TWMS J. App. and Eng. Math. V.16, N.2, 2026, pp. 154-168

VISCOUS DISSIPATION, HALL CURRENT AND ION SLIP EFFECTS ON AN UNSTEADY MHD FLOW OVER AN INCLINED PLATE

S. AISWARYA[1*], K. BHUVANESHWARI[1], §

ABSTRACT. In this study, we have examined the effects of Hall current and ion slip on an unsteady MHD flow of a viscous, electrically conducting incompressible fluid over a porous medium bounded by an infinite inclined plate in the presence of viscous dissipation. Closed-form solutions are obtained for velocity, temperature, and concentration profiles using the regular perturbation technique. The effect of various non dimensional parameters such as Hall current (β_e), Ion slip parameter(β_i), Eckert number (Ec), Grashof number (Gr), Magnetic parameter(M), Modified Grashof number (Gm), Heat source parameter(Q), Dufour number (Du), Schmidt number (Sc), Permeability parameter (K), Radiation parameter(N), and Chemical Reaction parameter (ν), on velocity, temperature and concentration were discussed graphically. The results reveal that the Hall current increases velocity and decreases temperature by creating a secondary flow that reduces resistance and redistributes energy. In contrast, the ion-slip effect reduces velocity while increasing temperature due to increased resistance and internal energy dissipation.

Keywords: Hall current, ion slip, inclined plate, porous medium, heat and mass transfer, MHD, chemical reaction, Dufour effect.

AMS Subject Classification: 80A20, 74K20, 76S05, 74E40, 76E25

1. Introduction

Magnetohydrodynamic (MHD) flow across inclined plates is of great importance for a number of geophysical and technical applications, including cooling technologies, plasma systems, and MHD generators. MHD flows under simple assumptions have been studied by several researchers; however, the combined effects of ion slip, Hall current, and viscous dissipation in an unsteady flow situation have not been extensively addressed. The properties of momentum and heat transport are significantly impacted by internal friction, electromagnetic cross-forces, and ion-electron interactions in high-temperature and partly ionised environments. The unique aspect of this study is the presentation of a thorough

[1] Department of Mathematics, PSGR Krishnammal College for Women, Coimbatore, India.
e-mail:aishwaryas@psgrkcw.ac.in; ORCID : https://orcid.org/0000-0001-9142-6266.
e-mail:bhuvana7430@gmail.com; ORCID : https://orcid.org/0009-0006-3482-0068.
∗ Corresponding Author.
§ Manuscript received: January 02, 2025; accepted: April 24, 2025.
TWMS Journal of Applied and Engineering Mathematics, Vol.16, No.2;

model that depicts how various physical systems interact in an unstable MHD flow along an incline. This method provides more insight into the dynamics of conducting fluids and improves forecasting capabilities for real-world thermal-fluid systems. Osalusi et al. [21] examined the influence of Hall current and ion slip on Bingham fluid in rotating fluid over a porous matrix in the presence of joule heating and viscous dissipation. They observed that an increase in the Hall parameter, ion-slip parameter, increases the temperature profiles, whereas an increase in the magnetic parameter and Eckert number decreases the rate of heat transfer coefficient. Jha et al.[11] analysed the influence of suction and injection on unsteady MHD Couette flow in a rotating system in the presence of Hall and ion slip parameters. Sigey et al. [27] numerically investigated the magnetohydrodynamics free convective flow past an infinite vertical porous plate with Joule's heating. They found that an increase in Joule's parameter leads to an increase in velocity and temperature, and found that it is uniformly distributed near the plate, but as we move away from the plate, the velocity and temperature profiles remain constant. Seth et al. [26] carried out a numerical study to analyse the influence of hall current and rotation effect on an unsteady MHD natural convective flow of chemically reacting and radiating fluid past a moving vertical plate. Maripala and Naikoti [18]considered a stretching sheet with variable viscosity and viscous dissipation under the influence of Hall current and found that the velocity profile decreases with an increase in Eckert number. Ojjela and Kumar [20] analysed the influence of hall and ion slip currents on micropolar fluid with velocity slip condition with expansion or contraction. Sandhya et al. [25]analysed the effects of viscosity and Dufour number on MHD free transport on a perforated plate with ion slip current, and it was found that the ion slip parameter increases the flow field and decreases the temperature. Increases in velocity and temperature were observed with an increase in Hall currents. Rajakumar et al. [23] carried out an analytical study to analyse the impact of Dufour number, Hall current, ion slip current and viscous dissipation on an unsteady free convective oscillatory flow through a semi-infinite porous inclined plate. Falodun and Ahmed [6] studied the effects of unsteady Casson magneto nanofluid flow and observed that as the non-Newtonian Casson nanofluid parameter increases, the fluid velocity decreases. The temperature profile increases with the Soret parameter, whereas concentration decreases with an increase in the Dufour parameter. Babu et al.[3] examined the combined effects of Joule's and viscous dissipation on unsteady natural convective MHD Casson fluid past a vertically inclined plate in the presence of heat and mass transfer. Veera Krishna et al.[31] studied the effects of radiation and hall current on an unsteady magnetohydrodynamics free convective flow in a vertical channel filled with a porous medium. They observed that the velocity component for the primary flow enhances with an increase in permeability parameter, Hall parameter and seems to have a reduction with an increase in the intensity of the magnetic field and radiation parameter. Anand et al. [16]examined the resultant effect of the Hall current on free convective flow of a micropolar fluid through a porous surface with an inclined magnetic field. Sadiq Basha et al. [24] considered the steady fully developed MHD free convection flow through a porous medium in a microchannel bounded by two infinite vertical plates due to asymmetric heating of plates, taking Hall and ion slip effects into account. Vedavathi et al.[30] carried out an analytical study to investigate the Hall Effect on Nanofluids in the presence of ion slip on an incompressible flow, and they found that increasing thermal convection of nanoparticles during drug delivery will help to destroy cancer cells. Islam et al. [9] studied mixed convective flow of electrically conducting fluid in the presence of Dufour and observed that the velocity decreases with an increase in heat source parameter, whereas found increasing nature with an increase in Dufour number. Krishna et al. [13] discussed about the influence of Hall and ion slip on a rotating magnetohydrodynamic nanofluid through a porous medium and found that due to the increased effect of thermal convection of a nanofluid, cancer cells can be destroyed during drug delivery. Krishna [15] investigated the impact of radiation-absorption, hall current, ion slip and chemical reaction on MHD free convective flow and observed that resultant velocity increases as the Hall and ion slip parameter increases. Nhial et al. [19] discussed about radiation and mass transfer effects on MHD free convective flow of an electrically conducting incompressible viscous fluid and found that velocity as well as concentration decreases with an increase in the Schmidt number and chemical reaction parameter. Alim and Ali [2] examined the impact of viscous dissipation, slip parameter and Joule's heat parameters on the flow of MHD nanofluids over a permeable wedge surface. Raghunath et al. [22] explored the impact of radiation, hall and ion slip, on Jeffrey fluid embedded on a porous medium in the presence of chemical reaction. They observed that an increase in the radiation parameter, Prandtl number and the heat source parameter increases the heat transfer coefficient. Bilal et al. [4] used Ag and AI2O3 nanoparticles to study the behaviour of Jeffrey fluid (blood) and discovered that when magnetic and viscoelastic components rise in steady state flow, heat transfer increases while velocity magnitude decreases. Ganjikunta et al. [7] investigated the effect of radiation absorption and chemical reaction combined with the effects of Hall and ion slip effects on a rotating flow of a heat-producing fluid across an inclined surface. The results of the study show that velocity is increased by increasing radiation absorption, Hall, and ion slip parameters, with thermal and solutal buoyant forces playing a role. While Hall and ion slip effects increase skin friction, the rotation parameter decreases it. Sridevi et al. [29] studied the effects of radiation absorption, diffusion thermodynamics, and Hall current on the unsteady magnetohydromagnetic free convection flow of a viscous incompressible fluid between inclined porous plates. The findings indicate that velocity is increased by higher radiation absorption and hall current parameters, which are influenced by thermal and solutal buoyancy forces. Alamirew et al. [1] in their work examined the significance of MHD Williamson nanofluid flow across a stretched plate with Hall and ion-slip effects while taking nonlinear radiation, viscous dissipation, chemical reaction, and Joule heating into account. Unsteady convection flow of a conducting viscous fluid across an inclined plate with Hall and ion-slip effects was investigated by Islam et al. [10]. An oscillating plate drives the flow, which is affected by magnetic and buoyant forces. By ignoring magnetic induction and applying boundary layer theory, the results demonstrate that tilt decreases velocity. Kanimozhi et al.[12] investigated the behaviour of Jeffrey fluids in a variety of circumstances, such as chemical influences, magnetic fields, heat effects, and slip situations. Soret and Hall currents, velocity, temperature, and concentration differences are taken into account in this study on mixed convective flow across an inclined vertical plate. An incompressible viscous electrically conducting non-Newtonian Casson hybrid nanofluid's radiative MHD flow across an exponentially accelerated vertical porous surface is examined by Krishna [14]. Slip velocity, Hall and ion slip impacts, and the development of titanium and silver nanoparticles in the base fluid are all taken into account in this work. Gomathi and De [8] analysed the transmission of mass and energy in a viscous dissipative Casson Williamson nanofluid while taking the Hall current and ion slip effect into account. The research solves non-linear ordinary differential equations using the fifth-order Runge–Kutta–Fehlberg technique. A thorough grasp of fluid behaviour and mass transport mechanisms is possible through contour analysis. The results indicate that injection and suction have higher Nusselt number sensitivity.Lakshmi and Murty [17] investigated how water-based Cu and TiO_2 nanofluids move in a magnetohydrodynamic (MHD) environment in relation to Hall current, rotational effects, and thermal diffusion.

