TWMS J. App. and Eng. Math. V.16, N.2, 2026, pp. 169-187

INTERVAL-VALUED INTUITIONISTIC QUADRI-PARTITIONED NEUTROSOPHIC SETS IN PSYCHOLOGICAL RESEARCH WITH EXAMPLES AND THEORETICAL INSIGHTS

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Abstract. This manuscript presents a comprehensive study of interval-valued intuitionistic quadri-partitioned neutrosophic sets (IVIQPNSs) and their combination with soft sets (IVIQPNSSs), focusing on the relationships between the membership grades of truth, falsity, contradiction, and indeterminacy. Specifically, the sum of the truth and falsity membership grades is constrained such that it does not exceed unity, while the sum of all four membership grades (truth, indeterminacy, contradiction, and falsity) is bounded by three. The manuscript also explores operators such as necessity and possibility, establishing several key properties. Given that many psychiatric disorders exhibit ambivalent behavior, representing membership grades with IVIQPNSs is essential. This work give effective approach to diagnose psychiatric disorder with robust flexible framework in fuzzy environment.

Keywords: neutrosophic set, soft set, quadri-partitioned, fuzzy set. (Four or five keywords)

AMS Subject Classification: 03E72, 91B50

1. Introduction

Zadeh [32] introduced fuzzy sets (FS), where the membership value is represented between 0 and 1. Later, Atanassov [2] extended this idea to intuitionistic fuzzy sets (IFS), which involve membership and non-membership grades, with the condition that the sum should not exceed 1. Atanassov [4] also defined some important properties of IFS. These two theories FS and IFS are widely used to address decision-making problems. However, in real-world decision-making, especially in complex scenarios, decision-makers (DMs) often struggles to selecting the best option when facing uncertain or incomplete informations. This challenge is addressed by neutrosophic sets (NS), a more flexible approach that considers three membership grades for each element. Smarandache [25] introduced NS to handle uncertain and ambiguous data, which is particularly useful in decision-making

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§ Manuscript received: January 09, 2025; accepted: April 10, 2025.

processes. Wang et al. [28] further developed this idea by introducing single-valued neutrosophic sets (SVNS), with specific conditions on the membership grades, which made them more applicable to real-life problems. Chinnadurai et al. [15] and Bobin et al. [16] explored ranking methods for solving MCDM problems. Saranya et al. [23] developed a program to calculate similarity values in neutrosophic environments, while Broumi and Smarandache [7] applied distance measures to solve SM problems.

In the field of psychology, Smarandache [26] used neutrosophic theory to study psychological concepts, and Christianto and Smarandache [17] reviewed its application in cultural psychology. Other works, like those of Chicaiza et al. [14] and Abdel-Basset et al. [1], have applied neutrosophic sets to explore emotional intelligence and diagnose psychiatric disorders, respectively. Given the success of NS and SVNS in psychology, this paper explores the application of neutrosophic theory in understanding psychological behaviors. Previous studies have explored the properties of INS, with Bhowmik and Pal [6] introducing the INS concept and Broumi and Smarandache [8] defining intuitionistic neutrosophic soft sets (INSS) for decision-making.

Multi-Criteria Decision-Making (MCDM) plays a crucial role in handling complex decision problems involving uncertainty and imprecise information. The Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) is a widely used MCDM approach that evaluates alternatives based on their relative closeness to an ideal solution. However, real-world decision-making often involves higher levels of uncertainty, requiring an extension of classical TOPSIS to more generalized frameworks.

Bobin and Chinnadurai [16] introduced the Interval-valued intuitionistic neutrosophic hypersoft TOPSIS (IVINH-TOPSIS) method, which incorporates the correlation coefficient to enhance decision accuracy in uncertain environments. This method leverages interval-valued intuitionistic neutrosophic hypersoft sets to provide a more refined representation of uncertainty and hesitancy in decision-making. Chinnadurai et al. [15] proposed a Simplified Intuitionistic Neutrosophic Hypersoft TOPSIS (SINH-TOPSIS) method, which streamlines the computational process while preserving the robustness of the correlation-based evaluation. The simplified approach ensures efficient ranking of alternatives while maintaining high precision in decision-making models. Both studies contribute to advancing MCDM techniques in neutrosophic environments by improving computational efficiency and decision accuracy, making them suitable for applications in engineering, healthcare, and other domains requiring robust decision analysis under uncertainty.

Motivation of the study: Uncertainty and inconsistency are inherent in many real-world problems, particularly in domains where subjective assessments play a crucial role. Psychiatric disorders, in particular, often exhibit ambivalent behaviors, making precise categorization and decision-making challenging. Traditional mathematical frameworks, including classical fuzzy and intuitionistic sets, struggle to capture the nuanced relationships between conflicting and uncertain factors in psychiatric diagnosis. IVIQPNSs provide a robust and flexible approach to modeling such complex uncertainty. By incorporating four distinct membership grades truth, falsity, contradiction, and indeterminacy IVIQPNSs allow a more refined representation of ambivalent psychiatric symptoms. The constraints imposed on these membership grades ensure a structured yet adaptable mathematical foundation, enabling improved analysis in uncertain environments. Furthermore, integrating IVIQPNSs with soft sets IVIQPNSSs enhances their applicability by introducing additional flexibility in handling imprecise and dynamic data. Operators such as necessity and possibility further strengthen the theoretical framework, enabling deeper insights into psychiatric decision-making.

This study contributes to the development of a mathematically rigorous, yet practically viable, framework for psychiatric disorder diagnosis. By leveraging IVIQPNSs, this approach not only enhances the precision of disorder classification but also facilitates a more comprehensive understanding of the interplay between conflicting symptoms. As psychiatric assessments often involve subjective judgment, this work provides a significant step towards a more structured, quantitative, and adaptable methodology in mental health diagnostics.

2. Preliminaries

We define some essential concepts interval-valued intuitionistic fuzzy set (IVIFS), interval-valued neutrosophic set (IVNS) and interval-valued quadri-partitioned neutrosophic set (IVQPNSS) that are integral to this manuscript.

Definition 2.1. [32] *A fuzzy set $\mathcal{A}$ in a universal set $\mathcal{U}$ is defined as a set of ordered pairs:* $\mathcal{A} = \{\langle u, \mu_{\mathcal{A}}(u)\rangle \mid u \in \mathcal{U}\}$, *where* $\mu_{\mathcal{A}} : \mathcal{U} \to [0,1]$ *represents the membership function of* $\mathcal{A}$, *assigning to each element u a degree of membership in the interval* $[0,1]$.

Definition 2.2. [2] *An intuitionistic fuzzy set (IFS) $\mathcal{A}$ in a universal set $\mathcal{U}$ is defined as:* $\mathcal{A} = \{\langle u, \mu_{\mathcal{A}}(u), \nu_{\mathcal{A}}(u)\rangle \mid u \in \mathcal{U}\}$, *where* $\mu_{\mathcal{A}}(u), \nu_{\mathcal{A}}(u) : \mathcal{U} \to [0,1]$ *represent the membership and non-membership functions, respectively, satisfying:* $0 \leq \mu_{\mathcal{A}}(u) + \nu_{\mathcal{A}}(u) \leq 1$. *The hesitation degree is given by:* $\pi_{\mathcal{A}}(u) = 1 - \mu_{\mathcal{A}}(u) - \nu_{\mathcal{A}}(u)$.

Definition 2.3. [5] *An IVIFS is of the form* $\mathcal{A} = \{\langle u, \mu_{\mathcal{A}}(u), \nu_{\mathcal{A}}(u)\rangle\}$, *where the membership and non-membership interval grades are given by* $\mu_{\mathcal{A}}(u), \nu_{\mathcal{A}}(u) : \mathcal{U} \to \mathcal{C}[0,1]$. *The following conditions must hold for the set:* $0 \leq \overline{\mu}_{\mathcal{A}}(u) + \overline{\nu}_{\mathcal{A}}(u) \leq 1$ *and* $\underline{\mu}_{\mathcal{A}}(u), \underline{\nu}_{\mathcal{A}}(u) \geq 0$.

Definition 2.4. [29] *An IVNS is of the form* $\mathcal{Z} = \{u, \langle \alpha_{\mathcal{Z}}(u), \beta_{\mathcal{Z}}(u), \gamma_{\mathcal{Z}}(u)\rangle\}$, *where the truth, indeterminacy and falsity interval grades are given by* $\alpha_{\mathcal{Z}}(u), \beta_{\mathcal{Z}}(u), \gamma_{\mathcal{Z}}(u) : \mathcal{U} \to \mathcal{C}[0,1]$. *The following conditions must hold for the set:* $0 \leq \overline{\alpha}_{\mathcal{Z}}(u) + \overline{\beta}_{\mathcal{Z}}(u) + \overline{\gamma}_{\mathcal{Z}}(u) \leq 3$.

Definition 2.5. [22] *An IVQPNSS is of the form* $\mathcal{T} = \{u, \langle \alpha_{\mathcal{T}}(u), \beta_{\mathcal{T}}(u), \gamma_{\mathcal{T}}(u), \delta_{\mathcal{T}}(u)\rangle\}$, *where the truth, indeterminacy, contradiction and falsity interval grades are given by* $\alpha_{\mathcal{T}}(u), \beta_{\mathcal{T}}(u) : \mathcal{U}, \gamma_{\mathcal{T}}(u) : \mathcal{U}, \delta_{\mathcal{T}}(u) : \mathcal{U} \to \mathcal{C}[0,1]$. *The following conditions must hold for the set:* $0 \leq \overline{\alpha}_{\mathcal{T}}(u) + \overline{\beta}_{\mathcal{T}}(u) + \overline{\gamma}_{\mathcal{T}}(u) + \overline{\delta}_{\mathcal{T}}(u) \leq 4$.

3. Interval-valued intuitionistic quadri-partitioned neutrosophic soft set

We introduce the concept of IVIQPNSS and explore its key properties, extending its operations and characteristics by building upon the ideas presented in [2] and [20].

Definition 3.1. *An IVIQPNS Ω is of the form* $\tau = \left\{\left\langle \Omega, \Xi^T_\tau(\mho), \Xi^I_\tau(\mho), \Xi^C_\tau(\mho), \Xi^F_\tau(\mho)\right\rangle\right\}$, *where* $\Xi^T_\tau(\mho), \Xi^I_\tau(\mho), \Xi^C_\tau(\mho)$ *and* $\Xi^F_\tau(\mho) : \mho \to D[0,1]$. *The lower and upper ends should meet the conditions outlined below:*
(i) $\underline{\Xi^T}_\tau(\mho), \underline{\Xi^I}_\tau(\mho), \underline{\Xi^C}_\tau(\mho), \underline{\Xi^F}_\tau(\mho) \geq 0$,
(ii) $0 \leq \overline{\Xi^T}_\tau(\mho) + \overline{\Xi^F}_\tau(\mho) \leq 1$,
(iii) $0 \leq \overline{\Xi^T}_\tau(\mho) + \overline{\Xi^I}_\tau(\mho) + \overline{\Xi^C}_\tau(\mho) + \overline{\Xi^F}_\tau(\mho) \leq 3$.

Example 3.1. *An IVIQPNS* $\Omega = \{\mho_1, \mho_2, \mho_3\}$ *can be given as,*

$$\tau = \{\, \langle \mho_1, [.2,.3], [.6,.7], [.4,.5], [.3,.4]\rangle, \langle \mho_2, [.1,.2], [.7,.8], [.5,.6], [.1,.2]\rangle,$$
$$\langle \mho_3, [.3,.4], [.8,.9], [.6,.7], [.2,.3]\rangle \,\}.$$

Definition 3.2. *A pair* $(\Delta, \wp)$ *is called IVIQPNSS over* $\mho$, *where* $\Delta : \wp \to 2^\tau$. *Thus for any parameter* $\wp \in \wp$, $\Delta(\wp)$ *is an IVIQPNSS.*

Example 3.2. *Let $\Omega = \{\mho_1, \mho_2, \mho_3\}$ represent patients undergoing post-traumatic stress evaluation (PTSE), and $\wp = \{\wp_1, \wp_2, \wp_3\}$ represent symptoms which stand for heightened anxiety (HA), frequent flashbacks (FF) and difficulty sleeping (DS), respectively. An IVIQPNSS $(\Delta, \wp)$ is a collection of subsets of $\mho$, determined by a clinical psychologist based on the evaluation provided in Table 1.*

TABLE 1. Shows patients with PTSE in IVIQPNSS $(\Delta, \wp)$ form.

