let $P = [P_{\mu L}, P_{vL}], [P_{\mu U}, P_{vU}]$*is an IV S-K ks*
Let $[P_{\mu L}, P_{vL}]$ *is an S-K ks*
$\Leftrightarrow N([P_{\mu L}, P_{vL}]) = N(KV[P_{\mu L}, P_{vL}]^T VK), N([P_{\mu U}, P_{vU}]) = N(KV[P_{\mu U}, P_{vU}]^T VK),$
$\Leftrightarrow N(KV[P_{\mu L}, P_{vL}]) = N(KV[P_{\mu L}, P_{vL}])^T, N([P_{\mu U}, P_{vU}]) = N(KV[P_{\mu U}, P_{vU}])^T$
$(ByP_{2.3})$
$\Leftrightarrow KVP =< KV[P_{\mu L}, P_{vL}], KV[P_{\mu U}, P_{vU}] >$ *is an IV kernal symmetric*
$\Leftrightarrow VP =< V[P_{\mu L}, P_{vL}], V[P_{\mu U}, P_{vU}] >$ *is an IV kernal symmetric*
As a conclusion (1) iff (2) iff (4) is true
(1) iff (3) iff (5)
Let $P = [P_{\mu L}, P_{vL}], [P_{\mu U}, P_{vU}]$ *is an IV S - K kernal symmetric*
$\Leftrightarrow N([P_{\mu L}, P_{vL}]) = N(KV[P_{\mu L}, P_{vL}]^T VK), N([P_{\mu U}, P_{vU}]) = N(KV[P_{\mu L}, P_{vL}]^T VK),$
$\Leftrightarrow N(KV[P_{\mu L}, P_{vL}]) = N(KV[P_{\mu L}, P_{vL}]^T VK), N([P_{\mu U}, P_{vU}]) = N(KV[P_{\mu U}, P_{vU}]^T),$
$(ByP_{2.3}) \Leftrightarrow N(VK[P_{\mu L}, P_{vL}])$
$= N((VK)[P_{\mu L}, P_{vL}]^T), N(VK(KV[P_{\mu U}, P_{vU}]) = N((VK)[P_{\mu U}, P_{vU}]^T VK(VK)^T),$
$\Leftrightarrow N([P_{\mu L}, P_{vL}]) = N[P_{\mu L}, P_{vL}]^T), N([P_{\mu U}, P_{vU}]KV) = N([P_{\mu U}, P_{vU}]KV^T),$
$(ByLemma2.2)$
$\Leftrightarrow PKV = [[P_{\mu L}, P_{vL}]KV, [P_{\mu U}, P_{vU}]KV)$ *is IV s - kernal symmetric*
$\Leftrightarrow PK = [[P_{\mu L}, P_{vL}]K, [P_{\mu U}, P_{vU}]K)$ *is IV s - kernal symmetric*
As a conclusion (1) Iff (3) iff (5) is true
(2) iff (9)
KVA *=[*$KV[P_{\mu L}, P_{vL}], KV[P_{\mu U}, P_{vU}]$*]is an interval valued ks*
$\Leftrightarrow N(KV[P_{\mu L}, P_{vL}]) = N((KV[P_{\mu L}, P_{vL}])^T), N(KV[P_{\mu U}, P_{vU}]) = N((KV[P_{\mu U}, P_{vU}])^T)$
(2) iff (9) is true.
(9) iff (7)
$KVP = KV[P_{\mu L}, P_{vL}], KV[P_{\mu U}, P_{vU}]$ *is an IV ks*
$\Leftrightarrow N(KV[P_{\mu L}, P_{vL}]) = N((KV[P_{\mu L}, P_{vL}])^T), N(KV[P_{\mu U}, P_{vU}]) = N((KV[P_{\mu U}, P_{vU}])^T)$
$\Leftrightarrow N([P_{\mu L}, P_{vL}]) = N((KV[P_{\mu L}, P_{vL}])^T), N([P_{\mu U}, P_{vU}]) = N((KV[P_{\mu U}, P_{vU}])^T)$
$\Leftrightarrow N([P_{\mu L}, P_{vL}]) = N([P_{\mu L}, P_{vL}])^T VK), N([P_{\mu U}, P_{vU}]) = N([P_{\mu U}, P_{vU}])^T VK)$