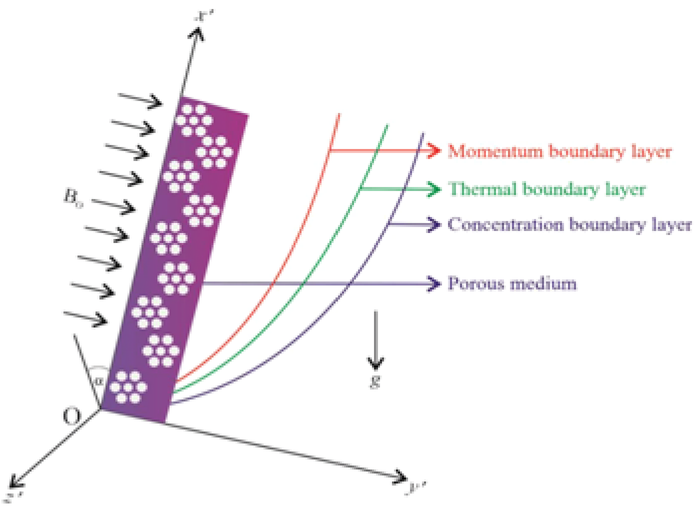

FIGURE 1. Physical model and coordinate system

It demonstrates that raising the Hall parameter causes the primary flow velocity to increase while the secondary velocity drops. While a stronger magnetic field slows fluid motion because of Lorentz force effects, a higher thermal Grashof number improves velocity. Titanium oxide-engine oil (TiO_2-EO) nanofluid transient hydromagnetic convective flow across a vertically oriented surface was investigated by Singh et al. [28] in relation to Hall and ion-slip. Based on the data, the major flow is stabilised by both ion slip and Hall current, whereas the secondary flow is destabilised by the Hall effect.

Inspired by the numerous applications of Hall current and ion slip parameters on the transport of flow of an incompressible fluid over an inclined plate, we have extended the work of Islam et al.[9] to integrate the effects of viscous dissipation, Hall current and ion slip on MHD free convection on an inclined plate embedded in a porous medium.

2. Mathematical Formulation

Consider an unsteady flow of an incompressible magnetohydrodynamic fluid through a porous medium in an infinite inclined plate with variable heat and mass transfer in the presence of Hall current, ion slip, viscous dissipation, along with heat source and Dufour effect.

The x-axis is considered along the plate with the angle of inclination α to the vertical, and the y-axis is taken normal to the plate. Assuming that the fluid is electrically conducting, a uniform magnetic field B_0 is provided transversely to the plate in the y direction. The electric field generated by the polarisation of charges is zero, since there is no applied voltage placed on the flow field.

At first, the temperature and concentration of the fluid and the plate are both kept constant (T_∞ and C_∞). The plate receives a tiny shock at time t > 0 and motion is produced with uniform velocity u_0 in the direction of flow counter to gravity. The plate's temperature and concentration rise linearly as time passes. Each physical variable is a function of y and t alone because the plate is infinite in the (x, z) plane.

The basic governing equations are as follows,

$$\nabla.\bar{q} = 0 \tag{1}$$

$$\rho\left(\frac{\partial \bar{q}}{\partial t} + (\bar{q}.\nabla)\bar{q}\right) = -\nabla p + \bar{j} \times \bar{B} + \rho g + \bar{\mu}\nabla^2\bar{q} - \frac{\mu\bar{q}}{k^*} \tag{2}$$

$$\rho C_p\left(\frac{\partial \bar{T}}{\partial t} + (\bar{q}.\nabla)T\right) = K\nabla^2 T + \bar{\nabla}.\bar{q}_r + \frac{\rho D_M K_T}{C_S}\nabla^2\bar{C} + \varphi \tag{3}$$

$$\frac{\partial C}{\partial t} + (\bar{q}.\nabla)C = D_M\nabla^2 C + \bar{k}(C - C_\infty) \tag{4}$$