Ω	*HA*$(\wp_1)$	*FF*$(\wp_2)$	*DS*$(\wp_3)$
$\mho_1$	$\langle[.3,.5],[.4,.6],[.7,.8],[.2,.4]\rangle$	$\langle[.2,.4],[.5,.7],[.7,.8],[.2,.3]\rangle$	$\langle[.3,.4],[.6,.8],[.7,.8],[.1,.2]\rangle$
$\mho_2$	$\langle[.4,.6],[.2,.4],[.7,.8],[.1,.3]\rangle$	$\langle[.5,.6],[.3,.5],[.7,.8],[.1,.2]\rangle$	$\langle[.2,.3],[.4,.6],[.7,.8],[.3,.5]\rangle$
$\mho_3$	$\langle[.2,.3],[.6,.8],[.6,.7],[.2,.3]\rangle$	$\langle[.3,.4],[.5,.7],[.6,.7],[.4,.5]\rangle$	$\langle[.4,.6],[.3,.5],[.6,.7],[.1,.2]\rangle$

Definition 3.3. *Consider $(\Delta, \wp)$ and $(\Delta', \wp')$ over Ω. Then,*
(i) $(\Delta, \wp)$ OR $(\Delta', \wp')$ is an IVIQPNSS represented as $(\Delta, \wp) \vee (\Delta', \wp') = (\Delta_\vee, \wp \times \wp')$, where $\Delta_\vee(\wp, \wp') = \Delta(\wp) \cup \Delta'(\wp')$, $\forall\ (\wp, \wp') \in \wp \times \wp'$.

$$\begin{aligned}\Delta_\vee(\wp, \wp') =&\Big\langle[\vee(\underline{\Xi^T}_{(\Delta,\wp)}(\mho), \underline{\Xi^T}_{\Delta'(\wp')}(\mho)), \vee(\overline{\Xi^T}_{(\Delta,\wp)}(\mho), \overline{\Xi^T}_{\Delta'(\wp')}(\mho))],\\ &[\vee(\underline{\Xi^I}_{(\Delta,\wp)}(\mho), \underline{\Xi^I}_{\Delta'(\wp')}(\mho)), \vee(\overline{\Xi^I}_{(\Delta,\wp)}(\mho), \overline{\Xi^I}_{\Delta'(\wp')}(\mho))],\\ &[\vee(\underline{\Xi^C}_{(\Delta,\wp)}(\mho), \underline{\Xi^C}_{\Delta'(\wp')}(\mho)), \vee(\overline{\Xi^C}_{(\Delta,\wp)}(\mho), \overline{\Xi^C}_{\Delta'(\wp')}(\mho))],\\ &[\wedge(\underline{\Xi^F}_{(\Delta,\wp)}(\mho), \underline{\Xi^F}_{\Delta'(\wp')}(\mho)), \wedge(\overline{\Xi^F}_{(\Delta,\wp)}(\mho), \overline{\Xi^F}_{\Delta'(\wp')}(\mho))], \Big\rangle, \forall\ (\wp, \wp') \in \wp \times \wp'.\end{aligned}$$

(ii) $(\Delta, \wp)$ AND $(\Delta', \wp')$ is an IVIQPNSS represented as $(\Delta, \wp) \wedge (\Delta', \wp') = (\Delta_\wedge, \wp \times \wp')$, where $\Delta_\wedge(\wp, \wp') = \Delta(\wp) \cap \Delta'(\wp')$, $\forall\ (\wp, \wp') \in \wp \times \wp'$.

$$\begin{aligned}\Delta_\wedge(\wp, \wp') =&\Big\langle[\wedge(\underline{\Xi^T}_{(\Delta,\wp)}(\mho), \underline{\Xi^T}_{\Delta'(\wp')}(\mho)), \wedge(\overline{\Xi^T}_{(\Delta,\wp)}(\mho), \overline{\Xi^T}_{\Delta'(\wp')}(\mho))],\\ &[\wedge(\underline{\Xi^I}_{(\Delta,\wp)}(\mho), \underline{\Xi^I}_{\Delta'(\wp')}(\mho)), \wedge(\overline{\Xi^I}_{(\Delta,\wp)}(\mho), \overline{\Xi^I}_{\Delta'(\wp')}(\mho))],\\ &[\wedge(\underline{\Xi^I}_{(\Delta,\wp)}(\mho), \underline{\Xi^I}_{\Delta'(\wp')}(\mho)), \wedge(\overline{\Xi^I}_{(\Delta,\wp)}(\mho), \overline{\Xi^I}_{\Delta'(\wp')}(\mho))],\\ &[\vee(\underline{\Xi^C}_{(\Delta,\wp)}(\mho), \underline{\Xi^C}_{\Delta'(\wp')}(\mho)), \vee(\overline{\Xi^C}_{(\Delta,\wp)}(\mho), \overline{\Xi^C}_{\Delta'(\wp')}(\mho))]\Big\rangle, \forall\ (\wp, \wp') \in \wp \times \wp'.\end{aligned}$$

Definition 3.4. *Consider $(\Delta, \wp)$ and $(\Delta', \wp')$ over Ω. Then,*
(i) $(\Delta, \wp)$ union $(\Delta', \wp')$ is an IVIQPNSS represented as $(\Delta, \wp) \cup (\Delta', \wp') = (\Delta_\cup, \wp_\cup)$, where $\wp_\cup = \wp \cup \wp'$ and $\forall\ \wp \in \wp_\cup$,
$\Delta_\cup(\wp) =$

$$\begin{cases} \left\{\left\langle \Omega, \left(\Xi^T_{\Delta(\wp)}(\mho), \Xi^I_{\Delta(\wp)}(\mho), \Xi^C_{\Delta(\wp)}(\mho), \Xi^F_{\Delta(\wp)}(\mho)\right)\right\rangle\right\}, & \text{if } \wp \in \wp - \wp', \\ \left\{\left\langle \Omega, \left(\Xi^T_{\Delta'(\wp)}(\mho), \Xi^I_{\Delta'(\wp)}(\mho), \Xi^C_{\Delta'(\wp)}(\mho), \Xi^F_{\Delta'(\wp)}(\mho)\right)\right\rangle\right\}, & \text{if } \wp \in \wp' - \wp, \\ \left\{\left\langle \Omega, \begin{matrix} \left[\vee\left(\underline{\Xi^T}_{\Delta(\wp)}(\mho), \underline{\Xi^T}_{\Delta'(\wp)}(\mho)\right), \vee\left(\overline{\Xi^T}_{\Delta(\wp)}(\mho), \overline{\Xi^T}_{\Delta'(\wp)}(\mho)\right)\right], \\ \left[\vee\left(\underline{\Xi^I}_{\Delta(\wp)}(\mho), \underline{\Xi^I}_{\Delta'(\wp)}(\mho)\right), \vee\left(\overline{\Xi^I}_{\Delta(\wp)}(\mho), \overline{\Xi^I}_{\Delta'(\wp)}(\mho)\right)\right], \\ \left[\vee\left(\underline{\Xi^C}_{\Delta(\wp)}(\mho), \underline{\Xi^C}_{\Delta'(\wp)}(\mho)\right), \vee\left(\overline{\Xi^C}_{\Delta(\wp)}(\mho), \overline{\Xi^C}_{\Delta'(\wp)}(\mho)\right)\right], \\ \left[\wedge\left(\underline{\Xi^F}_{\Delta(\wp)}(\mho), \underline{\Xi^F}_{\Delta'(\wp)}(\mho)\right), \wedge\left(\overline{\Xi^F}_{\Delta(\wp)}(\mho), \overline{\Xi^F}_{\Delta'(\wp)}(\mho)\right)\right] \end{matrix} \right\rangle\right\}, & \text{if } \wp \in \wp \cap \wp'. \end{cases}$$

(ii) $(\Delta, \wp)$ intersection $(\Delta', \wp')$ is an IVIQPNSS represented as $(\Delta, \wp) \cap (\Delta', \wp') = (\Delta_\cap, \wp_\cap)$, where $\wp_\cap = \wp \cup \wp'$ and $\forall\ \wp \in \wp_\cap$,

$$\Delta_{\cap}(\wp) =$$

$$\begin{cases} \left\{\left\langle \Omega, \left(\Xi^{T}_{\Delta(\wp)}(\mho), \Xi^{I}_{\Delta(\wp)}(\mho), \Xi^{C}_{\Delta(\wp)}(\mho), \Xi^{F}_{\Delta(\wp)}(\mho)\right)\right\rangle\right\}, & \textit{if } \wp \in \wp - \wp', \\ \left\{\left\langle \Omega, \left(\Xi^{T}_{\Delta'(\wp)}(\mho), \Xi^{I}_{\Delta'(\wp)}(\mho), \Xi^{C}_{\Delta'(\wp)}(\mho), \Xi^{F}_{\Delta'(\wp)}(\mho)\right)\right\rangle\right\}, & \textit{if } \wp \in \wp' - \wp, \\ \left\{\left\langle \Omega, \begin{matrix} \left[\wedge\left(\underline{\Xi^{T}}_{\Delta(\wp)}(\mho), \underline{\Xi^{T}}_{\Delta'(\wp)}(\mho)\right), \wedge\left(\overline{\Xi^{T}}_{\Delta(\wp)}(\mho), \overline{\Xi^{T}}_{\Delta'(\wp)}(\mho)\right)\right], \\ \left[\wedge\left(\underline{\Xi^{I}}_{\Delta(\wp)}(\mho), \underline{\Xi^{I}}_{\Delta'(\wp)}(\mho)\right), \wedge\left(\overline{\Xi^{I}}_{\Delta(\wp)}(\mho), \overline{\Xi^{I}}_{\Delta'(\wp)}(\mho)\right)\right], \\ \left[\wedge\left(\underline{\Xi^{C}}_{\Delta(\wp)}(\mho), \underline{\Xi^{C}}_{\Delta'(\wp)}(\mho)\right), \wedge\left(\overline{\Xi^{C}}_{\Delta(\wp)}(\mho), \overline{\Xi^{C}}_{\Delta'(\wp)}(\mho)\right)\right], \\ \left[\vee\left(\underline{\Xi^{F}}_{\Delta(\wp)}(\mho), \underline{\Xi^{F}}_{\Delta'(\wp)}(\mho)\right), \vee\left(\overline{\Xi^{F}}_{\Delta(\wp)}(\mho), \overline{\Xi^{F}}_{\Delta'(\wp)}(\mho)\right)\right] \end{matrix} \right\rangle\right\}, & \textit{if } \wp \in \wp \cap \wp'. \end{cases}$$

Definition 3.5. *The complement* $(\Delta, \wp)$ *is given as,*

$$(\Delta, \wp)^c = \big\{\big\langle \Omega, \Xi^{F}_{(\Delta,\wp)}(\mho), \big[(1 - \overline{\Xi}^{I}_{(\Delta,\wp)}(\mho)), (1 - \underline{\Xi}^{I}_{(\Delta,\wp)}(\mho))\big], \big[(1 - \overline{\Xi}^{C}_{(\Delta,\wp)}(\mho)), (1 - \underline{\Xi}^{C}_{(\Delta,\wp)}(\mho))\big], \Xi^{T}_{(\Delta,\wp)}(\mho)\big\rangle\big\}.$$

Example 3.3. *Consider an IVIQPNS* $(\Delta, \wp)$ *defined over a universal set* $U = \{u_1, u_2\}$, *where the membership functions are given as follows:*

For u_1*:*

- *Truth Membership:* $\Xi^{T}_{(\Delta,\wp)}(u_1) = [0.6, 0.7]$
- *Indeterminacy Membership:* $\Xi^{I}_{(\Delta,\wp)}(u_1) = [0.2, 0.3]$
- *Contradiction Membership:* $\Xi^{C}_{(\Delta,\wp)}(u_1) = [0.1, 0.2]$
- *Falsity Membership:* $\Xi^{F}_{(\Delta,\wp)}(u_1) = [0.4, 0.5]$

For u_2*:*

- *Truth Membership:* $\Xi^{T}_{(\Delta,\wp)}(u_2) = [0.5, 0.6]$
- *Indeterminacy Membership:* $\Xi^{I}_{(\Delta,\wp)}(u_2) = [0.3, 0.4]$
- *Contradiction Membership:* $\Xi^{C}_{(\Delta,\wp)}(u_2) = [0.2, 0.3]$
- *Falsity Membership:* $\Xi^{F}_{(\Delta,\wp)}(u_2) = [0.3, 0.4]$

Now, using the complement definition:

$$(\Delta, \wp)^c = \left\{ \left\langle \begin{matrix} u,\ \Xi^{F}_{(\Delta,\wp)}(u), \big[(1 - \overline{\Xi^{I}}_{(\Delta,\wp)}(u)),\ (1 - \underline{\Xi^{I}}_{(\Delta,\wp)}(u))\big], \\ \big[(1 - \overline{\Xi^{C}}_{(\Delta,\wp)}(u)),\ (1 - \underline{\Xi^{C}}_{(\Delta,\wp)}(u))\big],\ \Xi^{T}_{(\Delta,\wp)}(u) \end{matrix} \right\rangle \right\}$$

For u_1*, the complement values are computed as follows:*

- *Complement Indeterminacy Membership:*

$$(1 - \overline{\Xi}^{I}_{(\Delta,\wp)}(u_1), 1 - \underline{\Xi}^{I}_{(\Delta,\wp)}(u_1)) = (1 - 0.3, 1 - 0.2) = (0.7, 0.8)$$

- *Complement Contradiction Membership:*

$$(1 - \overline{\Xi}^{C}_{(\Delta,\wp)}(u_1), 1 - \underline{\Xi}^{C}_{(\Delta,\wp)}(u_1)) = (1 - 0.2, 1 - 0.1) = (0.8, 0.9)$$

For u_2*, the complement values are:*

- *Complement Indeterminacy Membership:*

$$(1 - \overline{\Xi}^{I}_{(\Delta,\wp)}(u_2), 1 - \underline{\Xi}^{I}_{(\Delta,\wp)}(u_2)) = (1 - 0.4, 1 - 0.3) = (0.6, 0.7)$$

- *Complement Contradiction Membership:*

$$(1-\overline{\Xi}^{C}_{(\Delta,\wp)}(u_2), 1-\underline{\Xi}^{C}_{(\Delta,\wp)}(u_2)) = (1-0.3, 1-0.2) = (0.7, 0.8)$$

Thus, the complement $(\Delta,\wp)^c$ is given as:

$$(\Delta,\wp)^c = \begin{cases} \langle u_1, [0.4,0.5], [0.7,0.8], [0.8,0.9], [0.6,0.7]\rangle, \\ \langle u_2, [0.3,0.4], [0.6,0.7], [0.7,0.8], [0.5,0.6]\rangle. \end{cases}$$

This example illustrates how the complement operation modifies the indeterminacy and contradiction membership grades while retaining the truth and falsity membership grades.

Theorem 3.1. *Consider $(\Delta,\wp)$ and $(\Delta',\wp')$ over (Ω). Then,*
(i) $\big((\Delta,\wp)\vee(\Delta',\wp')\big)^c = (\Delta,\wp)^c \wedge (\Delta',\wp')^c$;
(ii) $\big((\Delta,\wp)\wedge(\Delta',\wp')\big)^c = (\Delta,\wp)^c \vee (\Delta',\wp')^c$.

