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q^*-RUNG ORTHOPAIR NEUTROSOPHIC SUBSPACES AND NODEC SPACES

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Abstract. The study explores the concept of q*-rung orthopair neutrosophic topological spaces, beginning with foundational results on q*-rung orthopair neutrosophic sets. It defines subspace topology within these spaces and analyzes various properties, particularly q*-rung orthopair neutrosophic nodec spaces. These are examined under the condition that every q*-rung orthopair neutrosophic nowhere dense subset is q*-rung orthopair neutrosophic closed. Additionally, as specific examples of nodec spaces, the study investigates submaximal spaces and q*-rung orthopair neutrosophic doors. Relevant characteristics and behaviors are methodically examined. Interestingly, it shows that a q*-rung orthopair neutrosophic nodec space can be obtained by combining two discontinuous q*-rung orthopair neutrosophic closed and q*-rung orthopair neutrosophic dense (or open) spaces. Furthermore, the way these nodec spaces behave under different operations is examined.

Keywords: q*-rung orthopair neutrosophic set, q*-rung orthopair neutrosophic topological space, q*-rung orthopair neutrosophic point, q*-rung orthopair neutrosophic subspaces, q*-rung orthopair neutrosophic nodec space and q*-rung orthopair neutrosophic continuous.

AMS Subject Classification: 54A40

1. Introduction

Classical topology and set theory deal with the binary concepts of membership and non-membership. However, many real-world situations involve uncertainty, hesitation, or indeterminacy. To overcome this limitation, neutrosophic sets were introduced, and later, q-rung orthopair neutrosophic sets emerged as a more flexible and powerful extension. Uncertainty and indeterminacy are common in real-world data mining situations, rendering traditional mathematical frameworks inadequate. A strong tool for managing such complications is provided by neutrosophic sets. Neutosophic sets, a generalization of fuzzy, intuitionistic, Boolean, and paraconsistent sets, are special in that they explicitly include

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a degree of indeterminacy in addition to truth and falsehood. q-rung orthopair neutrosophic sets enhance the modeling of uncertainty by incorporating three degrees: truth, falsity, and indeterminacy. In this structure, truth and falsity components are dependent and bounded by the q-power constraint, while the indeterminacy component remains independent. Specifically, the constraint $T^q + F^q + I^q \leq 2$ where q is a positive integer ($q \geq 1$) allows for a more refined representation of uncertainty compared to traditional fuzzy, intuitionistic fuzzy, or Pythagorean fuzzy sets. Because of this, q-rung orthopair neutrosophic sets are highly useful for processing ambiguous or linguistically stated data, particularly in situations involving decision-making. The flexibility of the parameter q, which may be adjusted to account for different degrees of vagueness and hence model more complicated uncertainty, is their main advantage. Extending topological concepts into this framework allows for the definition of spaces such as q-rung orthopair neutrosophic nodec spaces. These spaces facilitate exploration of the behavior of neutrosophic nowhere dense sets and their relationships with neutrosophic closed sets.

1.1. **Literature review.** This literature review delves into existing research on q-rung orthopair neutrosophic sets and their application to nodec spaces. The concept of fuzzy sets, introduced by Zadeh L. [27] in 1965, marked a significant advancement in mathematical frameworks by allowing for partial membership. This concept was further expanded upon with the introduction of intuitionistic fuzzy sets by Atanassov K. T. [2] in 1986, which incorporate both membership and non-membership degrees. However, both fuzzy and intuitionistic fuzzy sets have limitations in dealing with high levels of uncertainty and indeterminacy. In 2013 Revathi G. K. and et. al [9] introduced the concept of Ordered (r, s) intuitionistic fuzzy quasi-uniform regular $G\delta$ extremally disconnected spaces. In a subsequent study, Revathi G. K. and et. al [10] presented the notion of regular $G\delta$-continuous mappings.The concept of the q-rung orthopair fuzzy set, where q is a positive integer, was later introduced by Yager R. R. [25]. In q-rung orthopair fuzzy set, the sum of the qth powers of membership and non-membership does not exceed 1. This generalization allows for greater flexibility in defining the degrees of truth and falsehood, providing a more comprehensive model for uncertainty. When $q = 1$, the structure reduces to an intuitionistic fuzzy set, while for $q = 2$, it forms a Pythagorean fuzzy set [24], and for $q = 3$, a Fermatean fuzzy set [12]. This led to the development of neutrosophic sets by Smarandache F. [15] in 1998. Neutrosophic sets introduced three components: truth (T), indeterminacy (I), and falsity (F), each of which can independently take values in the range between 0 and 1. This allows for the representation of incomplete, inconsistent, and ambiguous information. Neutrosophic topological space can be applied to many engineering problems. In 2024, Shyamaladevi V. and Revathi G. K. [13] conducted an extensive review of numerous papers exploring the practical applications of neutrosophic topology, a remarkable extension of classical topology. This innovative field addresses challenges rooted in uncertainty and indeterminacy, uncovering its relevance across diverse domains. Soft neutrosophic topology serves as an extension of neutrosophic topology, combining the strengths of soft sets and neutrosophic sets to offer a more adaptable toolset. Bera T. and Mahapatra N. K. [4], [5] introduced the concept of neutrosophic soft sets, exploring fundamental notions such as neutrosophic soft interior, neutrosophic soft closure, neutrosophic soft neighborhood, neutrosophic soft boundary, and regular neutrosophic soft sets. Leveraging this concept, many mathematicians have contributed to diverse mathematical structures. For instance, in 2024, Narmada Devi R. and Parthiban Y. [6] established the theoretical foundations of a groundbreaking framework and demonstrated its practical relevance in healthcare decision-making. Furthermore, [8] presented innovative ideas, including neutrosophic over soft semi-j open sets and neutrosophic over soft hyperconnected spaces, supported by a numerical illustration to identify the most effective approach for novel pharmaceutical applications using the neutrosophic over soft measure of correlation. The q-rung orthopair neutrosophic sets, introduced by Voskoglou M. G. and et. al [22] in 2024, extend both neutrosophic and q-rung orthopair sets. In q-rung orthopair sets, the sum of the q-th powers of membership and non-membership degrees is limited to 1. In contrast, q-rung orthopair neutrosophic sets define truth, falsehood, and indeterminacy degrees, allowing the total sum to reach 2, with the parameter q controlling the sum. This provides a more comprehensive framework for modeling uncertainty. This enables the model to handle even higher levels of uncertainty compared to traditional neutrosophic and intuitionistic fuzzy models. Nodec spaces are topological spaces where every nowhere dense set is closed; in fuzzy topology, fuzzy nodec spaces extend this by treating fuzzy nowhere dense sets as fuzzy closed sets. This concept was notably explored by Sostak A. P. in 1985 [18]. Neutrosophic nodec spaces generalize classical topology by incorporating uncertainty, indeterminacy, and hesitation through neutrosophic sets. These spaces are valuable in areas like decision-making, machine learning, and data clustering. Building on this, q*-rung orthopair neutrosophic nodec spaces introduce a three-valued logic framework, allowing more flexible representation of membership, non-membership, and indeterminacy under q*-rung orthopair constraints.