As a conclusion (2) iff (7) is true.
(3) iff (8)
$PVK = [[P_{\mu L}, P_{vL}]VK, [P_{\mu L}, P_{vL}]VK$
$\Leftrightarrow N([P_{\mu L}, P_{vL}]VK) = N(([P_{\mu L}, P_{vL}]VK)^T), N([P_{\mu U}, P_{vU}]VK) = N(([P_{\mu U}, P_{vU}]VK)^T)$
$\Leftrightarrow N([P_{\mu L}, P_{vL}]VK) = N(([P_{\mu L}, P_{vL}])^T), N([P_{\mu U}, P_{vU}]VK) = N([P_{\mu U}, P_{vU}])^T$
As a conclusion (3) iff (8 is true
(1) iff (6)
Let $P =< [P_{\mu L}, P_{vL}], [P_{\mu U}, P_{vU}] >$ *is an IV S - K KS*

$$\Leftrightarrow N([P_{\mu L}, P_{vL}]) = N(KV[P_{\mu L}, P_{vL}])^T VK), N([P_{\mu U}, P_{vU}]) = N(KV[P_{\mu U}, P_{vU}])^T VK),$$

(By Definition 3.3)
$(KVP)^T = (KV[P_{\mu L}, P_{vL}], KV[P_{\mu U}, P_{vU}])^T$ *is an IV kernal symmetric*
$\Leftrightarrow P^T VK = ([P_{\mu L}, P_{vL}]VK, [P_{\mu U}, P_{vU}]VK)$ *is an IV kernal symmetric*
$\Leftrightarrow P^T = ([P_{\mu L}, P_{vL}]^T, [P_{\mu U}, P_{vU}]^T)$ *is an IV S - K kernal symmetric*
As a conclusion (1) iff (6) is true
Let $P =< [P_{\mu L}, P_{vL}], [P_{\mu U}, P_{vU}] >$ *is an IV S -K ks*
Consider $[P_{\mu L}, P_{vL}]$ *is a S - K ks*
$\Leftrightarrow N([P_{\mu L}, P_{vL}]) = N(KV[P_{\mu L}, P_{vL}]^T VK), N([P_{\mu U}, P_{vU}]) = N(KV[P_{\mu U}, P_{vU}]^T VK)$
$\Leftrightarrow N([P_{\mu L}, P_{vL}]VK) = N([P_{\mu L}, P_{vL}]VK), N([P_{\mu U}, P_{vU}]VK) = N([P_{\mu U}, P_{vU}]^T VK)$
$By(P.2.3)$
$\Leftrightarrow PVK = [[P_{\mu L}, P_{vL}]VK), [P_{\mu U}, P_{vU}]VK)$ *is an IV kernal symmetric*
$\Leftrightarrow PV = [[P_{\mu L}, P_{vL}]V), [P_{\mu U}, P_{vU}]V)$ *is an IV k -kernal symmetric*
Therefore (1) iff (10) iff (11) is true
(1) iff (12) iff (13)
Let $P =< [P_{\mu L}, P_{vL}], [P_{\mu U}, P_{vU}]$ *is an IV s-k ks*
$\Leftrightarrow N([P_{\mu L}, P_{vL}]) = N(KV[P_{\mu L}, P_{vL}]^T VK), N([P_{\mu U}, P_{vU}]) = N(KV[P_{\mu U}, P_{vU}]^T VK)$
$(ByDefinition 3.3)$
$\Leftrightarrow N(VK[P_{\mu L}, P_{vL}]) = N(VK[P_{\mu L}, P_{vL}]^T), N(VK[P_{\mu U}, P_{vU}]) = N(VK[P_{\mu U}, P_{vU}]^T)$
$By(P.2.3)$
$\Leftrightarrow N(KV(VK[P_{\mu L}, P_{vL}]))$
$= N((KV)[P_{\mu L}, P_{vL}])^T KV(KV)^T, N(KV(VK[P_{\mu L}, P_{vL}]))$
$= N((KV[P_{\mu U}, P_{vU}]^T KV(KV)^T)$
$\Leftrightarrow N(VK[P_{\mu L}, P_{vL}])$
$= N(VK[P_{\mu L}, P_{vL}]^T), N(VK[P_{\mu L}, P_{vL}]) = N(VK[P_{\mu L}, P_{vL}]^T)^T [ByLemma 2.2]$
$\Leftrightarrow VKP = [VK[P_{\mu L}, P_{vL}], VK[P_{\mu U}, P_{vU}]]$ *is an IV kernal symmetric*
$\Leftrightarrow KP = [K[P_{\mu L}, P_{vL}], K[P_{\mu U}, P_{vU}]]$ *is an IV s - kernal symmetric*
As a conclusion (1) iff (12) iff (13) is true
The above statement can be reduced to the equivalent requirement that a matrix be an IV s- CS for K = I in particular.