Owing to flow assumption and using the Boussinesq approximation, the governing equations of the problem will be reduced to[23],

$$\frac{\partial u'}{\partial t'} = \nu\frac{\partial^2 u'}{\partial y'^2} + g\beta(T - T_\infty)cos\alpha + g\beta(C - C_\infty)cos\alpha$$
$$-\frac{B_0 J_z}{\rho} - \frac{\nu}{k^*}u' \tag{5}$$

$$\frac{\partial w'}{\partial t'} = \nu\frac{\partial^2 w'}{\partial y'^2} + \frac{B_0 J_x}{\rho} - \frac{\nu}{k^*}w' \tag{6}$$

$$\frac{\partial T'}{\partial t'} = \frac{K}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y} + \frac{D_M K_T}{C_p C_s}\frac{\partial^2 C'}{\partial y'^2} + \frac{Q_0}{\rho C_p}(T - T_\infty)$$
$$+\frac{\mu}{\rho C_p}\left[\left(\frac{\partial u'}{\partial y'}\right)^2 + \left(\frac{\partial w'}{\partial y'}\right)^2\right] \tag{7}$$

$$\frac{\partial C'}{\partial t'} = D_M\frac{\partial^2 C}{\partial y'^2} - K_1(C - C_\infty) \tag{8}$$

The boundary conditions are,

$$\begin{aligned} &t' \leq 0 : u' = 0, w' = 0, T = T_\infty, C = C_\infty \text{ for all } y' > 0 \\ &t' > 0 : u' = U_0, w' = 0, T = T_w + (T_w - T_\infty)e^{nt}, C = C_w + (C_w - C_\infty)e^{nt} \text{ at } y' = 0 \\ &u' = 0, w' = 0, T = T_\infty, C = C_\infty \text{ as } y' \to \infty \end{aligned} \tag{9}$$

The electron-atom collision frequency is assumed to be very high, so that Hall and ion slip currents cannot be neglected. Hence, the Hall and ion slip currents give rise to the velocity in the y-direction. When the strength of the magnetic field is very large, the generalised Ohm's law is modified to include the Hall and ion slip effect. The electron pressure gradient and thermoelectric effects are abandoned, i.e., the electric field E = 0. The Generalised Ohm's law is defined as

$$J = \sigma(E + q \times B) - \frac{\omega_e\tau_e}{B_0}(J \times B) + \frac{\omega_e\tau_e\beta_i}{B_0^2}((J \times B) \times B)$$

Upon solving, we get

$$J_x = \sigma B_0(\alpha_2 u - \alpha_1 w)$$
$$J_z = \sigma B_0(\alpha_2 w + \alpha_1 u)$$

where $\alpha_1 = \frac{1+\beta_e\beta_i}{(1+\beta_e\beta_i)^2+\beta_e{}^2}, \alpha_2 = \frac{\beta_e}{(1+\beta_e\beta_i)^2+\beta_e{}^2}$ $\beta_e = \omega_e\tau_e \sim O$ (1) (Hall parameter) and $\beta_i = \omega_i\tau_i \ll 1$, (ion slip parameter)

Using Rosseland approximation, the radiative heat flux q_r is defined as

$$q_r = \frac{-4\sigma^*}{3a^*}\frac{\partial T'^4}{\partial y}$$

Now omitting higher order terms and expanding T'^4 in Taylor series about T_∞ we obtain

$$T'^4 = 4T_\infty'^3 T' - 3T_\infty^4$$

$$\frac{\partial q_r}{\partial y} = \frac{-16\sigma^* T_\infty^3}{3a^*}\frac{\partial^2 T'}{\partial y'^2} \tag{10}$$

In view of equation (10), equation (7) will be rewritten as

$$\frac{\partial T'}{\partial t'} = \frac{K}{\rho C_p}\frac{\partial^2 T'}{\partial y^2} + \frac{1}{\rho C_p}\frac{16\sigma^2 T_\infty^3}{3a^2}\frac{\partial^2 T'}{\partial y^2} + \frac{D_M K_T}{C_p C_s}\frac{\partial^2 C'}{\partial y^2} + \frac{Q_0}{\rho C_p}(T - T_\infty)$$
$$+\frac{\mu}{\rho c_p}\left[\left(\frac{\partial u'}{\partial y'}\right)^2 + \left(\frac{\partial w'}{\partial y'}\right)^2\right] \tag{11}$$

Introducing non-dimensional quantities

$$t = \frac{t' u_0^2}{v}, u = \frac{u'}{u_0}, w = \frac{w'}{u_0}, y = \frac{y' u_0}{v}, \theta = \frac{T - T_\infty}{T_w - T_\infty}, \varphi = \frac{c - C_\infty}{C_w - C_\infty},$$

The non-dimensional form of the equations (5,6,8,11) are represented as follows.

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + Gr\theta\cos\alpha + Gm\varphi\cos\alpha - M^2(\alpha_1 w - i\alpha_2 u) - \frac{1}{k}u \tag{12}$$

$$\frac{\partial w}{\partial t} = \frac{\partial^2 w}{\partial y^2} + M^2(\alpha_1 u - i\alpha_2 w) - \frac{1}{k}w \tag{13}$$

$$\frac{\partial\theta}{\partial t} = \left(\frac{1+N}{Pr}\right)\frac{\partial^2\theta}{\partial y^2} + Du\frac{\partial^2\varphi}{\partial y^2} + \frac{Q\theta}{Pr} + Ec\left(\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right) \tag{14}$$

$$\frac{\partial\phi}{\partial t} = \frac{1}{sc}\frac{\partial^2\phi}{\partial y^2} - \gamma\varphi \tag{15}$$

The non-dimensional parameters are as follows
$Gr = \frac{vg\beta(T_w - T_\infty)}{v_0^3}$ (Grashof number);Gm = $\frac{vg\beta(C_w - C_\infty)}{v_0^3}$ (Modified Grashof number) Pr = $\frac{\mu c_p}{k}$ (Prandt1 number) $K = \frac{k^* U_o^2}{v^2}$ (Permeability); $Sc = \frac{v}{D_M}$ (Schmidt number) $M = \sqrt{\frac{\sigma B_0^2 v}{\rho U_0^2}}$ (Magnetic field parameter; $N = \frac{16\sigma T_\infty^3}{3ka*}$ (Radiation parameter); $Q = \frac{\gamma^2 Q_0}{kU_0^2}$ (Heat source parameter); Du = $\frac{D_T K_T (C_w - C_\infty)}{\gamma C_p C_s (T_w - T_\infty)}$ (Dufour number);Ec = $\frac{U_0^2}{c_p(T_w - T_\infty)}$ (Eckert number) $\gamma = \frac{K_1}{v_0^2}$ (Chemical reaction parameter)

The modified boundary conditions are
$t \le 0 : u = 0, w = 0, \theta = 0, \varphi = 0$ for all $y > 0$
$t > 0 : u = 1, w = 0, \theta = 1 + e^{nt}, \varphi = 1 + e^{nt}$ at $y = 0$
$u = 0, w = 0, \theta = 0, \varphi = 0$ as $y \to \infty$