Proof. (i) $(\Delta,\wp)\vee(\Delta',\wp') = (\Delta_\vee, \wp\times\wp')$.
$\big((\Delta,\wp)\vee(\Delta',\wp')\big)^c = (\Delta_\vee,\wp\times\wp')^c$.
$\Omega^c_\vee(\wp,\wp')$

$$\begin{aligned}
&= \big\langle[\wedge(\underline{\Xi^F}_{(\Delta,\wp)}(\mho), \underline{\Xi^F}_{\Delta'(\wp')}(\mho)), \wedge(\overline{\Xi^F}_{(\Delta,\wp)}(\mho), \overline{\Xi^F}_{\Delta'(\wp')}(\mho))],\\
&\quad [\wedge((1-\overline{\Xi^I}_{(\Delta,\wp)}(\mho)), (1-\overline{\Xi^I}_{\Delta'(\wp')}(\mho))), \wedge((1-\underline{\Xi^I}_{(\Delta,\wp)}(\mho)), (1-\underline{\Xi^I}_{\Delta'(\wp')}(\mho)))],\\
&\quad [\wedge((1-\overline{\Xi^C}_{(\Delta,\wp)}(\mho)), (1-\overline{\Xi^C}_{\Delta'(\wp')}(\mho))), \wedge((1-\underline{\Xi^C}_{(\Delta,\wp)}(\mho)), (1-\underline{\Xi^C}_{\Delta'(\wp')}(\mho)))],\\
&\quad [\vee(\underline{\Xi^T}_{(\Delta,\wp)}(\mho), \underline{\Xi^T}_{\Delta'(\wp')}(\mho)), \vee(\overline{\Xi^T}_{(\Delta,\wp)}(\mho), \overline{\Xi^T}_{\Delta'(\wp')}(\mho))]\big\rangle, \forall\ (\wp,\wp')\in\wp\times\wp'.
\end{aligned}$$

and $(\Delta,\wp)^c\wedge(\Delta',\wp')^c = (\Delta_\wedge,\wp\times\wp')$. $\Delta_\wedge(\wp,\wp')$

$$\begin{aligned}
&= \big\langle[\wedge(\underline{\Xi^F}_{(\Delta,\wp)}(\mho), \underline{\Xi^F}_{\Delta'(\wp')}(\mho)), \wedge(\overline{\Xi^F}_{(\Delta,\wp)}(\mho), \overline{\Xi^F}_{\Delta'(\wp')}(\mho))],\\
&\quad [\wedge((1-\overline{\Xi^I}_{(\Delta,\wp)}(\mho)), (1-\overline{\Xi^I}_{\Delta'(\wp')}(\mho))), \wedge((1-\underline{\Xi^I}_{(\Delta,\wp)}(\mho)), (1-\underline{\Xi^I}_{\Delta'(\wp')}(\mho)))],\\
&\quad [\wedge((1-\overline{\Xi^C}_{(\Delta,\wp)}(\mho)), (1-\overline{\Xi^C}_{\Delta'(\wp')}(\mho))), \wedge((1-\underline{\Xi^C}_{(\Delta,\wp)}(\mho)), (1-\underline{\Xi^C}_{\Delta'(\wp')}(\mho)))],\\
&\quad [\vee(\underline{\Xi^T}_{(\Delta,\wp)}(\mho), \underline{\Xi^T}_{\Delta'(\wp')}(\mho)), \vee(\overline{\Xi^T}_{(\Delta,\wp)}(\mho), \overline{\Xi^T}_{\Delta'(\wp')}(\mho))]\big\rangle, \forall\ (\wp,\wp')\in\wp\times\wp'.
\end{aligned}$$

Thus $\big((\Delta,\wp)\vee(\Delta',\wp')\big)^c = (\Delta,\wp)^c\wedge(\Delta',\wp')^c$. □

Definition 3.6. *Let $\wp,\wp'\subseteq\mathcal{Q}$. $(\Delta,\wp)$ is an interval-valued intuitionistic quadri partitioned neutrosophic soft subset (IVIQPNSSS) of $(\Delta',\wp')$ represented as $(\Delta,\wp)\subset(\Delta',\wp')$ if and only if (iff)*
(i) $\wp\subseteq\wp'$;
(ii) $\Delta(\wp)$ *is an IVIQPNSSS of* $\Delta'(\wp)$ *that is for all* $\wp\in\wp$,
$\underline{\Xi^T}_{\Delta(\wp)}(\mho)\le\underline{\Xi^T}_{\Delta'(\wp)}(\mho)$, $\overline{\Xi^T}_{\Delta(\wp)}(\mho)\le\overline{\Xi^T}_{\Delta'(\wp)}(\mho)$;
$\underline{\Xi^I}_{\Delta(\wp)}(\mho)\le\underline{\Xi^I}_{\Delta'(\wp)}(\mho)$, $\overline{\Xi^I}_{\Delta(\wp)}(\mho)\le\overline{\Xi^I}_{\Delta'(\wp)}(\mho)$;
$\underline{\Xi^C}_{\Delta(\wp)}(\mho)\le\underline{\Xi^C}_{\Delta'(\wp)}(\mho)$, $\overline{\Xi^C}_{\Delta(\wp)}(\mho)\le\overline{\Xi^C}_{\Delta'(\wp)}(\mho)$;
$\underline{\Xi^F}_{\Delta(\wp)}(\mho)\ge\underline{\Xi^F}_{\Delta'(\wp)}(\mho)$ *and* $\overline{\Xi^F}_{\Delta(\wp)}(\mho)\ge\overline{\Xi^F}_{\Delta'(\wp)}(\mho)$.
Also, $(\Delta',\wp')$ is called an interval-valued intuitionistic quadri partitioned neutrosophic soft superset of $(\Delta,\wp)$ and represented as $(\Delta',\wp')\supset(\Delta,\wp)$.

Definition 3.7. *Consider $(\Delta,\wp)$ and $(\Delta',\wp')$. Then $(\Delta,\wp)=(\Delta',\wp')$ iff $(\Delta,\wp)\subset(\Delta',\wp')$ and $(\Delta',\wp')\subset(\Delta,\wp)$.*

4. Necessity ($\oplus$) and possibility ($\ominus$) operators on IVIQPNSS

We present the definitions and properties of the $\oplus$ and $\ominus$ operators for IVIQPNSS.

Definition 4.1. *If $(\Omega, \wp)$ is an IVIQPNSS over $\mho$ and $\Omega : \wp \to 2^{\tau}$, then,*
(i) the necessity operator ($\oplus$) is represented as,

$$\oplus(\Omega, \wp) = \left\{ \left\langle \Omega, \Xi^{T}_{\oplus\Omega(\wp)}(\mho), \Xi^{I}_{\oplus\Omega(\wp)}(\mho), \Xi^{C}_{\oplus\Omega(\wp)}(\mho), \Xi^{F}_{\oplus\Omega(\wp)}(\mho) \right\rangle ;\ \wp \in \wp \right\}.$$

Here, $\Xi^{T}_{\oplus\Omega(\wp)}(\mho) = [\underline{\Xi^{T}}_{\Omega(\wp)}(\mho), \overline{\Xi^{T}}_{\Omega(\wp)}(\mho)]$, $\Xi^{I}_{\oplus\Omega(\wp)}(\mho) = [\underline{\Xi^{I}}_{\Omega(\wp)}(\mho), \overline{\Xi^{I}}_{\Omega(\wp)}(\mho)]$, $\Xi^{C}_{\oplus\Omega(\wp)}(\mho) = [\underline{\Xi^{C}}_{\Omega(\wp)}(\mho), \overline{\Xi^{C}}_{\Omega(\wp)}(\mho)]$ *and* $\Xi^{F}_{\oplus\Omega(\wp)}(\mho) = [(1 - \overline{\Xi^{T}}_{\Omega(\wp)}(\mho)), (1 - \underline{\Xi^{T}}_{\Omega(\wp)}(\mho))]$, *are the membership grades for the parameter e.*
(ii) the possibility operator ($\ominus$) is represented as,

$$\ominus(\Omega, \wp) = \left\{ \left\langle \Omega, \Xi^{T}_{\ominus\Omega(\wp)}(\mho), \Xi^{I}_{\ominus\Omega(\wp)}(\mho), \Xi^{C}_{\ominus\Omega(\wp)}(\mho), \Xi^{F}_{\ominus\Xi^{F}(\wp)}(\mho) \right\rangle ;\ \wp \in \wp \right\}.$$

Here, $\Xi^{T}_{\ominus\Omega(\wp)}(\mho) = [(1 - \overline{\Xi^{F}}_{\Omega(\wp)}(\mho)), (1 - \underline{\Xi^{F}}_{\Omega(\wp)}(\mho))]$,
$\Xi^{I}_{\ominus\Omega(\wp)}(\mho) = [\underline{\Xi^{I}}_{\Omega(\wp)}(\mho), \overline{\Xi^{I}}_{\Omega(\wp)}(\mho)]$ $\Xi^{C}_{\ominus\Omega(\wp)}(\mho) = [\underline{\Xi^{C}}_{\Omega(\wp)}(\mho), \overline{\Xi^{C}}_{\Omega(\wp)}(\mho)]$ *and*
$\Xi^{F}_{\ominus\Omega(\wp)}(\mho) = [\underline{\Xi^{F}}_{\Omega(\wp)}(\mho), \overline{\Xi^{F}}_{\Omega(\wp)}(\mho)]$, *are the membership grades for the parameter e.*

Example 4.1. *(i) The IVIQPNSS $\oplus(\Omega, \wp)$ for Example 1.4 is shown in Table 3.*

Table 2. Shows patients with PTSE using $\oplus$ operator.

Ω	*HA*($\wp_1$)	*FF*($\wp_2$)	*DS*($\wp_3$)
$\mho_1$	$\langle[.3,.5],[.4,.6],[.7,.8],[.5,.7]\rangle$	$\langle[.2,.4],[.5,.7],[.7,.8],[.6,.8]\rangle$	$\langle[.3,.4],[.6,.8],[.7,.8],[.6,.7]\rangle$
$\mho_2$	$\langle[.4,.6],[.2,.4],[.7,.8],[.4,.6]\rangle$	$\langle[.5,.6],[.3,.5],[.7,.8],[.4,.5]\rangle$	$\langle[.2,.3],[.4,.6],[.7,.8],[.7,.8]\rangle$
$\mho_3$	$\langle[.2,.3],[.6,.8],[.6,.7],[.7,.8]\rangle$	$\langle[.3,.4],[.5,.7],[.6,.7],[.6,.7]\rangle$	$\langle[.4,.6],[.3,.5],[.6,.7],[.4,.6]\rangle$

(ii) The IVIQPNSS $\ominus(\Omega, \wp)$ for Example 1.4 is shown in Table 4.

Table 3. Shows patients with PTSE using $\ominus$ operator.

Ω	*HA*($\wp_1$)	*FF*($\wp_2$)	*DS*($\wp_3$)
$\mho_1$	$\langle[.6,.8],[.4,.6],[.7,.8],[.2,.4]\rangle$	$\langle[.7,.8],[.5,.7],[.7,.8],[.2,.3]\rangle$	$\langle[.8,.9],[.6,.8],[.7,.8],[.1,.2]\rangle$
$\mho_2$	$\langle[.7,.9],[.2,.4],[.7,.8],[.1,.3]\rangle$	$\langle[.8,.9],[.3,.5],[.7,.8],[.1,.2]\rangle$	$\langle[.5,.7],[.4,.6],[.7,.8],[.3,.5]\rangle$
$\mho_3$	$\langle[.7,.8],[.6,.8],[.6,.7],[.2,.3]\rangle$	$\langle[.5,.6],[.5,.7],[.6,.7],[.4,.5]\rangle$	$\langle[.8,.9],[.3,.5],[.6,.7],[.1,.2]\rangle$

Theorem 4.1. *Consider $(\Delta, \wp)$ and $(\Delta', \wp')$ over $\mho$. Then,*
(i) $\oplus\left((\Delta, \wp) \cup (\Delta', \wp')\right) = \oplus(\Delta, \wp) \cup \oplus(\Delta', \wp')$;
(ii) $\oplus\left((\Delta, \wp) \cap (\Delta', \wp')\right) = \oplus(\Delta, \wp) \cap \oplus(\Delta', \wp')$;
(iii) $\oplus \oplus (\Delta, \wp) = \oplus(\Delta, \wp)$.

Proof. (i) Let $(\Delta,\wp)\cup(\Delta',\wp')=(\Delta_{\cup},\wp_{\cup})$, where $\wp_{\cup}=\wp\cup\wp'$, $\forall\ \wp\in\wp_{\cup}$.
Consider,

$$\oplus\Phi_{\cup}(\wp)=\begin{cases}\left\{\left\langle\Omega,\ \begin{array}{l}[\underline{\Xi^T}_{\Delta(\wp)}(\mho),\ \overline{\Xi^T}_{\Delta(\wp)}(\mho)],\\ [\underline{\Xi^I}_{\Delta(\wp)}(\mho),\ \overline{\Xi^I}_{\Delta(\wp)}(\mho)],\\ [\underline{\Xi^C}_{\Delta(\wp)}(\mho),\ \overline{\Xi^C}_{\Delta(\wp)}(\mho)],\\ [1-\overline{\Xi^T}_{\Delta(\wp)}(\mho),\ 1-\underline{\Xi^T}_{\Delta(\wp)}(\mho)]\end{array}\right\rangle;\ \text{if } \wp\in\wp-\wp'\right\},\\ \left\{\left\langle\Omega,\ \begin{array}{l}[\underline{\Xi^T}_{\Delta'(\wp)}(\mho),\ \overline{\Xi^T}_{\Delta'(\wp)}(\mho)],\\ [\underline{\Xi^I}_{\Delta'(\wp)}(\mho),\ \overline{\Xi^I}_{\Delta'(\wp)}(\mho)],\\ [\underline{\Xi^C}_{\Delta'(\wp)}(\mho),\ \overline{\Xi^C}_{\Delta'(\wp)}(\mho)],\\ [1-\overline{\Xi^T}_{\Delta'(\wp)}(\mho),\ 1-\underline{\Xi^T}_{\Delta'(\wp)}(\mho)]\end{array}\right\rangle;\ \text{if } \wp\in\wp'-\wp\right\},\\ \left\{\left\langle\Omega,\ \begin{array}{l}\vee\left(\Xi^T_{\Delta(\wp)}(\mho),\ \Xi^T_{\Delta'(\wp)}(\mho)\right),\\ \vee\left(\Xi^I_{\Delta(\wp)}(\mho),\ \Xi^I_{\Delta'(\wp)}(\mho)\right),\\ \vee\left(\Xi^C_{\Delta(\wp)}(\mho),\ \Xi^C_{\Delta'(\wp)}(\mho)\right),\\ \left[\begin{array}{l}\wedge\left(1-\overline{\Xi^T}_{\Delta(\wp)}(\mho),\ 1-\overline{\Xi^T}_{\Delta'(\wp)}(\mho)\right),\\ \wedge\left(1-\underline{\Xi^T}_{\Delta(\wp)}(\mho),\ 1-\underline{\Xi^T}_{\Delta'(\wp)}(\mho)\right)\end{array}\right]\end{array}\right\rangle;\ \text{if } \wp\in\wp\cap\wp'\right\}.\end{cases}$$