Corollary 3.1. *The following statements are equivalent for* $P \in IVIM_{nn}$

(1) $P =< [P_{\mu L}, P_{vL}], [P_{\mu U}, P_{vu}] >$ *is an IV s- ks.*

(2) $VP =< V[P_{\mu L}, P_{vL}], V[P_{\mu U}, P_{vU}] >$ *is an IV kernel symmetric.*

(3) $PV =< [P_{\mu L}, P_{vL}]V, [P_{\mu U}, P_{vu}]V >$ *is an IV s kernel symmetric.*

(4) $P^T =< [P_{\mu L}, P_{vL}]^T, [P_{\mu U}, P_{vu}]^T >$*is an IV s- kernel symmetric.*

(5) $N([P_{\mu L}, P_{vL}]) = N([P_{\mu L}, P_{vL}]^T V), N([P_{\mu U}, P_{vu}]) = N([P_{\mu U}, P_{vu}]^T V)$.

(6) $N([P_{\mu L}, P_{vL}]^T) = N([P_{\mu L}, P_{vL}]V), N([P_{\mu U}, P_{vu}]^T) = N([P_{\mu U}, P_{vu}]^T V)$.

(7) $N(KV[P_{\mu L}, P_{vL}]) = N(V[P_{\mu L}, P_{vL}]^T), N(KV[P_{\mu U}, P_{vu}]) = N(V[P_{\mu U}, P_{vu}]^T)$.

Theorem 3.2. *For* $P = [P_{\mu L}, P_{vL}], [P_{\mu U}, P_{vU}]$ *then any two of the conditions below imply the other*

1) $P =< [P_{\mu L}, P_{vL}], [P_{\mu U}, P_{vU}] >$ *is an IV K - ks.*

2) $P =< [P_{\mu L}, P_{vL}], [P_{\mu U}, P_{vU}] >$ *is an IV S-K- ks.*

3) $N([P_{\mu L}, P_{vL}]^T) = N(VK[P_{\mu L}, P_{vL}])^T, N([P_{\mu U}, P_{vU}])^T = N(VK[P_{\mu U}, P_{vU}]^T)$.

Proof: *(1) and (2) implies (3)*
Let $P =< [P_{\mu L}, P_{vL}], [P_{\mu U}, P_{vU}] >$ *is an IV S -K range symmetric.*
$\Rightarrow N([P_{\mu L}, P_{vL}]) = N([P_{\mu L}, P_{vL}]^T VK), N([P_{\mu U}, P_{vU}]) = N([P_{\mu U}, P_{vU}]^T VK)$
$(By\ Theorem 3.1)$
$\Rightarrow N(K[P_{\mu L}, P_{vL}]K) = N(K[P_{\mu L}, P_{vL}]^T K), N(K[P_{\mu U}, P_{vU}]K) = N(K[P_{\mu U}, P_{vU}]^T K)$
$(ByLemma 2.2)$
$\Rightarrow N([P_{\mu L}, P_{vL}]^T) = N((VK)[P_{\mu L}, P_{vL}]^T)), N([P_{\mu U}, P_{vU}]^T) = N(VK[P_{\mu U}, P_{vU}]^T)$
$(ByTheorem 3.1)$
(i) and (ii) implies (iii) is true
(i) and (iii) implies (ii)
$P =< [P_{\mu L}, P_{vL}], [P_{\mu U}, P_{vU}] >$ *is an IV k - ks*
$\Rightarrow N([P_{\mu L}, P_{vL}]) = N(K[P_{\mu L}, P_{vL}]^T K), N([P_{\mu L}, P_{vL}]) = N(K[P_{\mu L}, P_{vL}]^T K)$
$\Rightarrow N([P_{\mu L}, P_{vL}]K)$
$= N(([P_{\mu L}, P_{vL}])^T), N(K[P_{\mu U}, P_{vU}]K) = N(([P_{\mu L}, P_{vL}])^T)[ByLemma 2.5]$
Therefore, $(1) and (3)$
$\Rightarrow N([P_{\mu L}, P_{vL}]K) = N((V[P_{\mu L}, P_{vL}])^T), N(K[P_{\mu U}, P_{vU}]K) = N((V[P_{\mu U}, P_{vU}]K)^T)$
$N([P_{\mu L}, P_{vL}] = N((KV[P_{\mu L}, P_{vL}])^T), N([P_{\mu U}, P_{vU}]) = N((K[P_{\mu U}, P_{vU}])^T)$
$\Rightarrow (2)$ *is true*
$(2) and (3) implies (1)$
$P =< [P_{\mu L}, P_{vL}], [P_{\mu U}, P_{vU}] >$ *is an IV s- k - ks*
$\Rightarrow N([P_{\mu L}, P_{vL}]) = N([P_{\mu L}, P_{vL}]^T VK), N([P_{\mu U}, P_{vU}]) = N([P_{\mu L}, P_{vL}]^T VK)$
$\Rightarrow N(K[P_{\mu L}, P_{vL}]K) = N(K[P_{\mu L}, P_{vL}]^T K), N([P_{\mu U}, P_{vU}]) = N(K[P_{\mu L}, P_{vL}]^T K)$
$P =< [P_{\mu L}, P_{vL}], [P_{\mu U}, P_{vU}] >$ *is an IV k - kernal symmetric*
Therefore, (1) is true
Hence the theorem.