Let $q = u + iw$. Now combining equations (12) and (13) we have,

$$\frac{\partial q}{\partial t} = v\frac{\partial^2 q}{\partial y^2} + Gr\theta\cos\alpha + Gm\varphi\cos\alpha - M^2(\alpha_1 - i\alpha_2) - \frac{q}{k} \tag{16}$$

3. METHOD OF SOLUTION

In order to solve the dimensionless governing equations, we apply the regular perturbation technique. This can be done by representing the velocity, temperature and concentration as follows.

$$\left.\begin{aligned} q &= q_0(y) + \epsilon q_1(y)e^{nt} \\ \theta &= \theta_0(y) + \epsilon\theta_1(y)e^{nt} \\ \varphi &= \varphi_0(y) + \epsilon\varphi_1(y)e^{nt} \end{aligned}\right\} \tag{17}$$

Substituting equation (17) in equations (14), (15) and (16) and equating like terms and neglecting higher order terms of ϵ we get

$$q_0'' - \left(M^2(\alpha_1 - i\alpha_2) + \frac{1}{k}\right)q_0 = -Gr\theta_0 - Gm\varphi_0 \tag{18}$$

$$q_1'' - \left(M^2(\alpha_1 - i\alpha_2) + n + \frac{1}{k}\right)q_1 = -Gr\theta_1 - Gm\varphi_1 \tag{19}$$

$$\left(\frac{N+1}{Pr}\right)\theta_0'' + \frac{Q}{Pr}\theta_0 = -Du\varphi_0'' - Ec\left(q_0'\overline{q_0'}\right) \tag{20}$$

$$\left(\frac{N+1}{Pr}\right)\theta_1'' + \left(\frac{Q}{Pr} - n\right)\theta_1 = -Du\varphi_0'' - Ec\left(q_0'\overline{q_1'} - \overline{q_0'}q_1'\right) \tag{21}$$

$$\varphi_0'' - \mathrm{Scv}\,\varphi_0 = 0 \tag{22}$$

$$\varphi_1'' - \mathrm{Sc}(v+n)\varphi_1 = 0 \tag{23}$$

The modified boundary conditions are

$$q_0 = 1, q_1 = 0, \theta_0 = 1, \theta_1 = 1, \varphi_0 = \varphi_1 = 1 \text{ at y} = 0$$
$$q_0 = q_1 = \theta_0 = \theta_1 = \varphi_0 = \varphi_1 = 0 \text{ as } y \to \infty$$

On solving (22) & (23) with the help of the boundary condition, we get the solutions of φ_0 and φ_1 as follows.

$$\varphi_0 = e^{-a_1 y}$$
$$\varphi_1 = e^{-a_2 y}$$

To solve the non-linear coupled equation, we use the Eckert number (Ec) as the perturbation parameter and assuming Ec << 1, the asymptotic expansions are as follows:

$$\left.\begin{aligned} q_0 &= q_{00} + Ecq_{01} + o(Ec)^2 \\ q_1 &= q_{10} + Ecq_{11} + o(Ec)^2 \\ \theta_0 &= \theta_{00} + Ec\theta_{01} + o(Ec)^2 \\ \theta_1 &= \theta_{10} + Ec\theta_{11} + o(Ec)^2 \end{aligned}\right\} \tag{24}$$

Substituting equation (24) in equation (18) to (21) and equating like terms and neglecting higher order terms of Ec, we get the set of following equations

$$q''_{00} - \left(M^2(\alpha_1 - i\alpha_2) + \tfrac{1}{k}\right) q_{00} = -Gr\theta_{00} - Gm\varphi_0 \quad (25)$$

$$q''_{01} - \left(M^2(\alpha_1 - i\alpha_2) + \tfrac{1}{k}\right) q_{01} = -Gr\theta_{01} \quad (26)$$

$$q''_{10} - \left(M^2(\alpha_1 - i\alpha_2) + \tfrac{1}{k} + n\right) q_{10} = -\operatorname{Gr}\theta_{10} - Gm\varphi_1 \quad (27)$$

$$q''_{11} - \left(M^2(\alpha_1 - i\alpha_2) + \tfrac{1}{k} + n\right) q_{11} = -Gr\theta_{11} \quad (28)$$

$$\left(\tfrac{N+1}{Pr}\right)\theta''_{00} + \left(\tfrac{Q}{Pr}\right)\theta_{00} = -Du\varphi''_0 \quad (29)$$

$$\left(\tfrac{N+1}{Pr}\right)\theta''_{01} + \left(\tfrac{Q}{Pr}\right)\theta_{01} = -\left(q'_{10}\overline{q'_{00}}\right) \quad (30)$$

$$\left(\tfrac{N+1}{Pr}\right)\theta''_{10} + \left(\tfrac{Q}{Pr} - n\right)\theta_{10} = -Du\varphi''_1 \quad (31)$$

$$\left(\tfrac{N+1}{Pr}\right)\theta''_{11} + \left(\tfrac{Q}{Pr} - n\right)\theta_{11} = -\left(q'_{00}\overline{q'_{10}} + \overline{q'_{00}}q'_{10}\right) \quad (32)$$

The corresponding boundary conditions are as follows

$$\begin{aligned}
&q_{00} = 1, q_{01} = 0, \theta_{00} = 1, \theta_{01} = 0 \text{ at } y = 0\\
&q_{10} = 0, q_{11} = 0, \theta_{10} = 1, \theta_{11} = 0 \text{ at } y = 0\\
&q_{00} = q_{01} = q_{10} = q_{11} = 0 \text{ as } y \to \infty\\
&q_{10} = q_{11} = \theta_{10} = \theta_{11} = 0 \text{ as } y \to \infty
\end{aligned}$$

Upon solving equations (25) - (32) with the help of boundary condition we get the solutions of $\theta_{00}, q_{00}, q_{01}, \theta_{11}, q_{11}$ as follows.

$$\theta_{00} = A_4 e^{-a_2 y} + C_3 e^{-a_1 y}$$

$$q_{00} = A_6 e^{-a_4 y} + C_5 e^{-a_2 y} + C_8 e^{-a_1 y}$$

$$\theta_{01} = A_8 e^{-a_3 y} + C_{15} e^{-2a_1 y} e^{-2a_2 y} + C_{17} e^{-2a_4 y} + C_{18} e^{-(a_1+a_2)y} + C_{19} e^{-(a_1+a_4)y} + C_{20} e^{-(a_2+a_4)y}$$

$$q_{01} = A_{10} e^{-a_4 y} + C_{21} e^{-a_3 y} + C_{16} e^{-2a_2 y} + C_{22} e^{-2a_1 y} + C_{23} e^{-2a_2 y} + C_{24} e^{-2a_4 y} + C_{25} e^{-(a_1+a_2)y} + C_{26} e^{-(a_1+a_4)y} + C_{27} e^{-(a_2+a_4)y}$$