We know that,

$$\oplus(\Delta,\wp)=\big\{\big\langle\Omega,[\underline{\Xi^T}_{\Delta(\wp)}(\mho),\overline{\Xi^T}_{\Delta(\wp)}(\mho)],[\underline{\Xi^I}_{\Delta(\wp)}(\mho),\overline{\Xi^I}_{\Delta(\wp)}(\mho)],\\ [\underline{\Xi^C}_{\Delta(\wp)}(\mho),\overline{\Xi^C}_{\Delta(\wp)}(\mho)],[(1-\overline{\Xi^T}_{\Delta(\wp)}(\mho)),(1-\underline{\Xi^T}_{\Delta(\wp)}(\mho))]\big\rangle;\wp\in\wp\big\},$$

$$\oplus(\Delta',\wp')=\big\{\big\langle\Omega,[\underline{\Xi^T}_{\Delta'(\wp)}(\mho),\overline{\Xi^T}_{\Delta'(\wp)}(\mho)],[\underline{\Xi^I}_{\Delta'(\wp)}(\mho),\overline{\Xi^I}_{\Delta'(\wp)}(\mho)],\\ [\underline{\Xi^C}_{\Delta'(\wp)}(\mho),\overline{\Xi^C}_{\Delta'(\wp)}(\mho)],[(1-\overline{\Xi^T}_{\Delta'(\wp)}(\mho)),(1-\underline{\Xi^T}_{\Delta'(\wp)}(\mho))]\big\rangle;\wp\in\wp'\big\},$$

Let $\oplus(\Delta,\wp)\cup\oplus(\Delta',\wp')=(\Delta_{\oplus\cup},\wp_{\oplus\cup})$, where $\wp_{\oplus\cup}=\wp\cup\wp'$.
For $\wp\in\wp_{\oplus\cup}$,
$\Delta_{\oplus\cup}(\wp)=$

$$\begin{cases}\big\{\big\langle\Omega,[\underline{\Xi^T}_{\Delta(\wp)}(\mho),\overline{\Xi^T}_{\Delta(\wp)}(\mho)],[\underline{\Xi^I}_{\Delta(\wp)}(\mho),\overline{\Xi^I}_{\Delta(\wp)}(\mho)],\\ \quad[\underline{\Xi^C}_{\Delta(\wp)}(\mho),\overline{\Xi^C}_{\Delta(\wp)}(\mho)],[(1-\overline{\Xi^T}_{\Delta(\wp)}(\mho)),(1-\underline{\Xi^T}_{\Delta(\wp)}(\mho))]\big\rangle;\quad \text{if } \wp\in\wp-\wp'\big\},\\ \big\{\big\langle\Omega,[\underline{\Xi^T}_{\Delta'(\wp)}(\mho),\overline{\Xi^T}_{\Delta'(\wp)}(\mho)],[\underline{\Xi^I}_{\Delta'(\wp)}(\mho),\overline{\Xi^I}_{\Delta'(\wp)}(\mho)],\\ \quad[\underline{\Xi^C}_{\Delta'(\wp)}(\mho),\overline{\Xi^C}_{\Delta'(\wp)}(\mho)],[(1-\overline{\Xi^T}_{\Delta'(\wp)}(\mho)),(1-\underline{\Xi^T}_{\Delta'(\wp)}(\mho))]\big\rangle;\quad \text{if } \wp\in\wp'-\wp\big\},\\ \big\{\big\langle\Omega,\vee(\Xi^T_{\Delta(\wp)}(\mho),\Xi^T_{\Delta'(\wp)}(\mho)),\vee(\Xi^I_{\Delta(\wp)}(\mho),\Xi^I_{\Delta'(\wp)}(\mho)),\\ \quad\vee(\Xi^C_{\Delta(\wp)}(\mho),\Xi^C_{\Delta'(\wp)}(\mho)),[\wedge((1-\overline{\Xi^T}_{\Delta(\wp)}(\mho)),(1-\overline{\Xi^T}_{\Delta'(\wp)}(\mho))),\\ \quad\wedge((1-\underline{\Xi^T}_{\Delta(\wp)}(\mho)),(1-\underline{\Xi^T}_{\Delta'(\wp)}(\mho)))]\big\rangle;\quad \text{if } \wp\in\wp\cap\wp'\big\}.\end{cases}$$

Thus $\oplus((\Delta,\wp)\cup(\Delta',\wp')) = \oplus(\Delta,\wp)\cup\oplus(\Delta',\wp')$.
(iii) $\oplus\oplus(\Delta,\wp)$

$$
\begin{aligned}
&= \oplus\big\{\big\langle\Omega, [\underline{\Xi^T}_{\Delta(\wp)}(\mho), \overline{\Xi^T}_{\Delta(\wp)}(\mho)], [\underline{\Xi^I}_{\Delta(\wp)}(\mho), \overline{\Xi^I}_{\Delta(\wp)}(\mho)], [\underline{\Xi^C}_{\Delta(\wp)}(\mho), \overline{\Xi^C}_{\Delta(\wp)}(\mho)],\\
&\qquad [(1-\overline{\Xi^T}_{\Delta(\wp)}(\mho)), (1-\underline{\Xi^T}_{\Delta(\wp)}(\mho))]\big\rangle; \wp\in\wp\big\}\\
&= \big\{\big\langle\Omega, [\underline{\Xi^T}_{\Delta(\wp)}(\mho), \overline{\Xi^T}_{\Delta(\wp)}(\mho)], [\underline{\Xi^I}_{\Delta(\wp)}(\mho), \overline{\Xi^I}_{\Delta(\wp)}(\mho)], [\underline{\Xi^C}_{\Delta(\wp)}(\mho), \overline{\Xi^C}_{\Delta(\wp)}(\mho)],\\
&\qquad [(1-\overline{\Xi^T}_{\Delta(\wp)}(\mho)), (1-\underline{\Xi^T}_{\Delta(\wp)}(\mho))]\big\rangle; \wp\in\wp\big\}\\
&= \oplus(\Delta,\wp).
\end{aligned}
$$

□

Theorem 4.2. *Consider* $(\Delta,\wp)$ *and* $(\Delta',\wp')$ *over* $(\mho)$. *Then,*
(i) $\ominus((\Delta,\wp)\cup(\Delta',\wp')) = \ominus(\Delta,\wp)\cup\ominus(\Delta',\wp')$;
(ii) $\ominus((\Delta,\wp)\cap(\Delta',\wp')) = \ominus(\Delta,\wp)\cap\ominus(\Delta',\wp')$;
(iii) $\ominus\ominus(\Delta,\wp) = \ominus(\Delta,\wp)$.

Proof. (i) Let $(\Delta,\wp)\cup(\Delta',\wp') = (\Delta_\cup,\wp_\cup)$, where $\wp_\cup = \wp\cup\wp'$, $\forall\ \wp\in\wp_\cup$.
Consider, $\ominus\Xi^F_\cup(\wp)$

$$
= \left\{\begin{array}{l}
\left\{\left\langle\begin{array}{l}
\Omega,\ [(1-\overline{\Xi^F}_{\Delta(\wp)}(\mho)),\ (1-\underline{\Xi^F}_{\Delta(\wp)}(\mho))],\\
[\underline{\Xi^I}_{\Delta(\wp)}(\mho),\ \overline{\Xi^I}_{\Delta(\wp)}(\mho)],\\
[\underline{\Xi^C}_{\Delta(\wp)}(\mho),\ \overline{\Xi^C}_{\Delta(\wp)}(\mho)],\\
[\underline{\Xi^F}_{\Delta(\wp)}(\mho),\ \overline{\Xi^F}_{\Delta(\wp)}(\mho)]
\end{array}\right\rangle;\ \text{if } \wp\in\wp-\wp'\right\},\\
\left\{\left\langle\begin{array}{l}
\Omega,\ [(1-\overline{\Xi^F}_{\Delta'(\wp)}(\mho)),\ (1-\underline{\Xi^F}_{\Delta'(\wp)}(\mho))],\\
[\underline{\Xi^I}_{\Delta'(\wp)}(\mho),\ \overline{\Xi^I}_{\Delta'(\wp)}(\mho)],\\
[\underline{\Xi^C}_{\Delta'(\wp)}(\mho),\ \overline{\Xi^C}_{\Delta'(\wp)}(\mho)],\\
[\underline{\Xi^F}_{\Delta'(\wp)}(\mho),\ \overline{\Xi^F}_{\Delta'(\wp)}(\mho)]
\end{array}\right\rangle;\ \text{if } \wp\in\wp'-\wp\right\},\\
\left\{\left\langle\begin{array}{l}
\Omega,\ \Big[1-\wedge(\overline{\Xi^F}_{\Delta(\wp)}(\mho),\ \overline{\Xi^F}_{\Delta'(\wp)}(\mho)),\\
1-\wedge(\underline{\Xi^F}_{\Delta(\wp)}(\mho),\ \underline{\Xi^F}_{\Delta'(\wp)}(\mho))\Big],\\
\wedge(\Xi^I_{\Delta(\wp)}(\mho),\ \Xi^I_{\Delta'(\wp)}(\mho)),\\
\wedge(\Xi^C_{\Delta(\wp)}(\mho),\ \Xi^C_{\Delta'(\wp)}(\mho)),\\
\wedge(\Xi^F_{\Delta(\wp)}(\mho),\ \Xi^F_{\Delta'(\wp)}(\mho))
\end{array}\right\rangle;\ \text{if } \wp\in\wp\cap\wp'\right\}
\end{array}\right.
$$

$$
= \left\{ \begin{array}{l}
\left\{ \left\langle \begin{array}{l} \Omega,\ [(1-\overline{\Xi^F}_{\Delta(\wp)}(\mho)),\ (1-\underline{\Xi^F}_{\Delta(\wp)}(\mho))], \\ [\underline{\Xi^I}_{\Delta(\wp)}(\mho),\ \overline{\Xi^I}_{\Delta(\wp)}(\mho)], \\ [\underline{\Xi^C}_{\Delta(\wp)}(\mho),\ \overline{\Xi^C}_{\Delta(\wp)}(\mho)], \\ [\underline{\Xi^F}_{\Delta(\wp)}(\mho),\ \overline{\Xi^F}_{\Delta(\wp)}(\mho)] \end{array} \right\rangle;\quad \text{if } \wp \in \wp - \wp' \right\}, \\
\left\{ \left\langle \begin{array}{l} \Omega,\ [(1-\overline{\Xi^F}_{\Delta'(\wp)}(\mho)),\ (1-\underline{\Xi^F}_{\Delta'(\wp)}(\mho))], \\ [\underline{\Xi^I}_{\Delta'(\wp)}(\mho),\ \overline{\Xi^I}_{\Delta'(\wp)}(\mho)], \\ [\underline{\Xi^C}_{\Delta'(\wp)}(\mho),\ \overline{\Xi^C}_{\Delta'(\wp)}(\mho)], \\ [\overline{\Xi^F}_{\Delta'(\wp)}(\mho),\ \overline{\Xi^F}_{\Delta'(\wp)}(\mho)] \end{array} \right\rangle;\quad \text{if } \wp \in \wp' - \wp \right\}, \\
\left\{ \left\langle \begin{array}{l} \Omega,\ [\vee((1-\overline{\Xi^F}_{\Delta(\wp)}(\mho)),\ (1-\overline{\Xi^F}_{\Delta'(\wp)}(\mho))), \\ \vee\,((1-\underline{\Xi^F}_{\Delta(\wp)}(\mho)),\ (1-\underline{\Xi^F}_{\Delta'(\wp)}(\mho)))], \\ \wedge\,(\Xi^I_{\Delta(\wp)}(\mho),\ \Xi^I_{\Delta'(\wp)}(\mho)), \\ \wedge\,(\Xi^C_{\Delta(\wp)}(\mho),\ \Xi^C_{\Delta'(\wp)}(\mho)), \\ \wedge\,(\Xi^F_{\Delta(\wp)}(\mho),\ \Xi^F_{\Delta'(\wp)}(\mho)) \end{array} \right\rangle;\quad \text{if } \wp \in \wp \cap \wp' \right\}
\end{array} \right.
$$

We know that,

$$
\begin{aligned}
\ominus(\Delta,\wp) =& \big\{ \big\langle \Omega, [(1-\overline{\Xi^F}_{\Delta(\wp)}(\mho)), (1-\underline{\Xi^F}_{\Delta(\wp)}(\mho))], [\underline{\Xi^I}_{\Delta(\wp)}(\mho), \overline{\Xi^I}_{\Delta(\wp)}(\mho)], \\
& [\underline{\Xi^C}_{\Delta(\wp)}(\mho), \overline{\Xi^C}_{\Delta(\wp)}(\mho)], [\underline{\Xi^F}_{\Delta(\wp)}(\mho), \overline{\Xi^F}_{\Delta(\wp)}(\mho)] \big\rangle; \wp \in \wp \big\},
\end{aligned}
$$

$$
\begin{aligned}
\ominus(\Delta',\wp') =& \big\{ \big\langle \Omega, [(1-\overline{\Xi^F}_{\Delta'(\wp)}(\mho)), (1-\underline{\Xi^F}_{\Delta'(\wp)}(\mho))], [\underline{\Xi^I}_{\Delta'(\wp)}(\mho), \overline{\Xi^I}_{\Delta'(\wp)}(\mho)], \\
& [\underline{\Xi^C}_{\Delta'(\wp)}(\mho), \overline{\Xi^C}_{\Delta'(\wp)}(\mho)], [\underline{\Xi^F}_{\Delta'(\wp)}(\mho), \overline{\Xi^F}_{\Omega(\wp)}(\mho)] \big\rangle; \wp \in \wp' \big\},
\end{aligned}
$$

Let $\ominus(\Delta,\wp) \cup \ominus(\Delta',\wp') = (\Delta_{\ominus\cup}, \wp_{\ominus\cup})$, where $\wp_{\ominus\cup} = \wp \cup \wp'$.
For $\wp \in \wp_{\ominus\cup}$,
$\Delta_{\ominus\cup}(\wp)$

$$
= \left\{ \begin{array}{l}
\left\{ \left\langle \begin{array}{l} \Omega,\ [(1-\overline{\Xi^F}_{\Delta(\wp)}(\mho)),\ (1-\underline{\Xi^F}_{\Delta(\wp)}(\mho))], \\ [\underline{\Xi^I}_{\Delta(\wp)}(\mho),\ \overline{\Xi^I}_{\Delta(\wp)}(\mho)], \\ [\underline{\Xi^C}_{\Delta(\wp)}(\mho),\ \overline{\Xi^C}_{\Delta(\wp)}(\mho)], \\ [\underline{\Xi^F}_{\Delta(\wp)}(\mho),\ \overline{\Xi^F}_{\Delta(\wp)}(\mho)] \end{array} \right\rangle;\quad \text{if } \wp \in \wp - \wp' \right\}, \\
\left\{ \left\langle \begin{array}{l} \Omega,\ [(1-\overline{\Xi^F}_{\Delta'(\wp)}(\mho)),\ (1-\underline{\Xi^F}_{\Delta'(\wp)}(\mho))], \\ [\overline{\Xi^I}_{\Delta'(\wp)}(\mho),\ \overline{\Xi^I}_{\Delta'(\wp)}(\mho)], \\ [\overline{\Xi^C}_{\Delta'(\wp)}(\mho),\ \overline{\Xi^C}_{\Delta'(\wp)}(\mho)], \\ [\overline{\Xi^F}_{\Delta'(\wp)}(\mho),\ \overline{\Xi^F}_{\Delta'(\wp)}(\mho)] \end{array} \right\rangle;\quad \text{if } \wp \in \wp' - \wp \right\}, \\
\left\{ \left\langle \begin{array}{l} \Omega,\ [\vee((1-\overline{\Xi^F}_{\Delta(\wp)}(\mho)),\ (1-\overline{\Xi^F}_{\Delta'(\wp)}(\mho))), \\ \vee\,((1-\underline{\Xi^F}_{\Delta(\wp)}(\mho)),\ (1-\overline{\Xi^F}_{\Delta'(\wp)}(\mho)))], \\ \wedge\,(\Xi^I_{\Delta(\wp)}(\mho),\ \Xi^I_{\Delta'(\wp)}(\mho)), \\ \wedge\,(\Xi^C_{\Delta(\wp)}(\mho),\ \Xi^C_{\Delta'(\wp)}(\mho)), \\ \wedge\,(\Xi^F_{\Delta(\wp)}(\mho),\ \Xi^F_{\Delta'(\wp)}(\mho)) \end{array} \right\rangle;\quad \text{if } \wp \in \wp \cap \wp' \right\}
\end{array} \right.
$$

Thus $\ominus((\Delta,\wp)\cup(\Delta',\wp')) = \ominus(\Delta,\wp)\cup\ominus(\Delta',\wp')$.