1.2. **Motivation and Contribution of the Study.** Several main goals serve as the motivation for this paper. Initially, it seeks to create a new class of q^* -rung orthopair neutrosophic topological spaces, namely q^* -rung orthopair neutrosophic nodec spaces, in order to facilitate future studies in this field. Secondly, the framework showcased here incorporates and expands upon earlier ideas, including submaximal spaces, while bringing in fresh thoughts that may improve a number of advancements in mathematical and practical fields. With the introduction of q^* -rung orthopair neutrosophic sets and the extension of nodec spaces into the neutrosophic domain, this study aims to develop a flexible and reliable tool for practical applications that require advanced uncertainty management strategies. In finality, the investigation highlights the importance of topology as a starting point for recent advances in mathematics and applications. The following are the primary contributions of this paper:

(i) The innovative ideas, q*-rung orthopair neutrosophic topological space and q*-rung orthopair neutrosophic points are introduced.
(ii) The notion of q*-rung orthopair neutrosophic subspaces is defined in this paper along with the related theorems.
(iii) This study defines the concept of q*-rung orthopair neutrosophic nodec space and explores the associated theorems.

2. Preliminaries

This part introduces abbreviations and their expansions and also talk about the essential preliminary definitions in this section in order to comprehend the outcomes that follow.

Expansion	**Abbreviation**
Fuzzy set	FS
q-rung orthopair fuzzy set	q-ROFS
Neutrosophic set	NS

Fuzzy Topological	FT
Neutrosophic topological space	NTS
q-rung orthopair neutrosophic	q-RON
q*-rung orthopair neutrosophic topological space	q*-RONTS
q*-rung orthopair neutrosophic point	q*-RONP
q*-rung orthopair neutrosophic continuous	q*-RONC

Definition 2.1. [27] *Assume X is not an empty set. A FS A is derived from $A = \{(x, \mu_A(x)) : x \in X\}$, where $\mu_A(x) : X \longrightarrow [0, 1]$ represents the belongingness for the FS A. FS is a group of things with varying degrees of belongingness.*

Definition 2.2. [25] *An q-ROFS A in the universal discourse X is $A = \{(x, \mu_A(x), \nu_A(x)), x \in X\}$ where $0 \leq \mu_A^q(x) + \nu_A^q(x) \leq 1$, q is a positive integer ($q \geq 1$) is called q-ROFS and $\mu_A : X \longrightarrow [0, 1]$ is the degree of belongingness and $\nu_A : X \longrightarrow [0, 1]$ is the degree of non belongingness of the element $x \in X$ to the set A respectively.*

Definition 2.3. [17] *Assuming that X is not an empty set , then the set $A = \{(x, \mu_A(x), \pi_A(x), \nu_A(x)), x \in X\}$ is called NS on X, where $0 \leq \mu_A(x) + \pi_A(x) + \nu_A(x) \leq 3$ for all $x \in X$, $\nu_A : X \longrightarrow [0, 1]$ is the degree of non-belongingness and $\pi_A : X \longrightarrow [0, 1]$ is the degree of indeterminacy and $\mu_A : X \longrightarrow [0, 1]$ is the degree of belongingness of every $x \in X$ to the set A as well.*

Definition 2.4. [11] *A set X that is not empty. Then the set is NT that meets these three axioms:*

(i) $0_N, 1_N \in \tau$

(ii) $A_1 \bigcap A_2 \in \tau$ *for any* $A_1, A_2 \in \tau$,

(iii) $\bigcup A_i \in \tau$ *for all* $\{A_i : i \in J\} \subseteq \tau$.

The combination of two (X, τ) is referred to as NTS. Neutrosophic open sets make up the members of τ. $K^c \in \tau$ indicates that a set K is neutrosophic closed. In this NTS, K^c represents all Neutrosophic closed sets.

Definition 2.5. [22] *Assume that the set X is not empty, then the set $A_{qn} = \{(x, \mu_{A_{qn}}(x), \pi_{A_{qn}}(x), \nu_{A_{qn}}(x)), x \in X\}$ where $0 \leq \mu^q_{A_{qn}}(x) + \nu^q_{A_{qn}}(x) \leq 1$, $q \geq 1$, and for all $x \in X$ such that $0 \leq \mu^q_{A_{qn}}(x) + \pi^q_{A_{qn}}(x) + \nu^q_{A_{qn}}(x) \leq 2$ is called q-RONS where $\mu_{A_{qn}} : X \longrightarrow [0, 1]$ is the degree of belongingness , $\nu_{A_{qn}} : X \longrightarrow [0, 1]$ is the degree of non-belongingness and $\pi_{A_{qn}} : X \longrightarrow [0, 1]$ is the degree of indeterminacy of the element $x \in X$ to the set A_{qn} . The components $\mu^q_{A_{qn}}(x)$ and $\nu^q_{A_{qn}}(x)$ are dependent and the component $\pi^q_{A_{qn}}(x)$ is independent. Set of all q-RONS over X is denoted by $N_{qn}(X)$*

Example 2.1. *Let $X = \{x_1, x_2, x_3\}$ and A_{qn} be a q-RONS that can be written as*
$A_{qn} = \{(x_1, \mu_{A_{qn}}(x_1), \pi_{A_{qn}}(x_1), \nu_{A_{qn}}(x_1)), (x_2, \mu_{A_{qn}}(x_2), \pi_{A_{qn}}(x_2), \nu_{A_{qn}}(x_2)),$
$(x_3, \mu_{A_{qn}}(x_3), \pi_{A_{qn}}(x_3), \nu_{A_{qn}}(x_3))\} = \{(x_1, 0.4, 0.9, 0.5), (x_2, 0.5, 0.5, 0.5), (x_3, 0.5, 0.8, 0.4)\}$
Here, for all $\{x_1, x_2, x_3\} \in X, q \geq 1, 0 \leq \mu^q_{A_{qn}}(x) + \nu^q_{A_{qn}}(x) \leq 1$ and $0 \leq \mu^q_{A_{qn}}(x) + \nu^q_{A_{qn}}(x) + \pi^q_{A_{qn}}(x) \leq 2$.

Definition 2.6. [20] *A FS A in a FTS (X, τ), is called a fuzzy nowhere dense set if there exists no non zero fuzzy open set B in (X, τ) such that $B < cl(A)$. That is ,$int(cl(A)) = 0$, in (X, τ).*

Definition 2.7. [19] *Let (X, τ) be the FTS and (X, τ) is said to be fuzzy nodec space, if each non zero fuzzy nowhere dense set is fuzzy closed in (X, τ).*

Definition 2.8. [1] *Let (X, τ) be the FTS and (X, τ) is said to be fuzzy door space, if every fuzzy sub set of X is either fuzzy open or fuzzy closed in (X, τ).*

Definition 2.9. [3] *A FTS (X, τ) is termed a fuzzy submaximal space if, for every FS A in (X, τ) such that $cl(A) = 1$, it follows that $A \in \tau$. That is fuzzy submaximal space if each fuzzy dense set in (X, τ) is a fuzzy open set in (X, τ).*

Definition 2.10. [7] *A neutrosophic set A in NTS (X, τ) is called neutrosophic dense if there exists no neutrosophic closed set B in (X, τ) such that $A \subset B \subset 1_N$.*