$$\theta_{10} = A_{12} e^{-a_6 y} + A_{13} e^{-a_2 y}$$

$$q_{10} = A_{15} e^{-a_1 y} + A_{16} e^{-a_6 y} + A_{17} e^{-a_2 y}$$

$$\begin{aligned}
\theta_{11} &= A_{19} e^{-a_6 y} + A_{20} e^{-(a_4+a_8)y} + A_{21} e^{-(a_4+a_6)y} + A_{22} e^{-(a_2+a_4)y} + A_{23} e^{-(a_2+a_8)y}\\
&+ A_{24} e^{-(a_2+a_6)y} + A_{25} e^{-2a_2 y} + A_{26} e^{-(a_1+a_8)y} + A_{27} e^{-(a_1+a_6)y}\\
&+ A_{28} e^{-(a_1+a_2)y}
\end{aligned}$$

$$\begin{aligned}
q_{11} = A_{30} e^{-a_7 y} &+ A_{31} e^{-a_6 y} + A_{32} e^{-(a_4+a_8)y} + A_{33} e^{-(a_4+a_6)y} + A_{34} e^{-(a_2+a_4)y}\\
&+ A_{35} e^{-(a_2+a_8)y} + A_{36} e^{(a_2+a_6)y} + A_{37} e^{-2a_2 y} + A_{38} e^{-(a_1+a_8)y}\\
&+ A_{39} e^{-(a_1+a_6)y}\\
&+ A_{40} e^{-(a_1+a_2)y}
\end{aligned}$$

Hence the solution of velocity, temperature and concentration fields are represented as follows

$$q = A_6e^{-a_4y}+C_5e^{-a_2y} + C_8e^{-a_1y} + Ec\left(A_{10}e^{-a_4y} + C_{21}e^{-a_3y} + e^{-2a_2y} + C_{22}e^{-2a_1y}\right.$$
$$\left.+C_{23}e^{-2a_2y} + C_{24}e^{-2a_4y} + C_{25}e^{-(a_1+a_2)y} + C_{26}e^{-(a_1+a_4)y} + C_{27}e^{-(a_2+a_4)y}\right)$$
$$+ \epsilon\left(A_{15}e^{-a_8y} + A_{16}e^{-a_6y} + A_{17}e^{-a_2y}\right.$$
$$+ Ec\Big(A_{30}e^{-a_7y} + A_{31}e^{-a_6y} + A_{32}e^{-(a_4+a_8)y} + A_{33}e^{-(a_4+a_6)y}$$
$$+ A_{34}e^{-(a_2+a_4)y} + A_{35}e^{-(a_2+a_8)y} + A_{36}e^{(a_2+a_6)y} + A_{37}e^{-2a_2y}$$
$$\left.+A_{38}e^{-(a_1+a_8)y} + A_{39}e^{-(a_1+a_6)y} + A_{40}e^{-(a_1+a_2)y}\Big)\right)e^{nt}$$

$$\theta = A_4e^{-a_2y} + C_3e^{-a_1y} + Ec\left(A_8e^{-a_3y} + C_{15}e^{-2a_1y} + C_{16}e^{-2a_2y} + C_{17}e^{-2a_4y}\right.$$
$$\left.+C_{18}e^{-(a_1+a_2)y} + C_{19}e^{-(a_1+a_4)y} + C_{20}e^{-(a_2+a_4)y}\right) + \epsilon\left(A_8e^{-a_3y}\right.$$
$$+ C_{15}e^{-2a_1y} + C_{16}e^{-2a_2y} + C_{17}e^{-2a_4y} + C_{18}e^{-(a_1+a_2)y} + C_{19}e^{-(a_1+a_4)y}$$
$$+ C_{20}e^{-(a_2+a_4)y}$$
$$+ Ec\Big(A_{19}e^{-a_6y} + A_{20}e^{-(a_4+a_8)y} + A_{21}e^{-(a_4+a_6)y} + A_{22}e^{-(a_2+a_4)y}$$
$$+ A_{23}e^{-(a_2+a_8)y} + A_{24}e^{-(a_2+a_6)y} + A_{25}e^{-2a_2y} + A_{26}e^{-(a_1+a_8)y}$$
$$+ A_{27}e^{-(a_1+a_6)y} + A_{28}e^{-(a_1+a_2)y)})e^{nt}$$

$$\varphi = e^{-a_1y}+\epsilon\left(e^{-a_2y}\right)e^{nt}$$

4. RESULTS AND DISCUSSION

The influence of hall current, ion slip and viscous dissipation on free convective flow of an incompressible viscous fluid over an inclined plate embedded in a porous medium has been examined. To point out the effects of various physical parameters on the flow field like Hall current (β_e), ion slip (β_i), Eckert number (Ec), Magnetic parameter(M), Schmidt number (Sc), Grashof number (Gr), Dufour number (Du), chemical reaction parameter (Nu) and radiation parameter(N) were computed and presented in figures from (2) - (15).

In the present study, we have fixed a few parameter values such as $\epsilon = 0.01$, t = 1.5, n = 0.5 and i = 0.2.

Figures (2) and (3) show the effects of ion slip and Hall current on the velocity profile. Ion slip is the relative mobility of ions and neutral fluids driven by electromagnetic fields. A reduction in velocity suggests that ion slip is adding resistive forces (such as Joule heating or magnetic drag) that slow the fluid down. It also reduces the gravity-induced flow acceleration on an inclined plane, causing the fluid to move more slowly than expected. While velocity rises in the presence of a Hall current on an inclined plane, it suggests that the Hall effect lessens electromagnetic braking, allowing gravity to take over, increasing fluid velocity.

Figures (4) and (5) show the effect of Eckert number and Magnetic Parameter on velocity fields. A higher Eckert number creates more internal heat due to viscous dissipation, increasing fluid resistance through viscosity. A transverse magnetic field generates a Lorentz force, which acts as a resistive force, opposing fluid motion and reducing velocity.

When a chemical reaction progresses, it can increase fluid density or viscosity, lowering fluid velocity, as seen in figure (6). Figure (7) depicts the Dufour effect's effects on fluid velocity. The Dufour effect redistributes energy in the fluid, increasing thermal resistance and slowing along the wall (where gradients are steep), while invigorating the flow further out and boosting velocity in the core area.

Figures (8) and (9) illustrate how ion-slip and Hall current affect temperature profiles. Figure (8) illustrates how ion slip causes the ions to travel in the fluid differently than the electrons, increasing internal friction and producing additional heat. This results in a significant rise in the temperature. The charged particles are pushed in opposing directions by the magnetic field as the Hall current increases. As a result of less friction in the flow direction, the fluid experiences less internal heating. Consequently, the temperature of the fluid has significantly dropped.

The fluid temperature decreases overall as seen in figure (10), since higher Ec produces more heat that is drawn out faster via the wall (due to its variable and perhaps dropping temperature). Figure (11) illustrates the impact of Schmidt number on the temperature profile. Higher Schmidt number values block or deflect heat because they result in less thermal energy transfer via concentration gradient.