4. Interval valued s - k kernel symmetric regular Intuitionistic fuzzy matrices

In this section, it was discovered that there are various generalized inverses of matrices in IVIFM. The comparable standards for different g-inverses of an IV s-k ks Intuitionistic fuzzy matrix to be IV s-k ks are also established. The generalized inverses of an IV s-ks P corresponding to the sets $P\{1,2\}, P\{1,2,3\}$ and $P\{1,2,4\}$ are characterized.

Theorem 4.1. *Let* $P =< [P_{\mu L}, P_{vL}], [P_{\mu U}, P_{vU}] > IVIM_{nn}$, *X belongs to* $P\{1,2\}$ *and PX, XP are an IV s- k-Ks. Then P is an IV s - k -ks iff* $X =< [X_{\mu L}, X_{vL}], [P_{\mu U}, P_{vU}] >$ *is an s - k - ks.*

Proof: *Let* $N(KV[P_{\mu L}, P_{vL}]) = N(KV[P_{\mu L}, P_{vL}]X[P_{\mu L}, P_{vL}]) \subseteq N(X[P_{\mu L}, P_{vL}])$
$[Since[P_{\mu L}, P_{vL}] = [P_{\mu L}, P_{vL}]X[P_{\mu L}, P_{vL}]]$
$= N(XUV[P_{\mu L}, P_{vL}]) \subseteq M(XUKKV[P_{\mu L}, P_{vL}]) \subseteq N(KV[P_{\mu L}, P_{vL}])$
Hence, $N(KV[P_{\mu L}, P_{vL}]) = N(X[P_{\mu L}, P_{vL}])$
$= N(KV[XP_{\mu L}, P_{vL}]^T VK)[XPisIVs - k - ks]$
$= N([P_{\mu L}, P_{vL}]^T[[X_{\mu L}, X_{vL}]^T VVK])$
$= N([X_{\mu L}, X_{vL}]^T VK)$
$= N((KV[X_{\mu L}, X_{vL}])^T)$
$N((KV[P_{\mu L}, P_{vL}])^T) = N[P_{\mu L}, P_{vL}]^T VK$
$= N([X_{\mu L}, X_{vL}]^T [P_{\mu L}, P_{vL}]^T VK)$
$N((KV[P_{\mu L}, P_{vL}][X_{\mu L}, X_{vL}])^T)$
$= (KV[P_{\mu L}, P_{vL}][X_{\mu L}, X_{vL}])$ *[VP is s- k IV ks]*
$= N(KV[X_{\mu L}, X_{vL}])$
Similarly,
Hence, $N(KV[X_{\mu L}, X_{vL}]) = N((KV[P_{\mu L}, P_{vL}])^T)$ *(KVX is an IV ks)*
$\Leftrightarrow N(KV[P_{\mu L}, P_{vL}]) = N((KV[P_{\mu L}, P_{vL}])^T), N(KV[P_{\mu L}, P_{vL}]) = N((KV[P_{\mu U}, P_{vU}])^T)$
$\Leftrightarrow N(KV[X_{\mu L}, X_{vL}]) = N((KV[X_{\mu L}, X_{vL}])^T), N((KV[X_{\mu L}, X_{vL}])^T)$
$\Leftrightarrow KVX = [KV[X_{\mu L}, X_{vL}]], KV[x_{\mu L}, X_{vL}]$ *is an IV ks*
$X =< [X_{\mu L}, X_{vL}], [P_{\mu U}, P_{vU}] >$ *is an IV s- k -ks.*

Theorem 4.2. $P =< [P_{\mu L}, P_{vL}], [P_{\mu U}, P_{vU}] > \in IVIF_{nn}, X =< [X_{\mu L}, X_{vL}], [X_{\mu U}, X_{vU}] >$

$\in P\{1,2,3\}$,
$N(KV[P_{\mu L}, P_{vL}]) = N(KV[X_{\mu L}, X_{vL}])^T, N(KV[P_{\mu L}, P_{vL}]) = N(KV[X_{\mu L}, X_{vL}])^T$.
Then
$P =< [P_{\mu L}, P_{vL}], [P_{\mu U}, P_{vU}] >$ *is IV s- k- ks* $\Leftrightarrow X =< [X_{\mu U}, X_{vU}] >$ *is IV s- k - ks.*