(iii) $\ominus\ominus(\Delta,\wp)$

$$= \ominus\big\{\big\langle\Omega, [(1-\overline{\Xi^F}_{\Delta(\wp)}(\mho)), (1-\underline{\Xi^F}_{\Delta(\wp)}(\mho))], [\underline{\Xi^I}_{\Delta(\wp)}(\mho), \overline{\Xi^I}_{\Delta(\wp)}(\mho)], [\underline{\Xi^C}_{\Delta(\wp)}(\mho), \overline{\Xi^C}_{\Delta(\wp)}(\mho)], [\underline{\Xi^F}_{\Delta(\wp)}(\mho), \overline{\Xi^F}_{\Delta(\wp)}(\mho)]\big\rangle; \wp\in\wp\big\}$$

$$=\big\{\big\langle\Omega, [(1-\overline{\Xi^F}_{\Delta(\wp)}(\mho)), (1-\underline{\Xi^F}_{\Delta(\wp)}(\mho))], [\underline{\Xi^I}_{\Delta(\wp)}(\mho), \overline{\Xi^I}_{\Delta(\wp)}(\mho)], [\underline{\Xi^C}_{\Delta(\wp)}(\mho), \overline{\Xi^C}_{\Delta(\wp)}(\mho)], [\underline{\Xi^F}_{\Delta(\wp)}(\mho), \overline{\Xi^F}_{\Delta(\wp)}(\mho)]\big\rangle; \wp\in\wp\big\}$$

$$=\ominus(\Delta,\wp).$$

□

Theorem 4.3. *Consider* $(\Omega,\wp)$ *an IVIQPNSS over* $(\mho)$. *Then,*
(i) $\ominus\oplus(\Omega,\wp) = \oplus(\Omega,\wp)$;
(ii) $\oplus\ominus(\Omega,\wp) = \ominus(\Omega,\wp)$.

Proof. (i)$\ominus\oplus(\Omega,\wp)$

$$= \big\{\big\langle\Omega, [(1-(1-\underline{\Xi^T}_{\Omega(\wp)}(\mho)), (1-(1-\overline{\Xi^T}_{\Omega(\wp)}(\mho))], [\underline{\Xi^I}_{\Omega(\wp)}(\mho), \overline{\Xi^I}_{\Omega(\wp)}(\mho)], \underline{\Xi^C}_{\Omega(\wp)}(\mho), \overline{\Xi^C}_{\Omega(\wp)}(\mho)], [(1-\overline{\Xi^T}_{\Omega(\wp)}(\mho)), (1-\underline{\Xi^T}_{\Omega(\wp)}(\mho))]\big\rangle; \wp\in\wp\big\}$$

$$=\big\{\big\langle\Omega, [\underline{\Xi^T}_{\Omega(\wp)}(\mho), \overline{\Xi^T}_{\Omega(\wp)}(\mho)], [\underline{\Xi^I}_{\Omega(\wp)}(\mho), \overline{\Xi^I}_{\Omega(\wp)}(\mho)], \underline{\Xi^C}_{\Omega(\wp)}(\mho), \overline{\Xi^C}_{\Omega(\wp)}(\mho)], [(1-\overline{\Xi^T}_{\Omega(\wp)}(\mho)), (1-\underline{\Xi^T}_{\Omega(\wp)}(\mho))]\big\rangle; \wp\in\wp\big\}$$

$$=\oplus(\Omega,\wp).$$

(ii) $\oplus\ominus(\Omega,\wp)$

$$= \big\{\big\langle\Omega, [(1-\overline{\Xi^F}_{\Omega(\wp)}(\mho)), (1-\underline{\Xi^F}_{\Omega(\wp)}(\mho))], [\underline{\Xi^I}_{\Omega(\wp)}(\mho), \overline{\Xi^I}_{\Omega(\wp)}(\mho)], [\underline{\Xi^C}_{\Omega(\wp)}(\mho), \overline{\Xi^C}_{\Omega(\wp)}(\mho)], [(1-(1-\underline{\Xi^F}_{\Omega(\wp)}(\mho)), (1-(1-\overline{\Xi^F}_{\Omega(\wp)}(\mho))]\big\rangle; \wp\in\wp\big\}$$

$$=\big\{\big\langle\Omega, [(1-\overline{\Xi^F}_{\Omega(\wp)}(\mho)), (1-\underline{\Xi^F}_{\Omega(\wp)}(\mho))], [\underline{\Xi^I}_{\Omega(\wp)}(\mho), \overline{\Xi^I}_{\Omega(\wp)}(\mho)], [\underline{\Xi^C}_{\Omega(\wp)}(\mho), \overline{\Xi^C}_{\Omega(\wp)}(\mho)], [\underline{\Xi^F}_{\Omega(\wp)}(\mho), \overline{\Xi^F}_{\Omega(\wp)}(\mho)]\big\rangle; \wp\in\wp\big\}$$

$$=\ominus(\Omega,\wp).$$

□

Theorem 4.4. *Consider* $(\Delta,\wp)$ *and* $(\Delta',\wp')$ *over* $(\mho)$. *Then,*
(i) $\oplus((\Delta,\wp)\wedge(\Delta',\wp')) = \oplus(\Delta,\wp)\wedge\oplus(\Delta',\wp')$;
(ii) $\oplus((\Delta,\wp)\vee(\Delta',\wp')) = \oplus(\Delta,\wp)\vee\oplus(\Delta',\wp')$;
(iii) $\ominus((\Delta,\wp)\wedge(\Delta',\wp')) = \ominus(\Delta,\wp)\wedge\oplus(\Delta',\wp')$;
(iv) $\ominus((\Delta,\wp)\vee(\Delta',\wp')) = \ominus(\Delta,\wp)\vee\oplus(\Delta',\wp')$.

Proof. (i) $\oplus((\Delta,\wp)\wedge(\Delta',\wp'))$

$$= \left\{ \left\langle \begin{array}{l} \Omega,\ \Big[\wedge\big(\underline{\Xi^T}_{(\Delta,\wp)}(\mho),\ \underline{\Xi^T}_{\Delta'(\wp')}(\mho)\big),\ \wedge\big(\overline{\Xi^T}_{(\Delta,\wp)}(\mho),\ \overline{\Xi^T}_{\Delta'(\wp')}(\mho)\big)\Big], \\ \Big[\wedge\big(\underline{\Xi^I}_{(\Delta,\wp)}(\mho),\ \underline{\Xi^I}_{\Delta'(\wp')}(\mho)\big),\ \wedge\big(\overline{\Xi^I}_{(\Delta,\wp)}(\mho),\ \overline{\Xi^I}_{\Delta'(\wp')}(\mho)\big)\Big], \\ \Big[\wedge\big(\underline{\Xi^C}_{(\Delta,\wp)}(\mho),\ \underline{\Xi^C}_{\Delta'(\wp')}(\mho)\big),\ \wedge\big(\overline{\Xi^C}_{(\Delta,\wp)}(\mho),\ \overline{\Xi^C}_{\Delta'(\wp')}(\mho)\big)\Big], \\ \Big[1-\wedge\big(\underline{\Xi^T}_{(\Delta,\wp)}(\mho),\ \underline{\Xi^T}_{\Delta'(\wp')}(\mho)\big),\ 1-\wedge\big(\overline{\Xi^T}_{(\Delta,\wp)}(\mho),\ \overline{\Xi^T}_{\Delta'(\wp')}(\mho)\big)\Big] \end{array} \right\rangle; \right.$$

$$= \left\{ \left\langle \begin{array}{l} \Omega,\ \Big[\wedge\big(\underline{\Xi^T}_{(\Delta,\wp)}(\mho),\ \underline{\Xi^T}_{\Delta'(\wp')}(\mho)\big), \wedge\big(\overline{\Xi^T}_{(\Delta,\wp)}(\mho),\ \overline{\Xi^T}_{\Delta'(\wp')}(\mho)\big)\Big], \\ \Big[\wedge\big(\underline{\Xi^I}_{(\Delta,\wp)}(\mho),\ \underline{\Xi^I}_{\Delta'(\wp')}(\mho)\big), \wedge\big(\overline{\Xi^I}_{(\Delta,\wp)}(\mho),\ \overline{\Xi^I}_{\Delta'(\wp')}(\mho)\big)\Big], \\ \Big[\wedge\big(\underline{\Xi^C}_{(\Delta,\wp)}(\mho),\ \underline{\Xi^C}_{\Delta'(\wp')}(\mho)\big), \wedge\big(\overline{\Xi^C}_{(\Delta,\wp)}(\mho),\ \overline{\Xi^C}_{\Delta'(\wp')}(\mho)\big)\Big], \\ \Big[\vee\big(1-\overline{\Xi^T}_{(\Delta,\wp)}(\mho),\ 1-\overline{\Xi^T}_{\Delta'(\wp')}(\mho)\big), \vee\big(1-\underline{\Xi^T}_{(\Delta,\wp)}(\mho),\ 1-\underline{\Xi^T}_{\Delta'(\wp')}(\mho)\big)\Big] \end{array} \right\rangle \right.;$$

Also,

$$\oplus(\Delta,\wp) = \big\{\big\langle \Omega, [\underline{\Xi^T}_{(\Delta,\wp)}(\mho), \overline{\Xi^T}_{(\Delta,\wp)}(\mho)], [\underline{\Xi^I}_{(\Delta,\wp)}(\mho), \overline{\Xi^I}_{(\Delta,\wp)}(\mho)],$$
$$[\underline{\Xi^C}_{(\Delta,\wp)}(\mho), \overline{\Xi^C}_{(\Delta,\wp)}(\mho)], [(1-\overline{\Xi^T}_{(\Delta,\wp)}(\mho)), (1-\underline{\Xi^T}_{(\Delta,\wp)}(\mho))]\big\rangle;\ \wp \in \wp\big\},$$

$$\oplus(\Delta',\wp') = \big\{\big\langle \Omega, [\underline{\Xi^T}_{\Delta'(\wp')}(\mho), \overline{\Xi^T}_{\Delta'(\wp')}(\mho)], [\underline{\Xi^I}_{\Delta'(\wp')}(\mho), \overline{\Xi^I}_{\Delta'(\wp')}(\mho)],$$
$$[\underline{\Xi^C}_{\Delta'(\wp')}(\mho), \overline{\Xi^C}_{\Delta'(\wp')}(\mho)], [(1-\overline{\Xi^T}_{\Delta'(\wp')}(\mho)), (1-\underline{\Xi^T}_{\Delta'(\wp')}(\mho))]\big\rangle;\ \wp' \in \wp'\big\}.$$

Therefore, we have
$\oplus(\Delta,\wp) \wedge \oplus(\Delta',\wp')$

$$= \big\{\big\langle \Omega, [\wedge(\underline{\Xi^T}_{(\Delta,\wp)}(\mho), \underline{\Xi^T}_{\Delta'(\wp')}(\mho)), \wedge(\overline{\Xi^T}_{(\Delta,\wp)}(\mho), \overline{\Xi^T}_{\Delta'(\wp')}(\mho))],$$
$$[\wedge(\underline{\Xi^I}_{(\Delta,\wp)}(\mho), \underline{\Xi^I}_{\Delta'(\wp')}(\mho)), \wedge(\overline{\Xi^I}_{(\Delta,\wp)}(\mho), \overline{\Xi^I}_{\Delta'(\wp')}(\mho))],$$
$$(\underline{\Xi^C}_{(\Delta,\wp)}(\mho), \underline{\Xi^C}_{\Delta'(\wp')}(\mho)), \wedge(\overline{\Xi^C}_{(\Delta,\wp)}(\mho), \overline{\Xi^C}_{\Delta'(\wp')}(\mho))],$$
$$[\vee((1-\overline{\Xi^T}_{(\Delta,\wp)}(\mho)), (1-\overline{\Xi^T}_{\Delta'(\wp')}(\mho))), \vee((1-\underline{\Xi^T}_{(\Delta,\wp)}(\mho)), (1-\underline{\Xi^T}_{\Delta'(\wp')}(\mho)))]\big\}.$$
$$= \oplus\left((\Delta,\wp) \wedge (\Delta',\wp')\right).$$