As seen in figure (12), thermal energy moves from high concentration areas to low concentration areas as a function of the Dufour number. Strong concentration gradients close to the wall cause heat to be pulled away, lowering the temperature. Heat is essentially transferred from the wall to the fluid mass as this energy diffuses outward, raising the temperature in areas further away from the wall. A high radiation parameter results in radiative energy loss, which reduces the fluid's temperature, especially if the boundary emits less heat than the fluid, as shown in figure (13).

Figures (14) and (15) depict the impact of Schmidt number and chemical reaction parameter on the concentration profile. It has been discovered that when the Schmidt number increases, the species finds it more difficult to disseminate or penetrate the fluid, causing the concentration to drop faster along the wall and typically be lower in the flow. In contrast, when the chemical reaction parameter increases, species consumption accelerates, lowering the concentration profile.

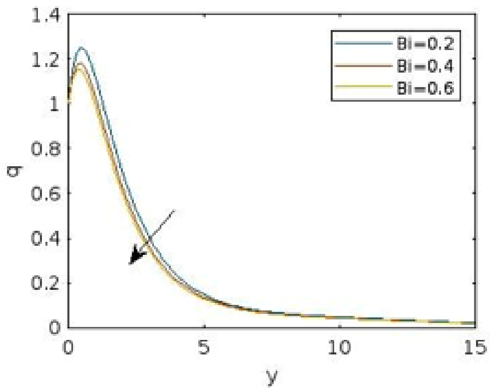

FIGURE 2. Velocity profile for various values of ion slip parameter (β_i)

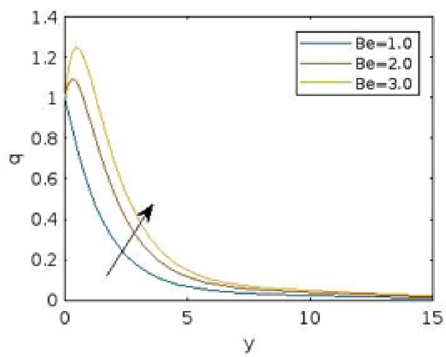

FIGURE 3. Velocity profile for various values of Hall Current parameter (β_e)

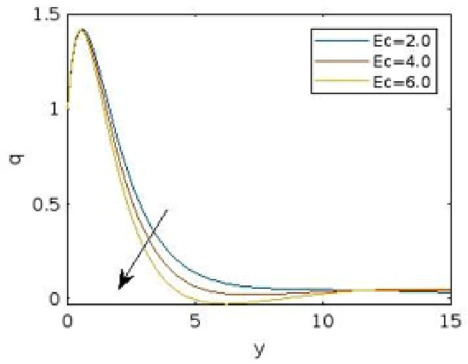

FIGURE 4. Velocity profile for various values of Eckert number (Ec)

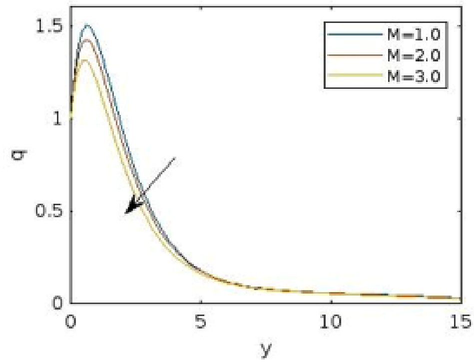

FIGURE 5. Velocity profile for various values of magnetic parameter (M)

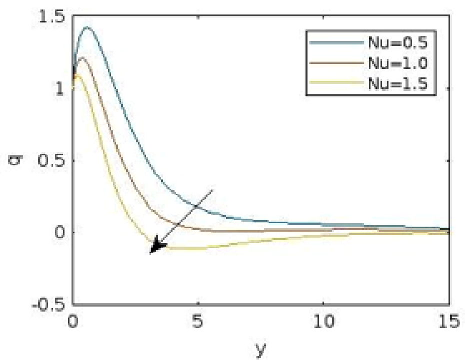

FIGURE 6. Velocity profile for various values of chemical reaction parameter (Nu)

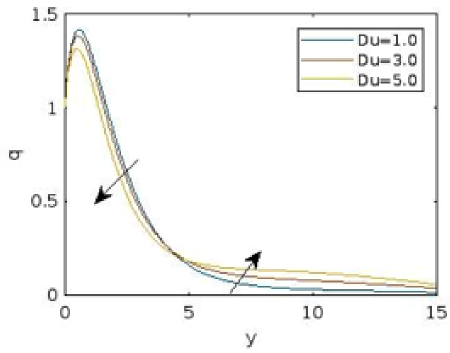

FIGURE 7. Velocity profile for various values of Dufour number (Du)

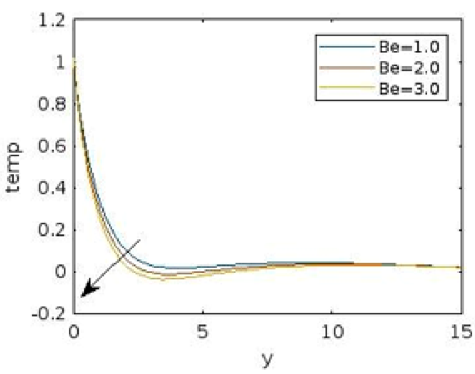

FIGURE 8. Temperature profile for various values of Hall Current parameter (β_e)

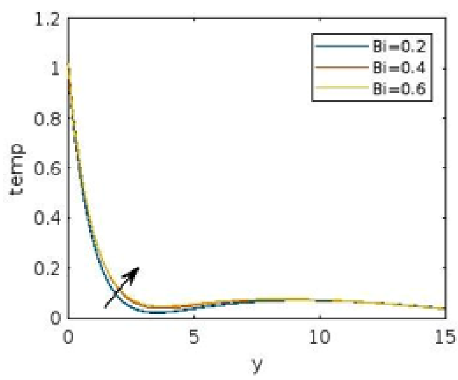

FIGURE 9. Temperature profile for various values of Ion slip parameter (β_i)

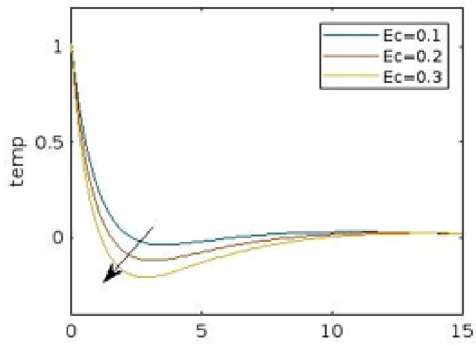

FIGURE 10. Temperature profile for various values of Eckert number (Ec)

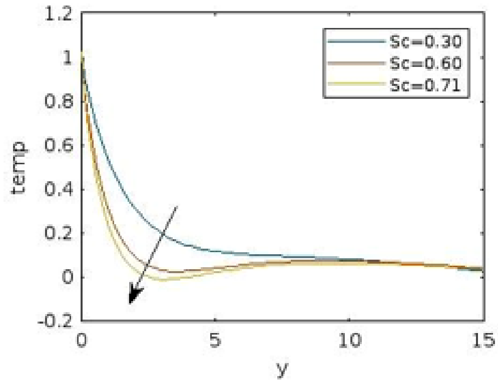

FIGURE 11. Temperature profile for various values of Schmidt number (Sc)