Definition 2.11. [7] *A neutrosophic set A in NTS (X, τ) is called neutrosophic nowhere dense set if there exists no neutrosophic open set U in (X, τ) such that $U \subset Ncl(A)$. That is $NintNcl(A) = 0_N$*

Definition 2.12. [21] *For a q-RON set*
$A_{qn} = \{(x, \mu_{A_{qn}}(x), \pi_{A_{qn}}(x), \nu_{A_{qn}}(x), x) \in X\}$ *of the non-empty fixed set X, the q^*-RON set A^*_{qn}, is defined to be the following triple structure:*
$A^*_{qn} = \{(\mu^*_{A^*_{qn}}, \pi^*_{A^*_{qn}}, \nu^*_{A^*_{qn}})\}$ *where* $\mu^*_{A^*_{qn}} = \min(\mu_{A_{qn}}, 1 - \max(\pi_{A_{qn}}, \nu_{A_{qn}}))$,
$\pi^*_{A^*_{qn}} = \min(\pi_{A_{qn}}, 1 - \max(\mu_{A_{qn}}, \nu_{A_{qn}}))$ $\nu^*_{A^*_{qn}} = \min(\nu_{A_{qn}}, 1 - \max(\mu_{A_{qn}}, \pi_{A_{qn}}))$ *and* $0 \leq \mu^{*q}_{A^*_{qn}}(x) + \nu^{*q}_{A^*_{qn}}(x) \leq 1$ *and for all $x \in X$ such that* $0 \leq \mu^{*q}_{A^*_{qn}}(x) + \nu^{*q}_{A^*_{qn}}(x) + \pi^{*q}_{A^*_{qn}}(x) \leq 2$ *is called q^*-RONS where $q \geq 1$, $\mu^*_{A^*_{qn}} : X \longrightarrow [0, 1]$ is the degree of belongingness , $\nu^*_{A^*_{qn}} : X \longrightarrow [0, 1]$ is the degree of non-belongingness and $\pi^*_{A^*_{qn}} : X \longrightarrow [0, 1]$ is the degree of indeterminacy of the element $x \in X$ to the set A^*_{qn} . Set of all q^*-RON set over X is denoted by $N^*_{qn}(X)$*

Example 2.2. *Let $X = \{x_{qn_1}, x_{qn_2}, x_{qn_3}\}$ and A_{qn} be a q-RONS that can be written as*
$A_{qn} = \{(x_{qn_1}, \mu_{A_{qn}}(x_{qn_1}), \pi_{A_{qn}}(x_{qn_1}), \nu_{A_{qn}}(x_{qn_1})), (x_{qn_2}, \mu_{A_{qn}}(x_{qn_2}), \pi_{A_{qn}}(x_{qn_2}), \nu_{A_{qn}}(x_{qn_2})),$
$(x_{qn_3}, \mu_{A_{qn}}(x_{qn_3}), \pi_{A_{qn}}(x_{qn_3}), \nu_{A_{qn}}(x_{qn_3}))\} = \{(x_{qn_1}, 0.4, 0.9, 0.5), (x_{qn_2}, 0.5, 0.1, 0.5),$
$(x_{qn_3}, 0.5, 0.8, 0.4)\}$. *Here, for all* $\{x_{qn_1}, x_{qn_2}, x_{qn_3}\} \in X_{qn}, 0 \leq \mu^q_{A_{qn}}(x_{qn}) + \nu^q_{A_{qn}}(x_{qn}) \leq 1, 0 \leq \mu^q_{A_{qn}}(x_{qn}) + \nu^q_{A_{qn}}(x_{qn}) + \pi^q_{A_{qn}}(x_{qn}) \leq 2$.
Then the q^*-RON set A^*_{qn} is $A^*_{qn} = \{(x_{qn_1}, 0.1, 0.5, 0.1), (x_{qn_2}, 0.5, 0.1, 0.5), (x_{qn_3}, 0.2, 0.5, 0.2)\}$.

Definition 2.13. [21] *Let $X \neq \phi$, A^*_{qn} and B^*_{qn} be the q^*-RON subsets in X with the notation* $A^*_{qn} = \{(x, \mu^*_{A^*_{qn}}(x), \pi^*_{A^*_{qn}}(x), \nu^*_{A^*_{qn}}(x)) : x \in X\}$ *and*
$B^*_{qn} = \{(x, \mu^*_{B^*_{qn}}(x), \pi^*_{B^*_{qn}}(x), \nu^*_{B^*_{qn}}(x)) : x \in X\}$. *If* $\mu^*_{A^*_{qn}}(x) \leq \mu^*_{B_{qn}}(x), \pi^*_{A_{qn}}(x) \leq \pi^*_{B_{qn}}(x)$ *and* $\nu^*_{A_{qn}}(x) \geq \nu^*_{B_{qn}}(x)$ *then it is denoted as* $A^*_{qn} \subseteq B^*_{qn}$.

Definition 2.14. [21] *For q^*-RONS A^*_{qn} and B^*_{qn}, $A^*_{qn} \bigcup B^*_{qn}$, $A^*_{qn} \bigcap B^*_{qn}$ and ${A^*_{qn}}^c$ will be defined as*

(i) $A^*_{qn} \bigcup B^*_{qn} = \{(x, max\{\mu^*_{A^*_{qn}(x)}, \mu^*_{B^*_{qn}(x)}\}, max\{\pi^*_{A^*_{qn}}(x), \pi^*_{B^*_{qn}}(x)\}, min\{\nu^*_{A^*_{qn}}(x), \nu^*_{B^*_{qn}}(x)\})\}$
(ii) $A^*_{qn} \bigcap B^*_{qn} = \{(x_{qn}, min\{\mu^*_{A^*_{qn}(x)}, \mu^*_{B^*_{qn}(x)}\}, min\{\pi^*_{A^*_{qn}}(x), \pi^*_{B^*_{qn}}(x)\}, max\{\nu^*_{A^*_{qn}}(x), \nu^*_{B^*_{qn}}(x)\})\}$
(iii) ${A^*_{qn}}^c = \{(x, \nu^*_{A^*_{qn}}(x), 1 - \pi^*_{A^*_{qn}}(x), \mu^*_{A^*_{qn}}(x) : x \in X)\}$.

Remark 2.1. (i) *All types of $\emptyset^*_{qn}$ and $\emptyset_{qn}$ are conceded , that is $\emptyset_{qn} = (0, 0, 1) = \emptyset^*_{qn}$.*
(ii) *All types of X^*_{qn} and X_{qn} are conceded, that is $X_{qn} = (1, 1, 0) = X^*_{qn}$.*

Definition 2.15. [21] *Let $X \neq \phi$ and τ^*_{qn} be a collection of q^*-RON subsets of X. If τ^*_{qn} satisfies the following properties then it is called a s q^*-RONT.*

(i) $\emptyset^*_{qn}, X^*_{qn} \in \tau^*_{qn}$, *where* $\emptyset^*_{qn} = (0,0,1)$ *and* $X^*_{qn} = (1,1,0)$.
(ii) *For any* $A^*_{qn_1}, A^*_{qn_2} \in \tau^*_{qn}$ *then* $A^*_{qn_1} \bigcap A^*_{qn_2} \in \tau^*_{qn}$
(iii) *For all* $i \in J, if\{A^*_{qn_i}\} \in \tau^*_{qn}$ *then* $(\bigcup A^*_{qn_i}) \in \tau^*_{qn}$.