Proof: *Given* $P\{1,2,3\}$. *Hence* $[P_{\mu L}, P_{vL}][X_{\mu L}, X_{vL}] = [P_{\mu L}, P_{vL}]$,
$[X_{\mu L}, X_{vL}][P_{\mu L}, P_{vL}][X_{\mu L}, X_{vL}] = [X_{\mu L}, X_{vL}]$,
$([P_{\mu L}, P_{vL}][X_{\mu L}, X_{vL}])^T = [P_{\mu L}, P_{vL}][X_{\mu L}, X_{vL}]$
Consider, $N((KB[P_{\mu L}, P_{vL}])^T) = N([X_{\mu L}, X_{vL}])[P_{\mu L}, P_{vL}]^T VK[ByusingP = PXP]$
$= N(KV([P_{\mu L}, P_{vL}][X_{\mu L}, X_{vL}])^T)$
$= N(([P_{\mu L}, P_{vL}][X_{\mu L}, X_{vL}])^T)[ByP.2.3]$
$= N([P_{\mu L}, P_{vL}][X_{\mu L}, X_{vL}]) \quad [([P_{\mu L}, P_{vL}][X_{\mu L}, X_{vL}])^T = [P_{\mu L}, P_{vL}][X_{\mu L}, X_{vL}]]$
$= N([X_{\mu L}, X_{vL}]) \quad [ByUsing[X_{\mu L}, X_{vL}] = [[X_{\mu L}, X_{vL}][P_{\mu L}, P_{vL}][X_{\mu L}, X_{vL}]]$
$N(KV[X_{\mu L}, X_{vL}]) \quad [ByP.2.3]$
Similarly, we can consider, $N((KV[P_{\mu L}, P_{vL}])^T) = N(X_U^T[P_{\mu L}, P_{vL}]^T VK)$
$[By\ usingP = PXP]$
$= N(KV([P_{\mu L}, P_{vL}][X_{\mu U}, X_{vU}])^T) = N([P_{\mu U}, P_{vU}][X_{\mu U}, X_{vU}]) \quad (P.2.3)$
$= N([P_{\mu U}, P_{vU}][X_{\mu U}, X_{vU}]) \quad [(AX)^T = AX]$
$= N([X_{\mu U}, X_{vU}]) \quad [ByusingX = XAX]$
$= NKV[X_{\mu U}, X_{vU}] \quad [ByP.2.3]$
If KVP is an Kernal Symmetric
$\Leftrightarrow N(KV[P_{\mu L}, P_{vL}]) = N((KV[P_{\mu L}, P_{vL}])^T), N(KV[P_{\mu L}, P_{vL}]) = N((KV[P_{\mu L}, P_{vL}])^T)$
$\Leftrightarrow N(KV[X_{\mu L}, X_{vL}])$

$= N((KV[X_{\mu L}, X_{vL}])^T), N(KV[X_{\mu L}, X_{vL}]) = N((KV[X_{\mu U}, X_{vU}])^T)$
$KVX = [KV[X_{\mu L}, X_{vL}], KV[X_{\mu U}, X_{vU}]]$ *is an IV Ks*
$X = [[X_{\mu L}, X_{vL}], [X_{\mu U}, X_{vU}]]$ *is an s-k-ks.*