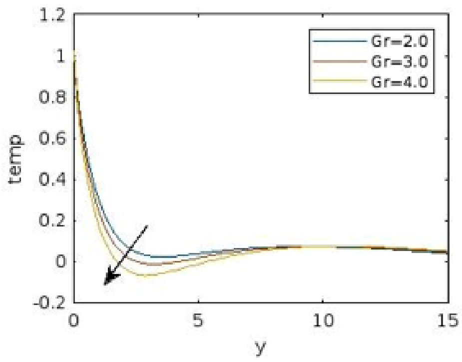

FIGURE 12. Temperature profile for various values of Grashof number (Gr)

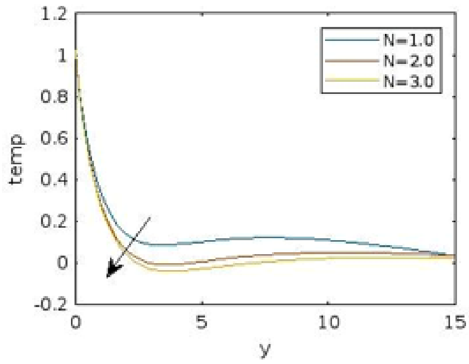

FIGURE 13. Temperature profile for various values of Radiation parameter(N)

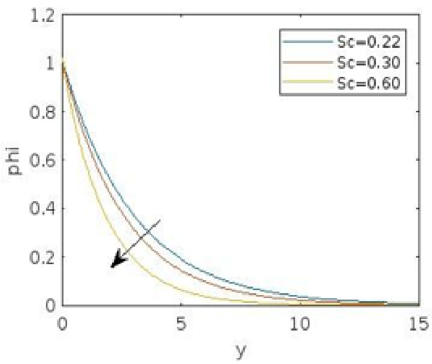

FIGURE 14. Concentration profile for various values of Schmidt number (Sc)

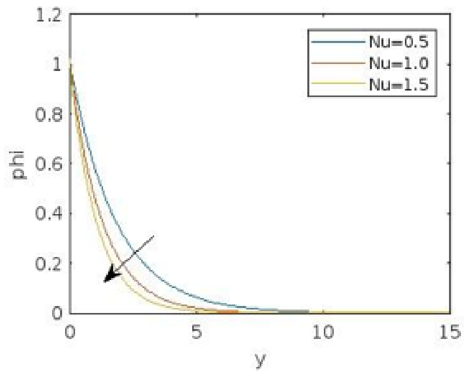

FIGURE 15. Concentration profile for various values of Chemical reaction (ν)

5. CONCLUSION

The effects of viscous dissipation, Hall current, and ion-slip currents are considered on an unsteady MHD flow through a porous medium in an infinite inclined plate. The reduced governing equations are analytically solved using the regular perturbation approach. The results are analysed through graphs for various values, and from those we conclude that:

- The Hall current increases fluid velocity by decreasing electromagnetic resistance while also lowering temperature due to reduced internal friction.
- The ion-slip effect, on the other hand, lowers velocity while increasing temperature due to extra resistive forces and heat production.
- Because of increased viscous and magnetic damping, the Eckert number and magnetic parameter reduce velocity.
- Chemical reaction and Schmidt number both contribute to a decrease in velocity and concentration due to increased resistance and restricted mass diffusivity.
- The Dufour effect redistributes thermal energy, lowering velocity near the wall while boosting velocity in the outside zone.
- Furthermore, increased Grashof and radiation parameters reduce fluid temperature through buoyancy-induced convection and radiative energy loss.

These findings provide insight into the dynamic behaviour of electrically conducting fluids in inclined geometries and lay the groundwork for better thermal and flow management in important industrial and engineering applications.

References

[1] Alamirew, W. D., Zergaw, G. A., and Gorfie, E. H., (2024), Effects of Hall, ion slip, viscous dissipation and nonlinear thermal radiation on MHD Williamson nanofluid flow past a stretching sheet, International Journal of Thermofluids, 22, pp. 100646.

[2] Ali, M., and Alim, M. A., (2022), Influence of slip parameter, viscous dissipation and joule heating effect on boundary layer flow and heat transfer over a power-law stretching wedge-shaped surface with the correlation coefficient and multiple regressions, International Journal of Applied Mechanics and Engineering, 27(2), pp. 1-21.

[3] Babu, N. V. N., Murali, G., and Bhati, S. M., (2018), Casson fluid performance on natural convective dissipative couette flow past an infinite vertically inclined plate filled in porous medium with heat transfer, mhd and hall current effects, International Journal of Pharmaceutical Research, 10(4), pp. 809-819.

[4] Bilal, M., Javid, K., and Ali, N., (2023), Unsteady flow of hybrid fluid via peristaltic pumping in wavy micro-channel under electric and magnetic field effects: An application of Hall device and slip boundary, Proceedings of the Institution of Mechanical Engineers, Part E: Journal of Process Mechanical Engineering, 237(6), pp. 2157-2170.

[5] Spalevic M.M., (2025), Error Bounds of Positive Interpolatory Quadrature Rules for Functions Analytic on Ellipses, TWMS J.Pure Appl. Math., V.16, N.2, pp.233-244 DOI: 10.30546/2219-1259.16.2.2025.233

[6] Falodun Bidemi, O., and Ahamed, M. S., (2019), Soret and Dufour effects on unsteady Casson magneto-nanofluid flow over an inclined plate embedded in a porous medium, World Journal of Engineering, 16(2), pp. 260-274.

[7] Ganjikunta, A., Kommaddi, H. B., Bhajanthri, V., and Kodi, R., (2023), An unsteady MHD flow of a second-grade fluid passing through a porous medium in the presence of radiation absorption exhibits Hall and ion slip effects, Heat Transfer, 52(1), pp. 780-806.

[8] Gomathi, N., and De, P., (2025), Contour and regression insights into Casson–Williamson nanofluid flow with suction/injection, hall currents, and ion slip effects. Multiscale and Multidisciplinary Modeling, Experiments and Design, 8(1), pp. 1-18.

[9] Islam, S. H., Begum, P., and Sarma, D., (2021), Dufour Effect on MHD Free Convection Heat and Mass Transfer Effects Flow over an Inclined Plate Embedded in a Porous Medium, Journal of Scientific Research, 13(1), pp. 111 - 123.

[10] Islam, M. R., Nasrin, S., and Alam, M. M., (2024), Unsteady MHD fluid flow over an inclined plate, inclined magnetic field and variable temperature with Hall and Ion-slip current, Ricerche di Matematica, 73(5), pp. 2477-2504.

[11] K. Jha, B., and A. Apere, C., (2011), Hall and ion-slip effects on unsteady MHD Couette flow in a rotating system with suction and injection, Journal of the Physical Society of Japan, 80(11), pp. 114401.