(iii) $\ominus((\Delta,\wp) \wedge (\Delta',\wp'))$

$$= \big\{\big\langle \Omega, [(1-\vee(\overline{\Xi^F}_{(\Delta,\wp)}(\mho), \overline{\Xi^F}_{\Delta'(\wp')}(\mho)), (1-\vee(\underline{\Xi^F}_{(\Delta,\wp)}(\mho), \underline{\Xi^T}_{\Delta'(\wp')}(\mho))],$$
$$[\vee(\underline{\Xi^I}_{(\Delta,\wp)}(\mho), \underline{\Xi^I}_{\Delta'(\wp')}(\mho)), \vee(\overline{\Xi^I}_{(\Delta,\wp)}(\mho), \overline{\Xi^I}_{\Delta'(\wp')}(\mho))],$$
$$[\vee(\underline{\Xi^C}_{(\Delta,\wp)}(\mho), \underline{\Xi^C}_{\Delta'(\wp')}(\mho)), \vee(\overline{\Xi^C}_{(\Delta,\wp)}(\mho), \overline{\Xi^C}_{\Delta'(\wp')}(\mho))],$$
$$[\vee(\underline{\Xi^F}_{(\Delta,\wp)}(\mho)), \underline{\Xi^F}_{\Delta'(\wp')}(\mho)), \vee(\overline{\Xi^F}_{(\Delta,\wp)}(\mho), \overline{\Xi^F}_{\Delta'(\wp')}(\mho))], \forall(\wp,\wp') \in \wp \times \wp'\big\}.$$
$$= \big\{\big\langle \Omega, [\wedge((1-\overline{\Xi^F}_{(\Delta,\wp)}(\mho), (1-\overline{\Xi^F}_{\Delta'(\wp')}(\mho))), \wedge((1-\underline{\Xi^F}_{(\Delta,\wp)}(\mho)), (1-\underline{\Xi^F}_{\Delta'(\wp')}(\mho)))],$$
$$[\vee(\underline{\Xi^I}_{(\Delta,\wp)}(\mho), \underline{\Xi^I}_{\Delta'(\wp')}(\mho)), \vee(\overline{\Xi^I}_{(\Delta,\wp)}(\mho), \overline{\Xi^I}_{\Delta'(\wp')}(\mho))],$$
$$[\vee(\underline{\Xi^C}_{(\Delta,\wp)}(\mho), \underline{\Xi^C}_{\Delta'(\wp')}(\mho)), \vee(\overline{\Xi^C}_{(\Delta,\wp)}(\mho), \overline{\Xi^C}_{\Delta'(\wp')}(\mho))],$$
$$[\vee(\underline{\Xi^F}_{(\Delta,\wp)}(\mho), \underline{\Xi^F}_{\Delta'(\wp')}(\mho)), \vee(\overline{\Xi^F}_{(\Delta,\wp)}(\mho), \overline{\Xi^F}_{\Delta'(\wp')}(\mho))], \forall(\wp,\wp') \in \wp \times \wp'\big\}.$$

Also,

$$\ominus(\Delta,\wp) = \big\{\big\langle \Omega, [(1-\overline{\Xi^F}_{(\Delta,\wp)}(\mho)), (1-\underline{\Xi^F}_{(\Delta,\wp)}(\mho))], [\underline{\Xi^I}_{(\Delta,\wp)}(\mho), \overline{\Xi^I}_{(\Delta,\wp)}(\mho)],$$
$$[\underline{\Xi^C}_{(\Delta,\wp)}(\mho), \overline{\Xi^C}_{(\Delta,\wp)}(\mho)], [\underline{\Xi^F}_{(\Delta,\wp)}(\mho), \overline{\Xi^F}_{(\Delta,\wp)}(\mho)]\big\rangle;\ \wp \in \wp\big\},$$

$$\ominus(\Delta',\wp') = \big\{\big\langle \Omega, [(1-\overline{\Xi^F}_{\Delta'(\wp')}(\mho)), (1-\underline{\Xi^F}_{\Delta'(\wp')}(\mho))], [\underline{\Xi^I}_{\Delta'(\wp')}(\mho), \overline{\Xi^I}_{\Delta'(\wp')}(\mho)],$$
$$[\underline{\Xi^C}_{\Delta'(\wp')}(\mho), \overline{\Xi^C}_{\Delta'(\wp')}(\mho)], [\underline{\Xi^F}_{\Delta'(\wp')}(\mho), \overline{\Xi^F}_{\Delta'(\wp')}(\mho)]\big\rangle;\ \wp' \in \wp'\big\}.$$

Therefore, we have
$\ominus(\Delta,\wp)\wedge\ominus(\Delta',\wp')$

$$
\begin{aligned}
&= \big\{\big\langle\Omega, [\wedge((1-\overline{\Xi^F}_{(\Delta,\wp)}(\mho)),(1-\overline{\Xi^F}_{\Delta'(\wp')}(\mho)),\wedge((1-\underline{\Xi^F}_{(\Delta,\wp)}(\mho)),(1-\underline{\Xi^F}_{\Delta'(\wp')}(\mho))],\\
&\quad [\vee(\underline{\Xi^I}_{(\Delta,\wp)}(\mho),\underline{\Xi^I}_{\Delta'(\wp')}(\mho)),\vee(\overline{\Xi^I}_{(\Delta,\wp)}(\mho),\overline{\Xi^I}_{\Delta'(\wp')}(\mho))],\\
&\quad [\vee(\underline{\Xi^C}_{(\Delta,\wp)}(\mho),\underline{\Xi^C}_{\Delta'(\wp')}(\mho)),\vee(\overline{\Xi^C}_{(\Delta,\wp)}(\mho),\overline{\Xi^C}_{\Delta'(\wp')}(\mho))],\\
&\quad [\vee(\underline{\Xi^F}_{(\Delta,\wp)}(\mho)),\underline{\Xi^F}_{\Delta'(\wp')}(\mho)),\vee(\overline{\Xi^F}_{(\Delta,\wp)}(\mho),\overline{\Xi^F}_{\Delta'(\wp')}(\mho)))],\forall(\wp,\wp')\in\wp\times\wp'\big\}.\\
&= \ominus\big((\Delta,\wp)\wedge(\Delta',\wp')\big).
\end{aligned}
$$

□

5. $\pm$ and $\mp$ operators on IVIQPNSS

We present the concept of two operators ($\pm$ and $\mp$) on IVIQPNSS and discuss its properties.

Definition 5.1. *Consider $(\Delta,\wp)$ and $(\Delta',\wp')$ over $(\mho)$. Then,*
(i) the operator $\pm$ is given as $(\Delta,\wp)\pm(\Delta',\wp')=(\Delta_\pm,\wp_\pm)$, where $\wp_\pm=\wp\cup\wp'$. $\forall\ \wp\in\wp_\pm$, $\Delta_\pm(\wp)$

$$
=\begin{cases}
\left\{\left\langle\begin{matrix}\Omega,\ [\underline{\Xi^T}_{\Delta(\wp)}(\mho)+\overline{\Xi^T}_{\Delta(\wp)}(\mho)],\ [\underline{\Xi^I}_{\Delta(\wp)}(\mho)+\overline{\Xi^I}_{\Delta(\wp)}(\mho)],\\ [\underline{\Xi^C}_{\Delta(\wp)}(\mho)+\overline{\Xi^C}_{\Delta(\wp)}(\mho)],\ [\underline{\Xi^F}_{\Delta(\wp)}(\mho)+\overline{\Xi^F}_{\Delta(\wp)}(\mho)]\end{matrix}\right\rangle\right\}; & \textit{if } \wp\in\wp-\wp',\\
\left\{\left\langle\begin{matrix}\Omega,\ [\underline{\Xi^T}_{\Delta'(\wp)}(\mho)+\overline{\Xi^T}_{\Delta'(\wp)}(\mho)],\ [\underline{\Xi^I}_{\Delta'(\wp)}(\mho)+\overline{\Xi^I}_{\Delta'(\wp)}(\mho)],\\ [\underline{\Xi^C}_{\Delta'(\wp)}(\mho)+\overline{\Xi^C}_{\Delta'(\wp)}(\mho)],\ [\underline{\Xi^F}_{\Delta'(\wp)}(\mho)+\overline{\Xi^F}_{\Delta'(\wp)}(\mho)]\end{matrix}\right\rangle\right\}; & \textit{if } \wp\in\wp'-\wp,\\
\left\{\left\langle\begin{matrix}\Omega,\ \left[\dfrac{\underline{\Xi^T}_{\Delta(\wp)}(\mho)+\underline{\Xi^T}_{\Delta'(\wp)}(\mho)}{2},\ \dfrac{\overline{\Xi^T}_{\Delta(\wp)}(\mho)+\overline{\Xi^T}_{\Delta'(\wp)}(\mho)}{2}\right],\\
\left[\dfrac{\underline{\Xi^I}_{\Delta(\wp)}(\mho)+\underline{\Xi^I}_{\Delta'(\wp)}(\mho)}{2},\ \dfrac{\overline{\Xi^I}_{\Delta(\wp)}(\mho)+\overline{\Xi^I}_{\Delta'(\wp)}(\mho)}{2}\right],\\
\left[\dfrac{\underline{\Xi^C}_{\Delta(\wp)}(\mho)+\underline{\Xi^C}_{\Delta'(\wp)}(\mho)}{2},\ \dfrac{\overline{\Xi^C}_{\Delta(\wp)}(\mho)+\overline{\Xi^C}_{\Delta'(\wp)}(\mho)}{2}\right],\\
\left[\dfrac{\underline{\Xi^F}_{\Delta(\wp)}(\mho)+\underline{\Xi^F}_{\Delta'(\wp)}(\mho)}{2},\ \dfrac{\overline{\Xi^F}_{\Delta(\wp)}(\mho)+\overline{\Xi^F}_{\Delta'(\wp)}(\mho)}{2}\right]\end{matrix}\right\rangle\right\}; & \textit{if } \wp\in\wp\cap\wp'.
\end{cases}
$$

(ii) the operator $\mp$ is represented as $(\Delta,\wp)\mp(\Delta',\wp')=(\Delta_\mp,\wp_\mp)$, where $\wp_\mp=\wp\cup\wp'$. $\forall\ \wp\in\wp_\mp$,

$$
\Delta_{\mp}(\wp) = \begin{cases}
\left\{ \left\langle \begin{array}{l} \Omega,\ \left[\underline{\Xi}^{T}{}_{\Delta(\wp)}(\mho) + \overline{\Xi}^{T}{}_{\Delta(\wp)}(\mho)\right],\ \left[\underline{\Xi}^{I}{}_{\Delta(\wp)}(\mho) + \overline{\Xi}^{I}{}_{\Delta(\wp)}(\mho)\right], \\ \left[\underline{\Xi}^{C}{}_{\Delta(\wp)}(\mho) + \overline{\Xi}^{C}{}_{\Delta(\wp)}(\mho)\right],\ \left[\underline{\Xi}^{F}{}_{\Delta(\wp)}(\mho) + \overline{\Xi}^{F}{}_{\Delta(\wp)}(\mho)\right] \end{array} \right\rangle \right\}; & \text{if } \wp \in \wp - \wp', \\
\left\{ \left\langle \begin{array}{l} \Omega,\ \left[\underline{\Xi}^{T}{}_{\Delta'(\wp)}(\mho) + \overline{\Xi}^{T}{}_{\Delta'(\wp)}(\mho)\right],\ \left[\underline{\Xi}^{I}{}_{\Delta'(\wp)}(\mho) + \overline{\Xi}^{I}{}_{\Delta'(\wp)}(\mho)\right], \\ \left[\underline{\Xi}^{C}{}_{\Delta'(\wp)}(\mho) + \overline{\Xi}^{C}{}_{\Delta'(\wp)}(\mho)\right],\ \left[\underline{\Xi}^{F}{}_{\Delta'(\wp)}(\mho) + \overline{\Xi}^{F}{}_{\Delta'(\wp)}(\mho)\right] \end{array} \right\rangle \right\}; & \text{if } \wp \in \wp' - \wp, \\
\left\{ \left\langle \begin{array}{l} \Omega,\ \left[\dfrac{2\,\underline{\Xi}^{T}{}_{\Delta(\wp)}(\mho)\cdot\underline{\Xi}^{T}{}_{\Delta'(\wp)}(\mho)}{\underline{\Xi}^{T}{}_{\Delta(\wp)}(\mho) + \underline{\Xi}^{T}{}_{\Delta'(\wp)}(\mho)},\ \dfrac{2\,\overline{\Xi}^{T}{}_{\Delta(\wp)}(\mho)\cdot\overline{\Xi}^{T}{}_{\Delta'(\wp)}(\mho)}{\overline{\Xi}^{T}{}_{\Delta(\wp)}(\mho) + \overline{\Xi}^{T}{}_{\Delta'(\wp)}(\mho)}\right], \\ \left[\dfrac{\underline{\Xi}^{I}{}_{\Delta(\wp)}(\mho) + \underline{\Xi}^{I}{}_{\Delta'(\wp)}(\mho)}{2},\ \dfrac{\overline{\Xi}^{I}{}_{\Delta(\wp)}(\mho) + \overline{\Xi}^{I}{}_{\Delta'(\wp)}(\mho)}{2}\right], \\ \left[\dfrac{\underline{\Xi}^{C}{}_{\Delta(\wp)}(\mho) + \underline{\Xi}^{C}{}_{\Delta'(\wp)}(\mho)}{2},\ \dfrac{\overline{\Xi}^{C}{}_{\Delta(\wp)}(\mho) + \overline{\Xi}^{C}{}_{\Delta'(\wp)}(\mho)}{2}\right], \\ \left[\dfrac{2\,\underline{\Xi}^{F}{}_{\Delta(\wp)}(\mho)\cdot\underline{\Xi}^{F}{}_{\Delta'(\wp)}(\mho)}{\underline{\Xi}^{F}{}_{\Delta(\wp)}(\mho) + \underline{\Xi}^{F}{}_{\Delta'(\wp)}(\mho)},\ \dfrac{2\,\overline{\Xi}^{F}{}_{\Delta(\wp)}(\mho)\cdot\overline{\Xi}^{F}{}_{\Delta'(\wp)}(\mho)}{\overline{\Xi}^{F}{}_{\Delta(\wp)}(\mho) + \overline{\Xi}^{F}{}_{\Delta'(\wp)}(\mho)}\right] \end{array} \right\rangle \right\}; & \text{if } \wp \in \wp \cap \wp'.
\end{cases}
$$

Example 5.1. *A Psychiatrist conducts two counseling sessions for clients, evaluating their conditions through IVIQPNSS representation. The values for the first session* $(\Delta, \wp)$ *are shown in Table 1, while the second session* $(\Delta', \wp')$ *is detailed in Table 4. Using these, combined results from the two sessions are computed with operations* $(\Delta, \wp) \pm (\Delta', \wp')$ *and* $(\Delta, \wp) \mp (\Delta', \wp')$*, as presented in Tables 5 and 6.*

TABLE 4. Shows patients with PSTE in IVIQPNSS $(\Delta', \wp')$ form.