Then (X, τ^*_{qn}) *is called a* q^**-RONTS.*
Every member of q^**-RONT is called as open* q^**-RONS, and its complement is closed* q^**-RONS. The set of all* q^**-RON set over* X *is denoted by* $N^*_{qn}(X_{qn})$.

Example 2.3. [21] *Let* $X = \{a_{qn}, b_{qn}, c_{qn}\}$ *for all* $k \in \{1,2,\}$, A_{qn_k} *be a q-RONS:*
$A_{qn_1} = \{(a_{qn}, 0.7, 0.5, 0.2), (b_{qn}, 0.3, 0.5, 0.2), (c_{qn}, 0.6, 0.3, 0.3)\}$,
$A_{qn_2} = \{(a_{qn}, 0.8, 0.5, 0.2), (b_{qn}, 0.5, 0.7, 0.2), (c_{qn}, 0.7, 0.5, 0.3)\}$
where for all $x \in X, 0 \leq \mu^q_{A_{qn_k}}(x_{qn}) + \nu^q_{A_{qn_k}}(x_{qn}) \leq 1$, $q \geq 1$ *and*
$0 \leq \mu^q_{A_{qn_k}}(x_{qn}) + \pi^q_{A_{qn_k}}(x_{qn}) + \nu^q_{A_{qn_k}}(x_{qn}) \leq 2$.
Then $A^*_{qn_1} = \{(a_{qn}, 0.5, 0.3, 0.2), (b_{qn}, 0.3, 0.5, 0.2), (c_{qn}, 0.6, 0.3, 0.3)\}$, *and*
$A^*_{qn_2} = \{(a_{qn}, 0.5, 0.2, 0.2), (b_{qn}, 0.3, 0.5, 0.2), (c_{qn}, 0.5, 0.3, 0.3)\}$ *are* q^**-RON set.*
Clearly $\tau^*_{qn} = \{\emptyset^*_{qn}, X^*_{qn}, A^*_{qn_1}, A^*_{qn_2},\}$ *is a* q^**-RONT.*

Definition 2.16. [21] *Let* $(A^*_{qn}, \tau^*_{qn_1})$ *and* $(B^*_{qn}, \tau^*_{qn_2})$ *be any two* q^**-RONTS and* $\tau^*_{qn_1} \subset \tau^*_{qn_2}$. *Then , the* $\tau^*_{qn_2}$ q^**-RONT is said to be finer than the* q^**-RONT* $\tau^*_{qn_1}$.

Definition 2.17. [21] *Let* (X, τ^*_{qn}) *be* q^**-RONTS and* $B^*_{qn} \subseteq X$ *be* q^**-RONS in* X. *If there is an open* q^**-RON subset* G^*_{qn} *such that* $B^*_{qn} \subset G^*_{qn} \subset X$, *it is said that* X *is a neighborhood of* B^*_{qn}.

Definition 2.18. [21] *Let* (X, τ^*_{qn}) *be* q^**-RONTS and let* $A^*_{qn} = \{(x, \mu^*_{A^*_{qn}}(x), \pi^*_{A^*_{qn}}(x), \nu^*_{A^*_{qn}}(x)) : x \in X\}$ *be a* q^**-RONS. In this case* q^**-rung orthopair interior and closure for* A^*_{qn} *are defined as*
(i) $int_{qn}(A^*_{qn}) = \bigcup\{O^*_{qn_i} : O^*_{qn_i} \subset A^*_{qn_i}, O^*_{qn_i}$ *is open* q^**-RONS* $\}$,
(ii) $cl_{qn}(A^*_{qn}) = \bigcap\{C^*_{qn_i} : A^*_{qn_i} \subset C^*_{qn_i}, C^*_{qn_i}$ *is closed* q^**-RONS* $\}$.

Definition 2.19. [21] *Let* $(X, \tau^*_{qn_1})$ *and* $(Y, \tau^*_{qn_2})$ *be any two* q^**-RONTS and let* $f_{qn} : X \longrightarrow Y$ *be a function. If for any open* q^**-RONS* B^*_{qn} *of* Y , $f^{-1}_{qn}(B^*_{qn})$ *is an open* q^**-RONS of* X, *then* f_{qn} *is said to be* q^**-RON continuous.*

Definition 2.20. [21] *Let* $(X, \tau^*_{qn_1})$ *and* $(Y, \tau^*_{qn_2})$ *be any two* q^**-RONTS and let* $f_{qn} : X_{qn} \longrightarrow Y$ *be a function. Then* f_{qn} *is said to be* q^**-RON open iff the image of each* q^**-RON in* $\tau^*_{qn_1}$ *is a* q^**-RON open in* $\tau^*_{qn_2}$.

Definition 2.21. [21] *Let* $(X, \tau^*_{qn_1})$ *and* $(Y, \tau^*_{qn_2})$ *be any two* q^**-RONTS and let* $f_{qn} : X \longrightarrow Y$ *be a function. Then* f_{qn} *is said to be* q^**-RON closed iff the image of each* q^**-RON in* $\tau^*_{qn_1}$ *is a* q^**-RON closed in* $\tau^*_{qn_2}$.

Definition 2.22. [14] *Let* $\alpha, \beta, \gamma \in [0,1]$ *and* $\alpha + \beta + \gamma \leq 2$. *A* q^**-RONP* $P^*_{qn}x(\alpha, \beta, \gamma)$ *of* X *defined by* $P^*_{qn}x(\alpha, \beta, \gamma) = \{x, \mu^*_{P^*_{qn}}, \pi^*_{P^*_{qn}}, \nu^*_{P^*_{qn}}\}$ *for* $y \in X$

$$\mu^*_{(P^*_{qn})} = \begin{cases} \alpha, & if y = x \\ 0, & if y \neq x \end{cases}$$

$$\pi^*_{(P^*_{qn})} = \begin{cases} \beta, & if y = x \\ 0, & if y \neq x \end{cases}$$

$$\nu^*_{(P^*_{qn})} = \begin{cases} \gamma, & if y = x \\ 1, & if y \neq x \end{cases}$$

.
The collection of all q^*-rung orthopair neutrosophic points (X, τ^*_{qn}) is denoted by P^*_{qn}.

3. q^*-Rung Orthopair Neutrosophic Subspaces

Several results pertaining to q*-rung orthopair neutrosophic sets are attempted to be established in this part. The q*-rung orthopair neutrosophic subspace is then defined with an example, and some of its properties are examined.