[12] Kanimozhi, N., Vijayaragavan, R., Rushi Kumar, B., and Chamkha, A. J., (2024), Investigation of Hall current and thermal diffusion effects on unsteady MHD mixed convective Jeffrey fluid flow over an inclined permeable surface with chemical reaction,The European Physical Journal Plus, 139(3), pp. 1-16.

[13] Krishna, M. V., (2021), Radiation-absorption, chemical reaction, Hall and ion slip impacts on magnetohydrodynamic free convective flow over semi-infinite moving absorbent surface, Chinese Journal of Chemical Engineering, 34, pp. 40-52.

[14] Krishna, M. V., (2024), Hall and ion slip effects on the MHD flow of Casson hybrid nanofluid past an infinite exponentially accelerated vertical porous surface,Waves in Random and Complex Media, 34(5), pp. 4658-4687.

[15] Krishna, M. V., Ahammad, N. A., and Chamkha, A. J., (2021), Radiative MHD flow of Casson hybrid nanofluid over an infinite exponentially accelerated vertical porous surface, Case Studies in Thermal Engineering, 27, pp. 101229.

[16] Krishna, M. V., Anand, P. V. S., and Chamkha, A. J., (2019), Heat and mass transfer on free convective flow of a micropolar fluid through a porous surface with inclined magnetic field and hall effects, Special Topics and Reviews in Porous Media: An International Journal, 10(3), pp. 203 - 223.

[17] Lakshmi, G. K., and Murty, P. S. R., (2025), Impact of hall current and rotational on MHD flow of nanofluid with joule heating and viscous dissipation, International Journal of Thermofluids, 27, pp. 101153.

[18] Maripala, S., and Naikoti, K., (2015), Hall effects on unsteady MHD free convection flow over a stretching sheet with variable viscosity and viscous dissipation, World Applied Sciences Journal, 33(6), pp. 1032-1041.

[19] Nhial, B. C., Vellanki, N., Babu, V. S. H., and Sagar, G. Y., (2022), Hall current, Rotation and chemical reaction effects on mhd free convection flow past an accelerated vertical plate through a porous medium, Journal of Positive School Psychology, pp. 707-716.

[20] Ojjela, O., and Kumar, N. N., (2016), Chemically reacting micropolar fluid flow and heat transfer between expanding or contracting walls with ion slip, Soret and Dufour effects, Alexandria Engineering Journal, 55(2), pp. 1683-1694.

[21] Osalusi, E., Side, J. C., Harris, R. E., and Johnston, B., (2007), On the effectiveness of viscous dissipation and Joule heating on steady MHD and slip flow of a Bingham fluid over a porous rotating disk in the presence of Hall and ion-slip currents, JP Journal of Heat and Mass Transfer, 1(3), pp. 303-330.

[22] Raghunath, K., Ganteda, C., and Lorenzini, G., (2022), Effects of Soret, rotation, Hall, and ion slip on unsteady MHD flow of a Jeffrey fluid through a porous medium in the presence of heat absorption and chemical reaction, Journal of Mechanical Engineering Research and Developments, 45(3), pp. 80-97.

[23] Rajakumar, K. V. B., Balamurugan, K. S., and Ramana Murthy, C., (2018), Viscous dissipation and Dufour effects on MHD free convection flow through an oscillatory inclined porous plate with Hall and ion-slip current, Int J Eng Technol, 7(4.5), pp. 410-415.

[24] Sadiq Basha, P. M., Veera Krishna, M., and Nagarathna, N., (2020), Hall and ion-slip effects on steady MHD free convective flow through a porous medium in a vertical microchannel, Heat Transfer, 49(8), pp. 4264-4280.

[25] Sandhya, A., Ramana Reddy, G. V., and Deekshitulu, V. S. R. G., (2020), Heat and mass transfer effects on MHD flow past an inclined porous plate in the presence of chemical reaction, International Journal of Applied Mechanics and Engineering, 25(3), pp. 86 -102.

[26] Seth, G. S., Sarkar, S., Hussain, S. M., and Mahato, G. K., (2014), Effects of hall current and rotation on hydromagnetic natural convection flow with heat and mass transfer of a heat absorbing fluid past an impulsively moving vertical plate with ramped temperature, Journal of Applied Fluid Mechanics, 8(1), pp. 159-171.

[27] Sigey, J. K., Okelo, J. A., Gatheri, F. K., and Ngesa, J. O., (2013), Magnetohydrodynamic (MHD) free convective flow past an infinite vertical porous plate with joule heating, Applied Mathematics, 4, pp. 825 - 833.

[28] Singh, J. K., Maurya, A. K., and Hussain, S. M., (2025), The physical significance of hall and ion-slip effects on second-grade nanofluid flow over a vertically oriented surface with oscillating free stream, International Journal of Advances in Engineering Sciences and Applied Mathematics, pp. 1-16.

[29] Sridevi, D., Murthy, C. V. R., and Varma, N. U. B., (2023), Hall current, radiation absorption, and diffusion thermoeffects on unsteady MHD rotating chemically reactive second-grade fluid flow past a porous inclined plate in the presence of an aligned magnetic field, thermal radiation, and chemical reaction, Heat Transfer, 52(6), pp. 4487-4511.

[30] Vedavathi. N., Baby Rani C. H., Balamurugan K. S., and Dharmaiah G., (2020), Hall and ion slip effects on MHD AG-H20 nanofluid biomedical engineering applications, Journal of critical reviews, 7, pp. 1270-1284.

[31] VeeraKrishna, M., Subba Reddy, G.,and Chamkha, A. J., (2018), Hall effects on unsteady MHD oscillatory free convective flow of second grade fluid through porous medium between two vertical plates, Physics of fluids, 30(2), pp. 023106.

[32] S.D. Akbarov, E.T. Bagirov, I.Z. Sardarova, (2025), The Responce Of an Iinfinite Elastic Medium with Inhomogeneous Initial Stresses To Moving Load Acting in a Cylindirical Cavity, Appl. Comput. Math., Vol.24, No.1, 2025, pp.3-15, DOI: 10.30546/1683-6154.24.1.2025.3.

S. Aiswarya is currently working as an Assistant Professor of Mathematics at PSGR Krishnammal College for Women, Coimbatore, with over 15 years of teaching experience. She received her Ph.D. in Mathematics from the same institution, affiliated with Bharathiar University, in 2019. With a strong academic background and a keen research interest, she has contributed significantly to the fields of stability theory, heat and mass transfer, and non-Newtonian fluids. She has authored 12 research papers in reputed peer journals and continues to be actively involved in advancing knowledge in her areas of expertise.

K. Bhuvaneshwari is an Assistant Professor at Rathinam Technical Campus, Coimbatore, specializing in fluid dynamics. She received her M.Phil in 2023 at PSGR Krishnammal College for Women affiliated to Bharathiar University, with a focus on fluid flow behavior, modeling, and simulation. Her work connects theory with practical applications in engineering. She is committed to fostering innovation and critical thinking through teaching and research.