$(\mho)$	$HA(\wp_1)$	$FF(\wp_2)$	$DS(\wp_3)$
v_1	$\langle[0, 0.2], [0.4, 0.6], [0.6, 0.8], [0, 0.1]\rangle$	$\langle[0, 0.1], [0.5, 0.7], [0.6, 0.8], [0, 0.2]\rangle$	$\langle[0, 0.1], [0.6, 0.7], [0.7, 0.9], [0, 0.2]\rangle$
v_2	$\langle[0, 0.2], [0.3, 0.5], [0.5, 0.7], [0, 0.1]\rangle$	$\langle[0, 0.2], [0.4, 0.6], [0.6, 0.8], [0, 0.1]\rangle$	$\langle[0, 0.1], [0.3, 0.5], [0.6, 0.7], [0, 0.2]\rangle$
v_3	$\langle[0, 0.1], [0.5, 0.6], [0.7, 0.8], [0, 0.1]\rangle$	$\langle[0, 0.1], [0.6, 0.7], [0.7, 0.9], [0, 0.2]\rangle$	$\langle[0, 0.1], [0.4, 0.5], [0.6, 0.8], [0, 0.1]\rangle$

(i) The IVIQPNSS $(\Delta, \wp) \pm (\Delta', \wp')$ *is shown in Table 5.*

TABLE 5. Final computed values in IVIQPNSS $(\Delta_{\pm}, \wp_{\pm})$ form.

$(\mho)$	$HA(\wp_1)$	$FF(\wp_2)$	$DS(\wp_3)$
$\mho_1$	$\langle[.15, .35], [.40, .60], [.65, .80], [.10, .25]\rangle$	$\langle[.10, .25], [.50, .70], [.65, .80], [.10, .25]\rangle$	$\langle[.15, .25], [.60, .75], [.70, .85], [.05, .20]\rangle$
$\mho_2$	$\langle[.20, .40], [.25, .45], [.60, .75], [.05, .20]\rangle$	$\langle[.25, .40], [.35, .55], [.65, .80], [.05, .15]\rangle$	$\langle[.10, .20], [.35, .55], [.65, .75], [.15, .35]\rangle$
$\mho_3$	$\langle[.10, .20], [.55, .70], [.65, .75], [.10, .20]\rangle$	$\langle[.15, .25], [.55, .70], [.65, .80], [.20, .35]\rangle$	$\langle[.20, .35], [.35, .50], [.65, .75], [.05, .15]\rangle$

(ii) The IVIQPNSS $(\Delta, \wp) \mp (\Delta', \wp')$ *is given in Table 6.*

TABLE 6. Final computed values in IVIQPNSS $(\Delta_{\mp}, \wp_{\mp})$, form.

$(\mho)$	$HA(\wp_1)$	$FF(\wp_2)$	$DS(\wp_3)$
$\mho_1$	$\langle[.00, .29], [.50, .60], [.70, .40], [.00, .16]\rangle$	$\langle[.00, .16], [.60, .65], [.70, .40], [.00, .24]\rangle$	$\langle[.00, .16], [.65, .70], [.80, .45], [.00, .20]\rangle$
$\mho_2$	$\langle[.00, .30], [.40, .50], [.60, .35], [.00, .15]\rangle$	$\langle[.00, .30], [.50, .60], [.70, .40], [.00, .13]\rangle$	$\langle[.00, .15], [.40, .55], [.65, .35], [.00, .29]\rangle$
$\mho_3$	$\langle[.00, .15], [.55, .65], [.75, .40], [.00, .15]\rangle$	$\langle[.00, .16], [.65, .70], [.80, .45], [.00, .29]\rangle$	$\langle[.00, .17], [.45, .55], [.70, .40], [.00, .13]\rangle$

Proposition 5.1. *Consider* $(\Delta, \wp)$ *and* $(\Delta', \wp')$ *over* $(\mho)$. *Then,*
(i) $(\Delta, \wp) \pm (\Delta', \wp') = (\Delta', \wp') \pm (\Delta, \wp)$;
(ii) $[(\Delta, \wp)^c \pm (\Delta', \wp')^c]^c = (\Delta, \wp) \pm (\Delta, \wp)$.

Proof. (i) Proof straightforward.
(ii) Consider two IVIQPNSS

$$(\Delta, \wp) = \big\{\big\langle \Omega, [\underline{\Xi^T}_{\Delta(\wp)}(\mho) + \overline{\Xi^T}_{\Delta(\wp)}(\mho)], [\underline{\Xi^I}_{\Delta(\wp)}(\mho) + \overline{\Xi^I}_{\Delta(\wp)}(\mho)], [\underline{\Xi^C}_{\Delta(\wp)}(\mho) + \overline{\Xi^C}_{\Delta(\wp)}(\mho)], [\underline{\Xi^F}_{\Delta(\wp)}(\mho) + \overline{\Xi^F}_{\Delta(\wp)}(\mho)]\big\rangle; \wp \in \wp\big\},$$

and

$$(\Delta', \wp') = \big\{\big\langle \Omega, [\underline{\Xi^T}_{\Delta'(\wp)}(\mho) + \overline{\Xi^T}_{\Delta'(\wp)}(\mho)], [\underline{\Xi^I}_{\Delta'(\wp)}(\mho) + \overline{\Xi^I}_{\Delta'(\wp)}(\mho)], [\underline{\Xi^C}_{\Delta'(\wp)}(\mho) + \overline{\Xi^C}_{\Delta'(\wp)}(\mho)], [\underline{\Xi^F}_{\Delta'(\wp)}(\mho) + \overline{\Xi^F}_{\Delta'(\wp)}(\mho)]\big\rangle; \wp \in \wp'\big\}$$

Then,
$[(\Delta, \wp)^c \pm (\Delta', \wp')^c]$

$$= \begin{cases} \big\{\big\langle \Omega, [\underline{\Xi}^F_{\Delta(\wp)}(\mho), \overline{\Xi}^F_{\Delta(\wp)}(\mho)], [\underline{\Xi}^I_{\Delta(\wp)}(\mho), \overline{\Xi}^I_{\Delta(\wp)}(\mho)], [\underline{\Xi}^C_{\Delta(\wp)}(\mho), \overline{\Xi}^C_{\Delta(\wp)}(\mho)], [\underline{\Xi}^T_{\Delta(\wp)}(\mho), \overline{\Xi}^T_{\Delta(\wp)}(\mho)]\big\rangle; \quad \text{if } \wp \in \wp - \wp'\big\}, \\ \big\{\big\langle \Omega, [\underline{\Xi}^F_{\Delta'(\wp)}(\mho), \overline{\Xi}^F_{\Delta'(\wp)}(\mho)], [\underline{\Xi}^I_{\Delta'(\wp)}(\mho), \overline{\Xi}^I_{\Delta'(\wp)}(\mho)], [\underline{\Xi}^C_{\Delta'(\wp)}(\mho), \overline{\Xi}^C_{\Delta'(\wp)}(\mho)], [\underline{\Xi}^T_{\Delta'(\wp)}(\mho), \overline{\Xi}^T_{\Delta'(\wp)}(\mho)]\big\rangle; \quad \text{if } \wp \in \wp' - \wp\big\}, \\ \Big\{\Big\langle \Omega, \Big[\frac{\underline{\Xi^F}_{\Delta(\wp)}(\mho) + \underline{\Xi^F}_{\Delta'(\wp)}(\mho)}{2}, \frac{\overline{\Xi^F}_{\Delta(\wp)}(\mho) + \overline{\Xi^F}_{\Delta'(\wp)}(\mho)}{2}\Big], \Big[\frac{(\underline{\Xi^I}_{\Delta(\wp)}(\mho)) + (\underline{\Xi^I}_{\Delta'(\wp)}(\mho))}{2}, \frac{(\overline{\Xi^I}_{\Delta(\wp)}(\mho)) + (\overline{\Xi^I}_{\Delta'(\wp)}(\mho))}{2}\Big], \\ \Big[\frac{\underline{\Xi^C}_{\Delta(\wp)}(\mho) + \underline{\Xi^C}_{\Delta'(\wp)}(\mho)}{2}, \frac{\overline{\Xi^C}_{\Delta(\wp)}(\mho) + \overline{\Xi^C}_{\Delta'(\wp)}(\mho)}{2}\Big], \Big[\frac{\underline{\Xi^T}_{\Delta(\wp)}(\mho) + \underline{\Xi^T}_{\Delta'(\wp)}(\mho)}{2}, \frac{\overline{\Xi^T}_{\Delta(\wp)}(\mho) + \overline{\Xi^T}_{\Delta'(\wp)}(\mho)}{2}\Big]\Big\rangle; \quad \text{if } \wp \in \wp \cap \wp'\Big\}. \end{cases}$$

Now consider,
$[(\Delta, \wp)^c \pm (\Delta', \wp')^c]^c$

$$= \begin{cases} \big\{\big\langle \Omega, [\underline{\Xi}^T_{\Delta(\wp)}(\mho), \overline{\Xi}^T_{\Delta(\wp)}(\mho)], [\underline{\Xi}^I_{\Delta(\wp)}(\mho), \overline{\Xi}^I_{\Delta(\wp)}(\mho)], [\underline{\Xi}^C_{\Delta(\wp)}(\mho), \overline{\Xi}^C_{\Delta(\wp)}(\mho)], [\underline{\Xi}^F_{\Delta(\wp)}(\mho), \overline{\Xi}^F_{\Delta(\wp)}(\mho)]\big\rangle; \quad \text{if } \wp \in \wp - \wp'\big\}, \\ \big\{\big\langle \Omega, [\underline{\Xi}^T_{\Delta'(\wp)}(\mho), \overline{\Xi}^T_{\Delta'(\wp)}(\mho)], [\underline{\Xi}^I_{\Delta'(\wp)}(\mho), \overline{\Xi}^I_{\Delta'(\wp)}(\mho)], [\underline{\Xi}^C_{\Delta'(\wp)}(\mho), \overline{\Xi}^C_{\Delta'(\wp)}(\mho)], [\underline{\Xi}^F_{\Delta'(\wp)}(\mho), \overline{\Xi}^F_{\Delta'(\wp)}(\mho)]\big\rangle; \quad \text{if } \wp \in \wp' - \wp\big\}, \\ \Big\{\Big\langle \Omega, \Big[\frac{\underline{\Xi^T}_{\Delta(\wp)}(\mho) + \underline{\Xi^T}_{\Delta'(\wp)}(\mho)}{2}, \frac{\overline{\Xi^T}_{\Delta(\wp)}(\mho) + \overline{\Xi^T}_{\Delta'(\wp)}(\mho)}{2}\Big], \Big[\frac{\underline{\Xi^I}_{\Delta(\wp)}(\mho) + \underline{\Xi^I}_{\Delta'(\wp)}(\mho)}{2}, \frac{\overline{\Xi^I}_{\Delta(\wp)}(\mho) + \overline{\Xi^I}_{\Delta'(\wp)}(\mho)}{2}\Big], \\ \Big[\frac{\underline{\Xi^C}_{\Delta(\wp)}(\mho) + \underline{\Xi^C}_{\Delta'(\wp)}(\mho)}{2}, \frac{\overline{\Xi^C}_{\Delta(\wp)}(\mho) + \overline{\Xi^C}_{\Delta'(\wp)}(\mho)}{2}\Big], \Big[\frac{\underline{\Xi^F}_{\Delta(\wp)}(\mho) + \underline{\Xi^F}_{\Delta'(\wp)}(\mho)}{2}, \frac{\overline{\Xi^F}_{\Delta(\wp)}(\mho) + \overline{\Xi^F}_{\Delta'(\wp)}(\mho)}{2}\Big]\Big\rangle; \quad \text{if } \wp \in \wp \cap \wp'\Big\}. \end{cases}$$

Hence $[(\Delta, \wp)^c \pm (\Delta', \wp')^c]^c = (\Delta, \wp) \pm (\Delta', \wp')$. ☐

Proposition 5.2. *Consider* $(\Delta, \wp)$ *and* $(\Delta', \wp')$ *over* $(\mho)$. *Then,*
(i) $(\Delta, \wp) \mp (\Delta', \wp') = (\Delta', \wp') \mp (\Delta, \wp)$;
(ii) $[(\Delta, \wp)^c \mp (\Delta', \wp')^c]^c = (\Delta, \wp) \mp (\Delta, \wp)$.