Theorem 4.3. *Let* $P =< [P_{\mu L}, P_{vL}], [P_{\mu U}, P_{vU}] >\in IVIFM_{nn}, X \in P\{1, 2, 4\}$,
$N(KV[P_{\mu L}, P_{vL}])^T = N(KV[X_{\mu L}, X_{vL}]), N(KV[P_{\mu U}, P_{vU}])^T = N(KV[X_{\mu U}, X_{vU}])$.
Then KVP *is an s - k - ks* $\Leftrightarrow X =< [X_{\mu L}, X_{vL}], [X_{\mu U}, X_{vU}] >$ *is an IV s - k - ks.*
Proof: *Given,* $P\{1, 2, 4\}$, *Hence* $[P_{\mu L}, P_{vL}]_L[X_{\mu L}, X_{vL}][P_{\mu L}, P_{vL}] = [P_{\mu L}, P_{vL}]$,
$[X_{\mu L}, X_{vL}][P_{\mu L}, P_{vL}][X_{\mu L}, X_{vL}] = [X_{\mu L}, X_{vL}]$,
$([X_{\mu L}, X_{vL}][P_{\mu L}, P_{vL}]^T) = [X_{\mu L}, X_{vL}][P_{\mu L}, P_{vL}]$
Consider, $N((KV[P_{\mu L}, P_{vL}])^T) - N([X_{\mu L}, X_{vL}]^T[P_{\mu L}, P_{vL}]^T VK)[ByusingP = PXP]$
$= N(KV([P_{\mu L}, P_{vL}][X_{\mu L}, X_{vL}])^T)$
$= N(([P_{\mu L}, P_{vL}][X_{\mu L}, X_{vL}])^T)$
$= N(([P_{\mu L}, P_{vL}][X_{\mu L}, X_{vL}])^T)[ByP.2.3]$
$= N([P_{\mu L}, P_{vL}][X_{\mu L}, X_{vL}])$
$= N([X_{\mu L}, X_{vL}])$
$= N(KV[X_{\mu L}, X_{vL}])[ByP.2.3]$
$N((KKV[P_{\mu U}, P_{vU}])^T) = N([X_{\mu U}, X_{vU}]^T[X_{\mu L}, X_{vL}]^T VK) \quad [ByusingP = PXP]$
$= N(KV([P_{\mu U}, P_{vU}][X_{\mu U}, X_{vU}])^T)$
$= N(([P_{\mu U}, P_{vU}][x_{\mu U}, X_{vU}])^T) \quad [ByP.2.3]$
$= N([P_{\mu U}, P_{vU}][X_{\mu U}, X_{vU}]) \quad [(PX)^T = PX]$
$= N([X_{\mu U}, X_{vU}])$
$= N(KV[X_{\mu U}, X_{vU}]) \quad [ByP.2.3.$
If KVP *is an kernal symmetric]*
$\Leftrightarrow N(KV[P_{\mu L}, P_{vL}]) = N((KV[P_{\mu L}, P_{vL}])^T), N(KV[P_{\mu U}, P_{vU}]) = N((KV[P_{\mu U}, P_{vU}])^T)$
$\Leftrightarrow N(KV[X_{\mu L}, X_{vL}])$
$= N((KV[X_{\mu L}, X_{vL}])^T), N(KV[X_{\mu U}, X_{vU}]) = N((KV[X_{\mu U}, X_{vU}])^T)$
$KVX = [KV[X_{\mu L}, X_{vL}], KV[X_{\mu U}, X_{vU}]]$ *is an Ks.*
$X = [[X_{\mu L}, X_{vL}], KV[X_{\mu U}, X_{vU}]]$ *is an IV s - k ks.*
The aforementioned Theorems reduce to comparable criteria, in particular for $K = I$, *for different g-inverses of IV s-ks to be IV secondary ks.*

Corollary 4.1. *For* $P =< [P_{\mu L}, P_{vL}], [P_{\mu U}, P_{vU}] >\in IVIFM_{nn}$ *X goes to* $P\{1, 2\}$ *and*
$PX =< [P_{\mu L}, P_{vL}][X_{\mu L}, X_{vL}], [P_{\mu U}, P_{vU}][X_{\mu U}, X_{vU}] >$,
$XP =< [X_{\mu L}, X_{vL}][P_{\mu L}, P_{vL}][P_{\mu U}, P_{vU}] >$, *are is an s-ks . Then P is an IV s- ks*
$\Leftrightarrow X =< [X_{\mu L}, X_{vL}][X_{\mu U}, X_{vU}] >$ *is an IV s - ks.*

Corollary 4.2. *For* $P =< [P_{\mu L}, P_{vL}], [P_{\mu U}, P_{vU}] >\in IVIFM_{nn}$ *X goes to* $P\{1, 2, 3\}$ *and* $N(KV[P_{\mu L}, P_{vL}]) = N(V[X_{\mu L}, x_{vL}])^T, N(KV[P_{\mu U}, P_{vU}]) = N([X_{\mu U}, X_{vU}]^T >$.
Then P is an IV s- ks $\Leftrightarrow X =< [X_{\mu L}, X_{vL}][X_{\mu U}, X_{vU}] >$ *is an IV s - ks.*

Corollary 4.3. *For* $P =< [P_{\mu L}, P_{vL}], [P_{\mu U}, P_{vU}] >\in IVIFM_{nn}$ *X goes to* $P\{1, 2, 4\}$ *and* $N(V[P_{\mu L}, P_{vL}])^T = N(V[X_{\mu L}, x_{vL}])^T, N(V[P_{\mu U}, P_{vU}])^T = N([X_{\mu U}, X_{vU}] >$.
Then P is an IV s- ks iff X is an IV s - ks.