Definition 3.1. *Let $(X, \tau^*_{qn_1})$ and $(Y, \tau^*_{qn_2})$ be any two q^*-RONTS , $f_{qn} : X \longrightarrow Y$ be a function and let A^*_{qn} and B^*_{qn} be q^*-RON sets and $A^*_{qn} \subseteq X$ and $B^*_{qn} \subseteq Y$. As shown by $f_{qn}(A^*_{qn})$, the grade of belongingness, non-belongingness, and indefiniteness of image of A^*_{qn} according to f_{qn} are defined by*

$$\mu^*_{A^*_{qn}, f_{qn}(A^*_{qn})}(y) = \begin{cases} \sup_{x \in f^{-1}_{qn}(x_2)} \mu^*_{A^*_{qn}}(x), & \text{if } f^{-1}_{qn}(y) \neq 0 \\ 0, & \text{if } f^{-1}_{qn}(y) = 0 \end{cases}$$

$$\pi^*_{A^*_{qn}, f_{qn}(A^*_{qn})}(y) = \begin{cases} \sup_{x \in f^{-1}_{qn}(y)} \pi^*_{A^*_{qn}}(x), & \text{if } f^{-1}_{qn}(y) \neq 0 \\ 0, & \text{if } f^{-1}_{qn}(y) = 0 \end{cases}$$

$$\nu^*_{A^*_{qn}, f_{qn}(A^*_{qn})}(y) = \begin{cases} \inf_{x \in f^{-1}_{qn}(y)} \nu^*_{A^*_{qn}}(x), & \text{if } f^{-1}_{qn}(y) \neq 0 \\ 1, & \text{if } f^{-1}_{qn}(y) = 0 \end{cases}$$

Here , $f_{qn}(A^*_{qn})$ is a q^*-RON subsets. According to f_{qn}, the degree of belongingness, indefiniteness, and non-belongingness of the pre-image of A^*_{qn} as shown by $f^{-1}_{qn}(A^*_{qn})$ is defined by
$\mu^*_{A^*_{qn}, f^{-1}_{qn}(A^*_{qn})}(x) = \mu^*_{A^*_{qn,B^*_{qn}}}(f_{qn}(x))$, $\pi^*_{A^*_{qn}, f^{-1}_{qn}(A^*_{qn})}(x) = \pi^*_{A^*_{qn,B^*_{qn}}}(f_{qn}(x))$,
and $\nu^*_{A^*_{qn}, f^{-1}_{qn}(A^*_{qn})}(x) = \nu^*_{A^*_{qn,B^*_{qn}}}(f_{qn}(x))$.
At the same time, the set $f^{-1}_{qn}(A^*_{qn})$ is also a q^*-RON subset.

Definition 3.2. *Let X and Y be any two crisp sets such that $Y \neq \emptyset$ and $Y \subseteq X$. Then we define $A^*_{qn} = \{(x, \mu^*(x), \pi^*(x), \nu^*(x)) : x \in X\}$, where $\mu^* = 1, \pi^* = 1, \nu^* = 0$ if $x \in Y$ and $\mu^* = 0, \pi^* = 0, \nu^* = 1$ if $x \in X \mid_Y$. The set of all q^*-RON sets over Y will be denoted by $N^*_{qn}(Y)$.*

Definition 3.3. *Let X and Y be any two crisp sets such that $Y \neq \emptyset$ and $Y \subseteq X$, then for a q^*-RON set $A^*_{qn} \in N_{qn}(X)$, we define $A^*_{qn} \mid_Y = \{(x, \mu^*_{A^*_{qn}|_Y}, \pi^*_{A^*_{qn}|_Y}, \nu^*_{A^*_{qn}|_Y}) : x \in X\}$, where $\mu^*_{A^*_{qn}|_Y}(x) = \mu^*_{A^*_{qn}}, \pi^*_{A^*_{qn}|_Y}(x) = \pi^*_{A^*_{qn}}, \nu^*_{A^*_{qn}|_Y}(x) = \nu^*_{A^*_{qn}}$ if $x \in Y$ and $\mu^*_{A^*_{qn}|_Y}(x) = 0, \pi^*_{A^*_{qn}|_Y}(x) = 0, \nu^*_{A^*_{qn}|_Y}(x) = 1$ if $x \in X \mid_Y$.*

Remark 3.1. *From the Definition 3.2 and 3.3*
- (i) *$A^*_{qn} \mid_Y \in N^*_{qn}(Y)$ for every $A^*_{qn} \in N^*_{qn}(X)$.*
- (ii) *Every q^*-RON set A^*_{qn} over Y can be considered as an q^*-RON set over X by taking $\mu^*_{A^*_{qn}} = 0, \pi^*_{A^*_{qn}} = 0, \nu^*_{A^*_{qn}} = 1$ for all $x \in X \mid Y$ and $\mu^*_{A^*_{qn}} = 1, \pi^*_{A^*_{qn}} = 1, \nu^*_{A^*_{qn}} = 0$ for all $x \in X \mid_Y$*
- (iii) *$X^*_{qn} \mid_Y = Y^*_{qn}$ and $\emptyset^*_{qn} \mid_Y = \emptyset^*_{qn}$*

Proposition 3.1. *Let X, Y, Z be three sets such that $\emptyset \neq Z \subseteq Y \subseteq X$. Let $A^*_{qn} \in N^*_{qn}(X)$ and $\{A^*_{qn_\lambda}; \lambda \in \triangle\} \subseteq N^*_{qn}(X)$, where $\triangle$ is an index set. Then*

(i) $(\bigcup_{\lambda\in\triangle} A^*_{qn_\lambda})|_Y = \bigcup_{\lambda\in\triangle}(A^*_{qn_\lambda}|_Y)$.
(ii) $(\bigcap_{\lambda\in\triangle} A^*_{qn_\lambda})|_Y = \bigcap_{\lambda\in\triangle}(A^*_{qn_\lambda}|_Y)$.
(iii) $A^{*c}_{qn}|_Y = (A^*_{qn}|_Y)^c$.
(iv) $(A^*_{qn}|_Y)|_Z = (A^*_{qn}|_Z)$.

Proof. (i)

$$
\begin{aligned}
(\bigcup_{\lambda\in\triangle} A^*_{qn_\lambda})|_Y &= \{(x, \mu^*_{(\cup_{\lambda\in\triangle} A^*_{qn_\lambda})|_Y}, \pi^*_{(\cup_{\lambda\in\triangle} A^*_{qn_\lambda})|_Y}, \nu^*_{(\cup_{\lambda\in\triangle} A^*_{qn_\lambda})|_Y}(x)); x \in X\} \\
&= \{(x, \mu^*_{(\cup_{\lambda\in\triangle} A^*_{qn_\lambda})|_Y}, \pi^*_{(\cup_{\lambda\in\triangle} A^*_{qn_\lambda})|_Y}, \nu^*_{(\cup_{\lambda\in\triangle} A^*_{qn_\lambda})|_Y}(x)); x \in Y\} \\
&\bigcup\{(x, \mu^*_{(\cup_{\lambda\in\triangle} A^*_{qn_\lambda})|_Y}, \pi^*_{(\cup_{\lambda\in\triangle} A^*_{qn_\lambda})|_Y}, \nu^*_{(\cup_{\lambda\in\triangle} A^*_{qn_\lambda})|_Y}(x)); x \in X\backslash Y\} \\
&= \{(x, \mu^*_{\cup_{\lambda\in\triangle} A^*_{qn_\lambda}}(x), \pi^*_{\cup_{\lambda\in\triangle} A^*_{qn_\lambda}}(x), \nu^*_{\cup_{\lambda\in\triangle} A^*_{qn_\lambda}}(x)) : x \in Y\} \\
&\bigcup\{(x, 0, 0, 1) : x \in X\backslash Y\} \\
&= \{(x, \vee_{\lambda\in\triangle}\mu^*_{A^*_{qn}\lambda}(x), \vee_{\lambda\in\triangle}\pi^*_{A^*_{qn}\lambda}(x), \wedge_{\lambda\in\triangle}\nu^*_{A^*_{qn}\lambda}(x); x \in Y\} \\
&= \{(x, \vee_{\lambda\in\triangle}\mu^*_{A^*_{qn}\lambda|Y}(x), \vee_{\lambda\in\triangle}\pi^*_{A^*_{qn}\lambda|Y}(x), \wedge_{\lambda\in\triangle}\nu^*_{A^*_{qn}\lambda|Y}(x); x \in Y\} \\
&\bigcup_{\lambda\in\triangle}[\{(x, \mu^*_{A^*_{qn}\lambda|Y}(x), \pi^*_{A^*_{qn}\lambda|Y}(x), \nu^*_{A^*_{qn}\lambda|Y}(x); x \in Y\} \\
&\bigcup\{(x, 0, 0, 1) : x \in X\backslash Y\}] \\
&= \bigcup_{\lambda\in\triangle}(A^*_{qn_{\lambda|Y}})
\end{aligned}
$$