Proof. (i) Consider, $(\Delta, \wp) \mp (\Delta', \wp')$

$$= \begin{cases} \Big\{\Big\langle \Omega, \big[\underline{\Xi^T}_{\Delta(\wp)}(\mho), \overline{\Xi^T}_{\Delta(\wp)}(\mho)\big], \big[\underline{\Xi^I}_{\Delta(\wp)}(\mho), \overline{\Xi^I}_{\Delta(\wp)}(\mho)\big], \\ \quad \big[\underline{\Xi^C}_{\Delta(\wp)}(\mho), \overline{\Xi^C}_{\Delta(\wp)}(\mho)\big], \big[\underline{\Xi^F}_{\Delta(\wp)}(\mho), \overline{\Xi^F}_{\Delta(\wp)}(\mho)\big]\Big\rangle; \qquad \text{if } \wp \in \wp - \wp'\Big\}, \\ \Big\{\Big\langle \Omega, \big[\underline{\Xi^T}_{\Delta'(\wp)}(\mho), \overline{\Xi^T}_{\Delta'(\wp)}(\mho)\big], \big[\underline{\Xi^I}_{\Delta'(\wp)}(\mho), \overline{\Xi^I}_{\Delta'(\wp)}(\mho)\big], \\ \quad \big[\underline{\Xi^C}_{\Delta'(\wp)}(\mho), \overline{\Xi^C}_{\Delta'(\wp)}(\mho)\big], \big[\underline{\Xi^F}_{\Delta'(\wp)}(\mho), \overline{\Xi^F}_{\Delta'(\wp)}(\mho)\big]\Big\rangle; \qquad \text{if } \wp \in \wp' - \wp\Big\}, \\ \Big\{\Big\langle \Omega, \left[\frac{2\underline{\Xi^T}_{\Delta(\wp)}(\mho)\cdot\underline{\Xi^T}_{\Delta'(\wp)}(\mho)}{\underline{\Xi^T}_{\Delta(\wp)}(\mho)+\underline{\Xi^T}_{\Delta'(\wp)}(\mho)}, \frac{2\overline{\Xi^T}_{\Delta(\wp)}(\mho)\cdot\overline{\Xi^T}_{\Delta'(\wp)}(\mho)}{\overline{\Xi^T}_{\Delta(\wp)}(\mho)+\overline{\Xi^T}_{\Delta'(\wp)}(\mho)}\right], \\ \quad \left[\frac{\underline{\Xi^I}_{\Delta(\wp)}(\mho)+\underline{\Xi^I}_{\Delta'(\wp)}(\mho)}{2}, \frac{\overline{\Xi^I}_{\Delta(\wp)}(\mho)+\overline{\Xi^I}_{\Delta'(\wp)}(\mho)}{2}\right], \\ \quad \left[\frac{\underline{\Xi^C}_{\Delta(\wp)}(\mho)+\underline{\Xi^C}_{\Delta'(\wp)}(\mho)}{2}, \frac{\overline{\Xi^C}_{\Delta(\wp)}(\mho)+\overline{\Xi^C}_{\Delta'(\wp)}(\mho)}{2}\right], \\ \quad \left[\frac{2\underline{\Xi^F}_{\Delta(\wp)}(\mho)\cdot\underline{\Xi^F}_{\Delta'(\wp)}(\mho)}{\underline{\Xi^F}_{\Delta(\wp)}(\mho)+\underline{\Xi^F}_{\Delta'(\wp)}(\mho)}, \frac{2\overline{\Xi^F}_{\Delta(\wp)}(\mho)\cdot\overline{\Xi^F}_{\Delta'(\wp)}(\mho)}{\overline{\Xi^F}_{\Delta(\wp)}(\mho)+\overline{\Xi^F}_{\Delta'(\wp)}(\mho)}\right]\Big\rangle; \qquad \text{if } \wp \in \wp \cap \wp'\Big\}. \end{cases}$$

$(\Delta, \wp) \mp (\Delta', \wp')$

$$= \begin{cases} \Big\{\Big\langle \Omega, \big[\underline{\Xi^T}_{\Delta(\wp)}(\mho), \overline{\Xi^T}_{\Delta(\wp)}(\mho)\big], \big[\underline{\Xi^I}_{\Delta(\wp)}(\mho), \overline{\Xi^I}_{\Delta(\wp)}(\mho)\big], \\ \quad \big[\underline{\Xi^C}_{\Delta(\wp)}(\mho), \overline{\Xi^C}_{\Delta(\wp)}(\mho)\big], \big[\underline{\Xi^F}_{\Delta(\wp)}(\mho), \overline{\Xi^F}_{\Delta(\wp)}(\mho)\big]\Big\rangle; \qquad \text{if } \wp \in \wp - \wp'\Big\}, \\ \Big\{\Big\langle \Omega, \big[\underline{\Xi^T}_{\Delta'(\wp)}(\mho), \overline{\Xi^T}_{\Delta'(\wp)}(\mho)\big], \big[\underline{\Xi^I}_{\Delta'(\wp)}(\mho), \overline{\Xi^I}_{\Delta'(\wp)}(\mho)\big], \\ \quad \big[\underline{\Xi^C}_{\Delta'(\wp)}(\mho), \overline{\Xi^C}_{\Delta'(\wp)}(\mho)\big], \big[\underline{\Xi^F}_{\Delta'(\wp)}(\mho), \overline{\Xi^F}_{\Delta'(\wp)}(\mho)\big]\Big\rangle; \qquad \text{if } \wp \in \wp' - \wp\Big\}, \\ \Big\{\Big\langle \Omega, \left[\frac{2\underline{\Xi^T}_{\Delta'(\wp)}(\mho)\cdot\underline{\Xi^T}_{\Delta(\wp)}(\mho)}{\underline{\Xi^T}_{\Delta'(\wp)}(\mho)+\underline{\Xi^T}_{\Delta(\wp)}(\mho)}, \frac{2\overline{\Xi^T}_{\Delta'(\wp)}(\mho)\cdot\overline{\Xi^T}_{\Delta(\wp)}(\mho)}{\overline{\Xi^T}_{\Delta'(\wp)}(\mho)+\overline{\Xi^T}_{\Delta(\wp)}(\mho)}\right], \\ \quad \left[\frac{\underline{\Xi^I}_{\Delta'(\wp)}(\mho)+\underline{\Xi^I}_{\Delta(\wp)}(\mho)}{2}, \frac{\overline{\Xi^I}_{\Delta'(\wp)}(\mho)+\overline{\Xi^I}_{\Delta(\wp)}(\mho)}{2}\right], \\ \quad \left[\frac{\underline{\Xi^C}_{\Delta'(\wp)}(\mho)+\underline{\Xi^C}_{\Delta(\wp)}(\mho)}{2}, \frac{\overline{\Xi^C}_{\Delta'(\wp)}(\mho)+\overline{\Xi^C}_{\Delta(\wp)}(\mho)}{2}\right], \\ \quad \left[\frac{2\underline{\Xi^F}_{\Delta'(\wp)}(\mho)\cdot\underline{\Xi^F}_{\Delta(\wp)}(\mho)}{\underline{\Xi^F}_{\Delta'(\wp)}(\mho)+\underline{\Xi^F}_{\Delta(\wp)}(\mho)}, \frac{2\overline{\Xi^F}_{\Delta'(\wp)}(\mho)\cdot\overline{\Xi^F}_{\Delta(\wp)}(\mho)}{\overline{\Xi^F}_{\Delta'(\wp)}(\mho)+\overline{\Xi^F}_{\Delta(\wp)}(\mho)}\right]\Big\rangle; \qquad \text{if } \wp \in \wp \cap \wp'\Big\}. \end{cases}$$

Hence $(\Delta, \wp) \mp (\Delta', \wp') = (\Delta', \wp') \mp (\Delta, \wp)$.
(ii) Consider, $(\Delta, \wp)^c \mp (\Delta', \wp')^c$

$$= \begin{cases} \Big\{\Big\langle \Omega, \big[\underline{\Xi^F}_{\Delta(\wp)}(\mho), \overline{\Xi^F}_{\Delta(\wp)}(\mho)\big], \big[\underline{\Xi^I}_{\Delta(\wp)}(\mho), \overline{\Xi^I}_{\Delta(\wp)}(\mho)\big], \\ \quad \big[\underline{\Xi^C}_{\Delta(\wp)}(\mho), \overline{\Xi^C}_{\Delta(\wp)}(\mho)\big], \big[\underline{\Xi^T}_{\Delta(\wp)}(\mho), \overline{\Xi^T}_{\Delta(\wp)}(\mho)\big]\Big\rangle; \qquad \text{if } \wp \in \wp - \wp'\Big\}, \\ \Big\{\Big\langle \Omega, \big[\underline{\Xi^F}_{\Delta'(\wp)}(\mho), \overline{\Xi^F}_{\Delta'(\wp)}(\mho)\big], \big[\underline{\Xi^I}_{\Delta'(\wp)}(\mho), \overline{\Xi^I}_{\Delta'(\wp)}(\mho)\big], \\ \quad \big[\underline{\Xi^C}_{\Delta'(\wp)}(\mho), \overline{\Xi^C}_{\Delta'(\wp)}(\mho)\big], \big[\underline{\Xi^T}_{\Delta'(\wp)}(\mho), \overline{\Xi^T}_{\Delta'(\wp)}(\mho)\big]\Big\rangle; \qquad \text{if } \wp \in \wp' - \wp\Big\}, \\ \Big\{\Big\langle \Omega, \left[\frac{2\underline{\Xi^F}_{\Delta(\wp)}(\mho)\cdot\underline{\Xi^F}_{\Delta'(\wp)}(\mho)}{\underline{\Xi^F}_{\Delta(\wp)}(\mho)+\underline{\Xi^F}_{\Delta'(\wp)}(\mho)}, \frac{2\overline{\Xi^F}_{\Delta(\wp)}(\mho)\cdot\overline{\Xi^F}_{\Delta'(\wp)}(\mho)}{\overline{\Xi^F}_{\Delta(\wp)}(\mho)+\overline{\Xi^F}_{\Delta'(\wp)}(\mho)}\right], \\ \quad \left[\frac{\underline{\Xi^I}_{\Delta(\wp)}(\mho)+\underline{\Xi^I}_{\Delta'(\wp)}(\mho)}{2}, \frac{\overline{\Xi^I}_{\Delta(\wp)}(\mho)+\overline{\Xi^I}_{\Delta'(\wp)}(\mho)}{2}\right], \\ \quad \left[\frac{\underline{\Xi^C}_{\Delta(\wp)}(\mho)+\underline{\Xi^C}_{\Delta'(\wp)}(\mho)}{2}, \frac{\overline{\Xi^C}_{\Delta(\wp)}(\mho)+\overline{\Xi^C}_{\Delta'(\wp)}(\mho)}{2}\right], \\ \quad \left[\frac{2\underline{\Xi^T}_{\Delta(\wp)}(\mho)\cdot\underline{\Xi^T}_{\Delta'(\wp)}(\mho)}{\underline{\Xi^T}_{\Delta(\wp)}(\mho)+\underline{\Xi^T}_{\Delta'(\wp)}(\mho)}, \frac{2\overline{\Xi^T}_{\Delta(\wp)}(\mho)\cdot\overline{\Xi^T}_{\Delta'(\wp)}(\mho)}{\overline{\Xi^T}_{\Delta(\wp)}(\mho)+\overline{\Xi^T}_{\Delta'(\wp)}(\mho)}\right]\Big\rangle; \qquad \text{if } \wp \in \wp \cap \wp'\Big\}. \end{cases}$$

Then, $[(\Delta,\wp)^c \mp (\Delta',\wp')^c]^c$

$$= \begin{cases} \Big\{\Big\langle \Omega, \big[\underline{\Xi^T}_{\Delta(\wp)}(\mho), \overline{\Xi^T}_{\Delta(\wp)}(\mho)\big], \big[\underline{\Xi^I}_{\Delta(\wp)}(\mho), \overline{\Xi^I}_{\Delta(\wp)}(\mho)\big], \\ \quad \big[\underline{\Xi^C}_{\Delta(\wp)}(\mho), \overline{\Xi^C}_{\Delta(\wp)}(\mho)\big], \big[\underline{\Xi^F}_{\Delta(\wp)}(\mho), \overline{\Xi^F}_{\Delta(\wp)}(\mho)\big]\Big\rangle; \quad \text{if } \wp \in \wp - \wp'\Big\}, \\ \Big\{\Big\langle \Omega, \big[\underline{\Xi^T}_{\Delta'(\wp)}(\mho), \overline{\Xi^T}_{\Delta'(\wp)}(\mho)\big], \big[\underline{\Xi^I}_{\Delta'(\wp)}(\mho), \overline{\Xi^I}_{\Delta'(\wp)}(\mho)\big], \\ \quad \big[\underline{\Xi^C}_{\Delta'(\wp)}(\mho), \overline{\Xi^C}_{\Delta'(\wp)}(\mho)\big], \big[\underline{\Xi^F}_{\Delta'(\wp)}(\mho), \overline{\Xi^F}_{\Delta'(\wp)}(\mho)\big]\Big\rangle; \quad \text{if } \wp \in \wp' - \wp\Big\}, \\ \Bigg\{\Bigg\langle \Omega, \left[\frac{2\underline{\Xi^T}_{\Delta(\wp)}(\mho)\cdot\underline{\Xi^T}_{\Delta'(\wp)}(\mho)}{\underline{\Xi^T}_{\Delta(\wp)}(\mho)+\underline{\Xi^T}_{\Delta'(\wp)}(\mho)}, \frac{2\overline{\Xi^T}_{\Delta(\wp)}(\mho)\cdot\overline{\Xi^T}_{\Delta'(\wp)}(\mho)}{\overline{\Xi^T}_{\Delta(\wp)}(\mho)+\overline{\Xi^T}_{\Delta'(\wp)}(\mho)}\right], \\ \quad \left[\frac{\underline{\Xi^I}_{\Delta(\wp)}(\mho)+\underline{\Xi^I}_{\Delta'(\wp)}(\mho)}{2}, \frac{\overline{\Xi^I}_{\Delta(\wp)}(\mho)+\overline{\Xi^I}_{\Delta'(\wp)}(\mho)}{2}\right], \\ \quad \left[\frac{\underline{\Xi^C}_{\Delta(\wp)}(\mho)+\underline{\Xi^C}_{\Delta'(\wp)}(\mho)}{2}, \frac{\overline{\Xi^C}_{\Delta(\wp)}(\mho)+\overline{\Xi^C}_{\Delta'(\wp)}(\mho)}{2}\right], \\ \quad \left[\frac{2\underline{\Xi^F}_{\Delta(\wp)}(\mho)\cdot\underline{\Xi^F}_{\Delta'(\wp)}(\mho)}{\underline{\Xi^F}_{\Delta(\wp)}(\mho)+\underline{\Xi^F}_{\Delta'(\wp)}(\mho)}, \frac{2\overline{\Xi^F}_{\Delta(\wp)}(\mho)\cdot\overline{\Xi^F}_{\Delta'(\wp)}(\mho)}{\overline{\Xi^F}_{\Delta(\wp)}(\mho)+\overline{\Xi^F}_{\Delta'(\wp)}(\mho)}\right]\Bigg\rangle; \quad \text{if } \wp \in \wp \cap \wp'\Bigg\}. \end{cases}$$

Hence $[(\Delta,\wp)^c \mp (\Delta',\wp')^c]^c = (\Delta,\wp) \mp (\Delta',\wp')$. □

6. Conclusions

This paper introduces the concept of IVIQPNSs and establishes several key properties associated with them. We also investigate the properties of two novel operators, $\pm$ and $\mp$, alongside the classical necessity and possibility operators. The study of these operators enhances the flexibility and applicability of IVIQPNSs in various decision-making contexts. Additionally, we propose a novel method for diagnosing psychiatric disorders using IVIQPNSs, where individuals' psychiatric behaviors are effectively modeled within this framework. The proposed methodology provides a robust mathematical foundation for handling uncertainty and contradictions in psychiatric diagnosis. Given the promising results, future research can extend this work by integrating IVIQPNSs with other hybrid set theories to refine psychiatric disorder classification further. Moreover, the application of IVIQPNSs can be explored in broader domains such as sociology, healthcare, and cognitive sciences, where managing conflicting and ambiguous information is crucial. Future studies may also focus on computational implementations and real-world validations to enhance the practical applicability of the proposed approach [27].

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