4.1. **Graphical Representation of kernel symmetric Adjacency IFM.**

Definition 4.1. Adjacency IFM : An adjacency Intuitionistic Fuzzy Matrix is a square matrix that serves as a representation for a finite graph. The matrix's elements convey information regarding whether pairs of vertices within the graph are connected or not. In the specific scenario of a finite simple graph, the adjacency matrix can be described as a binary matrix, often denoted as a $(1, 0)$ and $(0, 1)$ -matrix, where the diagonal elements are uniformly set to $(0,1)$. If $G(V,E)$ denote a simple graph with n vertices. The adjacency matrix $A=[a_{ij}]$ is a symmetric matrix defined

$$A=[a_j]=\begin{cases}(1,0)\ when\ v, is\ adiancent\ to\ V_j\\(0,1)\ otherwise\end{cases}\quad denoted\ by\ A(G)\ or\ A_G.$$

Example 4.1. Consider an adjacency IFM and a corresponding graph

$$A=\begin{bmatrix} v1 & v3 & v4 & v2 & v5\\ v1<0,1> & <1,0> & <1,0> & <0,1> & <0,1>\\ v3<1,0> & <0,1> & <0,1> & <0,1> & <1,0>\\ v4<1,0> & <0,1> & <0,1> & <1,0> & <0,1>\\ v2<0,1> & <0,1> & <1,0> & <0,1> & <1,0>\\ v5<0,1> & <1,0> & <0,1> & <1,0> & <0,1>\end{bmatrix}$$

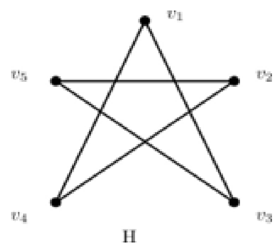

FIGURE 1

Definition 4.2. Incidence IFM: *If $G(V,E)$ represent a simple graph with n vertices. Let $V=V_1,V_2,\ldots,V_n$ and $E=e_1,e_2,...,e_m$. Then, the incidence IFM $I=[m_{ij}]$ is a matrix defined by*

$$A=[a_j]=\begin{cases}(1,0)\ when\ v\ is\ incidence\ to\ e_j\\(0,1)\ otherwise\end{cases}\quad denoted\ by\ A(G)\ or\ A_G.$$

Example 4.2. *Consider an incidence IFM and a corresponding graph. The incidence NFM is*

$$A=\begin{bmatrix} <1,0> & <1,0> & <0,1 & <0,1> & <0,1> & <0,1> & <1,0>\\ <1,0> & <1,0> & <0,1> & <1,0> & <0,1> & <1,0> & <0,1>\\ <0,1> & <1,0> & <1,0> & <0,1> & <0,1> & <0,1> & <0,1>\\ <0,1> & <0,1> & <1,0> & <1,0> & <1,0> & <0,1> & <0,1>\\ <0,1> & <0,1> & <0,1> & <0,1> & <1,0> & <1,0> & <1,0>\end{bmatrix}$$

Corresponding Graph

4.2. **Relation between isomorphism, non-isomorphism and KS.**

Graph A

Consider the graph G and name as follows

Let us consider adjacency matrix of the given graph is

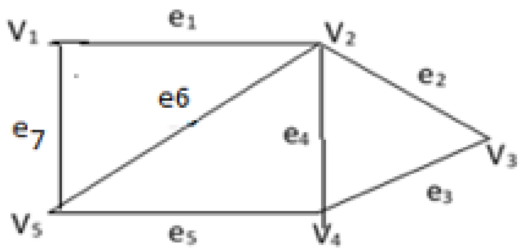

FIGURE 2

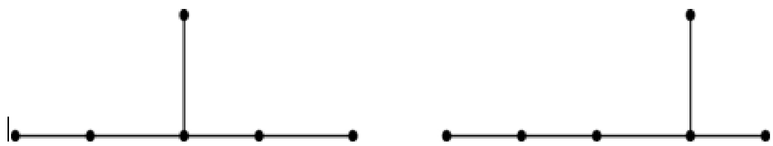

FIGURE 3

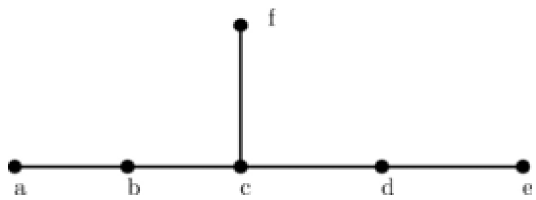

FIGURE 4

$$A = \begin{bmatrix} <0,1> & <0,1> & <0,1> & <0,1> & <0,1> & <0,1> \\ <1,0> & <0,1> & <1,0> & <0,1> & <0,1> & <1,0> \\ <0,1> & <1,0> & <0,1> & <1,0> & <0,1> & <1,0> \\ <0,1> & <0,1> & <1,0> & <0,1> & <1,0> & <0,1> \\ <0,1> & <0,1> & <0,1> & <1,0> & <0,1> & <0,1> \\ <0,1> & <0,1> & <1,0> & <0,1> & <0,1> & <0,1> \end{bmatrix}$$