(ii)

$$
\begin{aligned}
(\bigcap_{\lambda\in\triangle} A^*_{qn_\lambda})|_Y &= \{(x, \mu^*_{(\cap_{\lambda\in\triangle} A^*_{qn_\lambda})|_Y}, \pi^*_{(\cap_{\lambda\in\triangle} A^*_{qn_\lambda})|_Y}, \nu^*_{(\cap_{\lambda\in\triangle} A^*_{qn_\lambda})|_Y}(x)); x \in X\} \\
&= \{(x, \mu^*_{(\cap_{\lambda\in\triangle} A^*_{qn_\lambda})|_Y}, \pi^*_{(\cap_{\lambda\in\triangle} A^*_{qn_\lambda})|_Y}, \nu^*_{(\cap_{\lambda\in\triangle} A^*_{qn_\lambda})|_Y}(x)); x \in Y\} \\
&\bigcup\{(x, \mu^*_{(\cap_{\lambda\in\triangle} A^*_{qn_\lambda})|_Y}, \pi^*_{(\cap_{\lambda\in\triangle} A^*_{qn_\lambda})|_Y}, \nu^*_{(\cap_{\lambda\in\triangle} A^*_{qn_\lambda})|_Y}(x)); x \in X\backslash Y\} \\
&= \{(x, \mu^*_{\cap_{\lambda\in\triangle} A^*_{qn_\lambda}}(x), \pi^*_{\cap_{\lambda\in\triangle} A^*_{qn_\lambda}}(x), \nu^*_{\cap_{\lambda\in\triangle} A^*_{\lambda}}(x)) : x \in Y\} \\
&\bigcup\{(x, 0, 0, 1) : x \in X\backslash Y\} \\
&= \{(x, \wedge_{\lambda\in\triangle}\mu^*_{A^*_{qn}\lambda}(x), \wedge_{\lambda\in\triangle}\pi^*_{A^*_{qn}\lambda}(x), \vee_{\lambda\in\triangle}\nu^*_{A^*_{qn}\lambda}(x); x \in Y\} \\
&= \{(x, \wedge_{\lambda\in\triangle}\mu^*_{A^*_{qn}\lambda|Y}(x), \wedge_{\lambda\in\triangle}\pi^*_{A^*_{qn}\lambda|Y}(x), \vee_{\lambda\in\triangle}\nu^*_{A^*_{qn}\lambda|Y}(x); x \in Y\} \\
&\bigcap_{\lambda\in\triangle}[\{(x, \mu^*_{A^*_{qn}\lambda|Y}(x), \pi^*_{A^*_{qn}\lambda|Y}(x), \nu^*_{A^*_{qn}\lambda|Y}(x); x \in Y\} \\
&\bigcup\{(x, 0, 0, 1) : x \in X\backslash Y\}] \\
&= \bigcap_{\lambda\in\triangle}(A^*_{qn_{\lambda|Y}})
\end{aligned}
$$

(iii)

$$\begin{aligned}
A^{*c}_{qn}|_Y &= \{(x, \mu^*_{A^{*c}_{qn}|Y}, \pi^*_{A^{*c}_{qn}|Y}, \nu^*_{A^{*c}_{qn}|Y}) : x \in X\} \\
&= \{(x, \mu^*_{A^{*c}_{qn}}, \pi^*_{A^{*c}_{qn}}, \nu^*_{A^{*c}_{qn}}) : x \in Y\} \bigcup \{(x, 0, 0, 1) : x \in X \backslash Y\} \\
&= \{(x, \mu^*_{A^{*c}_{qn}}, \pi^*_{A^{*c}_{qn}}, \nu^*_{A^{*c}_{qn}}) : x \in Y\} \\
&= \{(x, \mu^*_{A^*_{qn}}, \pi^*_{A^*_{qn}}, \nu^*_{A^*_{qn}}) : x \in Y\}^c \\
&= \{(x, \mu^*_{A^*_{qn}|_Y}, \pi^*_{A^*_{qn}|_Y}, \nu^*_{A^*_{qn}|_Y}) : x \in Y\}^c \\
&= (\{(x, \mu^*_{A^*_{qn}|_Y}, \pi^*_{A^*_{qn}|_Y}, \nu^*_{A^*_{qn}|_Y}) : x \in Y\} \bigcup \{(x, 0, 0, 1) : x \in Y\})^c \\
&= \{(x, \mu^*_{(A^*_{qn}|_Y)^c}, \pi^*_{(A^*_{qn}|_Y)^c}, \nu^*_{(A^*_{qn}|_Y)^c}) : x \in X\} \\
&= (A^*_{qn}|_Y)^c
\end{aligned}$$

(iv)

$$\begin{aligned}
(A^*_{qn}|_Y)|_Z &= \{(x, \mu^*_{(A^*_{qn}|_Y)|Z}, \pi^*_{(A^*_{qn}|_Y)|Z}, \nu^*_{(A^*_{qn}|_Y)|Z}) : x \in X\} \\
&= \{(x, \mu^*_{A^*_{qn}|_Y}(x), \pi^*_{A^*_{qn}|_Y}(x), \nu^*_{A^*_{qn}|_Y}(x)) : x \in Z\} \bigcup \{(x, 0, 0, 1) : x \notin Z\} \\
&= \{(x, \mu^*_{A^*_{qn}}(x), \pi^*_{A^*_{qn}}(x), \nu^*_{A^*_{qn}}(x)) : x \in Y \bigcap Z\} \bigcup \{(x, 0, 0, 1) : x \notin Y \bigcap Z\} \\
&= \{(x, \mu^*_{A^*_{qn}}(x), \pi^*_{A^*_{qn}}(x), \nu^*_{A^*_{qn}}(x)) : x \in Z\} \bigcup \{(x, 0, 0, 1) : x \notin Z\} \\
&= \{(x, \mu^*_{A^*_{qn}|Z}(x), \pi^*_{A^*_{qn}|Z}(x), \nu^*_{A^*_{qn}|Z}(x)) : x \in Z\} \bigcup \{(x, 0, 0, 1) : x \notin Z\} \\
&= \{(x, \mu^*_{A^*_{qn}|Z}(x), \pi^*_{A^*_{qn}|Z}(x), \nu^*_{A^*_{qn}|Z}(x)) : x \in X\} \\
&= (A^*_{qn}|_Z)
\end{aligned}$$