Consider the graph H and name as follows

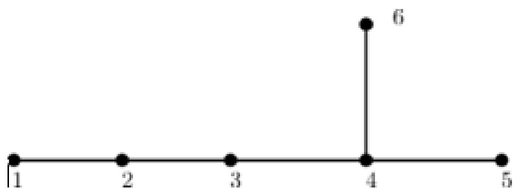

FIGURE 5

Consider the graph H and name as follows

Let us consider adjacency matrix of the given graph is

$$A = \begin{bmatrix} <0,1> & <0,1> & <0,1> & <0,1> & <0,1> & <0,1> \\ <1,0> & <0,1> & <1,0> & <0,1> & <0,1> & <1,0> \\ <0,1> & <1,0> & <0,1> & <1,0> & <0,1> & <1,0> \\ <0,1> & <0,1> & <1,0> & <0,1> & <1,0> & <0,1> \\ <0,1> & <0,1> & <0,1> & <1,0> & <0,1> & <0,1> \\ <0,1> & <0,1> & <1,0> & <0,1> & <0,1> & <0,1> \end{bmatrix}$$

The two graphs have the same number of vertices, same number of edges and same degree sequence. Though both the graphs have 3 pendent vertices, 2 vertices of degree 2 and 1 vertex of degree 3, the incidence relation of 3 pendent vertices are not preserved because in graph G 2 pendent vertices are attached to vertices of degree 2 and 1 pendent vertex is attached to vertex of degree 3 but in graph H only 1 pendent vertex is attached to vertex of degree 2 an 2 pendent vertices are attached to vertex of degree 3. Therefore, the isomorphism between the two graphs cannot be established.
Thus, the given two graphs are non-isomorphic.

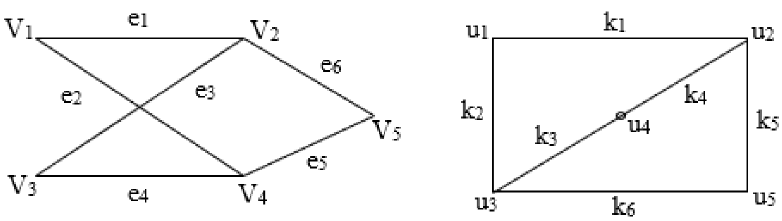

FIGURE 6

Let us consider adjacency matrix of the given graph is

$$G = \begin{bmatrix} <0,1> & <1,0> & <0,1> & <1,0> & <0,1> \\ <1,0> & <0,1> & <1,0> & <0,1> & <1,0> \\ <0,1> & <1,0> & <0,1> & <1,0> & <0,1> \\ <1,0> & <0,1> & <1,0> & <0,1> & <1,0> \\ <0,1> & <1,0> & <0,1> & <1,0> & <0,1> \end{bmatrix}$$

$$H = \begin{bmatrix} <0,1> & <1,0> & <0,1> & <1,0> & <0,1> \\ <1,0> & <0,1> & <1,0> & <0,1> & <1,0> \\ <0,1> & <1,0> & <0,1> & <1,0> & <0,1> \\ <1,0> & <0,1> & <1,0> & <0,1> & <1,0> \\ <0,1> & <1,0> & <0,1> & <1,0> & <0,1> \end{bmatrix}$$

There is a 1-1 correspondence between the vertices and edges. Therefore, the two graphs G and H are isomorphic.
The given two graphs have same number of vertices, edges and degree sequence and also the adjacency matrices are equal. Therefore the given Graph is isomorphic and also KS IFM.
Every isomorphic and non-isomorphic graph is KS adjacency IFM but converse need not be true.

5. Conclusions and Future Work:

In this study, we have examined the characterization of interval-valued secondary k-kernel symmetric Intuitionistic fuzzy matrices. We have explored the relationships between different types of matrices, including interval-valued s-kernel symmetric, s-kernel symmetric, interval-valued k-kernel symmetric, and interval-valued kernel symmetric matrices. Through this examination, we have provided a thorough understanding of the interplay among these matrix types and established the necessary and sufficient conditions for interval-valued s-kernel symmetric Intuitionistic fuzzy matrices. The findings contribute to the broader field of fuzzy matrices, offering a deeper insight into their structural properties and applications.

Future research could focus on extending the results of this study to more complex fuzzy structures, such as higher-order fuzzy matrices or fuzzy relational systems. Investigating the practical applications of interval-valued secondary k-kernel symmetric Intuitionistic fuzzy matrices in various fields, such as decision-making, optimization, and data analysis, would be a valuable direction. Moreover, exploring the computational aspects and algorithmic development for efficiently handling these matrices could lead to advancements in their practical implementation. Further studies could also examine the stability, robustness, and sensitivity of these matrices in real-world scenarios, potentially contributing to improved decision-making models in uncertain environments..

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