□

Proposition 3.2. *Let Y, Z be two non-empty subset of X and let $A^*_{qn} \in N^*_{qn}(X)$. Then $A^*_{qn} |_{(Y \bigcap Z)} = (A^*_{qn} |_Y) \bigcap (A^*_{qn} |_Z)$.*

Proof.

$$\begin{aligned}
A^*_{qn} |_{(Y \bigcap Z)} &= \{(x, \mu^*_{A^*_{qn}|(Y \bigcap Z)}(x), \pi^*_{A^*_{qn}|(Y \bigcap Z)}(x), \nu^*_{A^*_{qn}|(Y \bigcap Z)}(x)) : x \in X\} \\
&= \{(x, \mu^*_{A^*_{qn}}(x), \pi^*_{A^*_{qn}}(x), \nu^*_{A^*_{qn}}(x)) : x \in Y \bigcap Z\} \bigcup \{(x, 0, 0, 1) : x \notin Y \bigcap Z\} \\
&= \{(x, \mu^*_{A^*_{qn}}(x), \pi^*_{A^*_{qn}}(x), \nu^*_{A^*_{qn}}(x)) : x \in Y \bigcap Z\} \\
&= \{(x, \mu^*_{A^*_{qn}}(x), \pi^*_{A^*_{qn}}(x), \nu^*_{A^*_{qn}}(x)) : x \in Y\} \\
&\bigcap \{(x, \mu^*_{A^*_{qn}}(x), \pi^*_{A^*_{qn}}(x), \nu^*_{A^*_{qn}}(x)) : x \in Z\} \\
&= [\{(x, \mu^*_{A^*_{qn}|Y}(x), \pi^*_{A^*_{qn}|Y}(x), \nu^*_{A^*_{qn}|Y}(x)) : x \in Y\} \bigcup \{(x, 0, 0, 1) : x \notin Y\} \\
&\bigcap \{(x, \mu^*_{A^*_{qn}|Z}(x), \pi^*_{A^*_{qn}|Z}(x), \nu^*_{A^*_{qn}|Z}(x)) : x \in Z\} \bigcup \{(x, 0, 0, 1) : x \notin Z\}] \\
&= \{(x, \mu^*_{A^*_{qn}|Y}(x), \pi^*_{A^*_{qn}|Y}(x), \nu^*_{A^*_{qn}|Y}(x)) : x \in X\} \\
&\bigcap \{(x, \mu^*_{A^*_{qn}|Z}(x), \pi^*_{A^*_{qn}|Z}(x), \nu^*_{A^*_{qn}|Z}(x)) : x \in X\}\} \\
&= (A^*_{qn} |_Y) \bigcap (A^*_{qn} |_Z)
\end{aligned}$$

□

Proposition 3.3. *Let (X, τ^*_{qn}) be an q^*-RONTS. Let $\emptyset \neq Y \subseteq X$ and $\tau^*_{qn}|_Y = \{G^*_{qn}|_Y : G^*_{qn} \in \tau^*_{qn}\}$. Then $(Y, \tau^*_{qn}|_Y)$ is a q^*-RONTS.*

Proof. (i) $X^*_{qn}, \emptyset^*_{qn} \in \tau^*_{qn} \Longrightarrow X^*_{qn}|_Y, \emptyset^*_{qn}|_Y \in \tau^*_{qn}|_Y$. As $Y^*_{qn} = X^*_{qn}|_Y$ and $\emptyset^*_{qn} = \emptyset^*_{qn}|_Y$, so $Y^*_{qn}, \emptyset^*_{qn} \in \tau^*_{qn}|_Y$.

(ii) Let $\{G^*_{qn_i} : i \in \triangle\} \subseteq \tau^*_{qn}|_Y$ then for each $i \in \triangle, G^*_{qn_i} = G^{*'}_{qn_i}|_Y$ for some $G^{*'}_{qn_i} \in \tau^*_{qn}$. Now $\bigcup_{i\in\triangle} G^*_{qn_i} = \bigcup_{i\in\triangle}(G^{*'}_{qn}|_Y) = (\bigcup_{i\in\triangle} G^{*'}_{qn})|_Y \in \tau^*_{qn}|_Y$. Therefore $\bigcup_{i\in\triangle} G^{*'}_{qn} \in \tau^*_{qn}$.(By 3.1$(i)$)

(iii) Let $G^*_{qn}, H^*_{qn} \in \tau^*_{qn}|_Y$. Then $G^*_{qn} = G^{*'}_{qn}|_Y$ and $H^*_{qn} = H^{*'}_{qn}|_Y$ for some $G^{*'}_{qn}, H^{*'}_{qn} \in \tau^*_{qn}$. Now $G^*_{qn} \bigcap H^*_{qn} = (G^{*'}_{qn}|_Y) \bigcap H^{*'}_{qn}|_Y = (G^{'*}_{qn} \bigcap H^{*'}_{qn})|_Y \in \tau^*_{qn}|_Y$. Therefore $G^{*'}_{qn} \bigcap H^{*'}_{qn} \in \tau^*_{qn}$ by (3.1(ii)).

□

Definition 3.4. *Let (X, τ^*_{qn}) be a q^*-RONTS. Let $\emptyset \neq Y \subseteq X$ and $\tau^*_{qn}|_Y = \{G^*_{qn}|_Y : G^*_{qn} \in \tau^*_{qn}\}$. Then $(Y, \tau^*_{qn}|_Y)$ is a q^*-RONTS. The topology $\tau^*_{qn}|_Y$ is called q^*-RON relative topology of τ^*_{qn} on Y or the q^*-RON subspace topology of Y and the q^*-RONTS $(Y, \tau^*_{qn}|_Y)$ is called q^*-RON subspace of the q^*-RONTS (X, τ^*_{qn}). Members of $\tau^*_{qn}|_Y$ are called $\tau^*_{qn}|_Y$-open sets in Y. An q^*-RON set $A^*_{qn} \in N^*_{qn}(Y)$ such that $A^{*c}_{qn} \in \tau^*_{qn}|_Y$ is called a $\tau^*_{qn}|_Y$-closed set in Y. $(Y, \tau^*_{qn}|_Y)$ is called a q^*-RON open subspace or q^*-RON closed subspace of (X, τ^*_{qn}) according as $Y^*_{qn} \in \tau^*_{qn}$ or $Y^*_{qn} \in \tau^{*c}_{qn}$.*

Example 3.1. *Let $X = \{x_1, x_2\}$, and $\tau^*_{qn} = \{\emptyset^*_{qn}, X^*_{qn}, A^*_{qn}, B^*_{qn}, A^*_{qn} \cup B^*_{qn}, A^*_{qn} \cap B^*_{qn}\}$, where $q \geq 1$, $A^*_{qn} = \{(x_1, 0.4, 0.6, 0.4), (x_2, 0.4, 0.5, 0.4)\}$ and $B^*_{qn} = \{(x_1, 0.3, 0.4, 0.5), (x_2, 0.4, 0.5, 0.5)\}$. Clearly (X, τ^*_{qn}) is a q^*-RONTS. Let $Y = \{x_1\}$. Then $X^*_{qn}|_Y = \{(x_1, 1, 1, 0), (x_2, 0, 0, 1)\} = Y^*_{qn}$, $\emptyset^*_{qn}|_Y = \{(x_1, 0, 0, 1), (x_2, 0, 0, 1)\} = \emptyset^*_{qn}$, $A^*_{qn}|_Y = \{(x_1, 0.4, 0.6, 0.4), (x_2, 0, 0, 1)\}$, $B^*_{qn}|_Y = \{(x_1, 0.3, 0.4, 0.5), (x_2, 0, 0, 1)\}$, $A^*_{qn} \cup B^*_{qn}|_Y = \{(x_1, 0.4, 0.6, 0.4), (x_2, 0, 0, 1)\}$,$A^*_{qn} \cap B^*_{qn}|_Y = \{(x_1, 0.3, 0.4, 0.5), (x_2, 0, 0, 1)\}$. Clearly $\tau^*_{qn}|_Y = \{\emptyset^*_{qn}, Y^*_{qn}, A^*_{qn}|_Y, B^*_{qn}|_Y, (A^*_{qn} \cup B^*_{qn})|_Y, (A^*_{qn} \cap B^*_{qn})|_Y\}$ is a q^*-RON subspace of Y, that is $(Y, \tau^*_{qn}|_Y)$ is a q^*-RON subspace of (X, τ^*_{qn}).*

Proposition 3.4. *Let (Y, σ^*_{qn}) be a subspace of an q^*-RONTS (X, τ^*_{qn}) and (Z, ς^*_{qn}) is a subspace of (X, τ^*_{qn}).*

Proof. Since $Z \subseteq Y \subseteq X$, so $Z \subseteq X$. Therefore to prove taht $\tau^*_{qn}|_Z = \varsigma^*_{qn}$. Let $G^*_{qn} \in \varsigma^*_{qn}$. Since (Z, ς^*_{qn}) is a subspace of (Y, σ^*_{qn}), so there exists $H^*_{qn} \in \sigma^*_{qn}$ such that $G^*_{qn} = H^*_{qn}|_Z$. Again (Y, σ^*_{qn}) is a subspace of (X, τ^*_{qn}), so there exists $K^*_{qn} \in \tau^*_{qn}$ such taht $H^*_{qn} = K^*_{qn}|_Y$. Then $G^*_{qn} = H^*_{qn}|_Z = (K^*_{qn}|_Y)|_Z = K^*_{qn}|_Z$ (by(3.1(iv)). Since $K^*_{qn}|_Z \in \tau^*_{qn}|_Z$, so $G^*_{qn} \in \tau^*_{qn}|_Z$. Therfore $\varsigma^*_{qn} \subseteq \tau^*_{qn}|_Z$. Next suppose that $U^*_{qn} \in \tau^*_{qn}|_Z$. Then there exists $V^*_{qn} \in \tau^*_{qn}$ such taht $U^*_{qn} \in V^*_{qn}|_Z$. Since (Y, σ^*_{qn}) is a subspace of (X, τ^*_{qn}), so $V^*_{qn}|_Y \in \sigma^*_{qn}$. Again since (Z, ς^*_{qn}) is a subspace of (Y, σ^*_{qn}) , so $(V^*_{qn}|_Y)|_Z \in \varsigma^*_{qn} \Rightarrow V^*_{qn}|_Z \in \varsigma^*_{qn} \Rightarrow U^*_{qn} \in \varsigma^*_{qn}$. Therefore $\tau^*_{qn}|_Z = \varsigma^*_{qn}$. Hence (Z, ς^*_{qn}) is a subspace of (X, τ^*_{qn}). □

Proposition 3.5. *Let Y and Z be two subspaces of a q^*-RONTS (X, τ^*_{qn}). If $Y \subseteq Z$ then Y is a subspace of Z.*

Proof. Let (Y, σ^*_{qn}) and (Z, ς^*_{qn}) be the subspace of the q*-RONTS (X, τ^*_{qn}) . Then $\tau^*_{qn}|_Y = \sigma^*_{qn}$ and $\tau^*_{qn}|_Z = \varsigma^*_{qn}$. Now $\varsigma^*_{qn}|_Y = \{A^*_{qn}|_Y : A^*_{qn} \in \varsigma^*_{qn}\} = \{(B^*_{qn}|_Z)|_Y : B^*_{qn} \in \tau^*_{qn}$ and $B^*_{qn}|_Z = A^*_{qn} \in \varsigma^*_{qn}\} = \tau^*_{qn}|_Y = \sigma^*_{qn}$. Since $\varsigma^*_{qn}|_Y = \sigma^*_{qn}$, so Y is a subspace of Z. □

Proposition 3.6. *Let $(Y, \tau^*_{qn}|_Y)$ be a subspace of a q^*-RONTS (X, τ^*_{qn}) and $A^*_{qn} \in N^*_{qn}(Y)$. Then A^*_{qn} is $\tau^*_{qn}|_Y$-closed iff $A^*_{qn} = F^*_{qn}|_Y$ for some τ^*_{qn}-closed set F^*_{qn} in X.*

Proof. A^*_{qn} is $\tau^*_{qn}\mid_Y$-closed in $Y \iff A^{*c}_{qn}$ is $\tau^*_{qn}\mid_Y$-open in $Y \iff A^{*c}_{qn} = G^*_{qn}\mid_Y$ for some $G^*_{qn} \in \tau^*_{qn} \iff A^{*c}_{qn} = (G^*_{qn}\mid_Y)^c \iff A^{*c}_{qn} = G^{*c}_{qn}\mid_Y$ by (3.1(*iii*)) $\iff A^{*c}_{qn} = F^*_{qn}\mid_Y$, where $F^*_{qn} = G^{*c}_{qn}$ is a τ^*_{qn}-closed set in X. □

Remark 3.2. *From Proposition 3.6, it is easy to conclude that if* $(Y, \tau^*_{qn}\mid_Y)$ *is a subspace of a* q^**-RONTS* (X, τ^*_{qn}) *then* $(\tau^*_{qn}\mid_Y)^c = \tau^{*c}_{qn}\mid_Y$.

Proposition 3.7. *Let* $(Y, \tau^*_{qn}\mid_Y)$ *be a subspace of a* q^**-RONTS* (X, τ^*_{qn}) *and let* B^*_{qn} *be a base for* τ^*_{qn}*. Then* $B^*_{qn}\mid_Y = \{V^*_{qn}\mid_Y : V^*_{qn} \in B^*_{qn}\}$ *is a base for* $\tau^*_{qn}\mid_Y$.

Proof. Let H^*_{qn} be a $\tau^*_{qn}\mid_Y$-open set in Y. Also let $x_{\mu^*,\pi^*,\nu^*} \in H^*_{qn}$ be an arbitrary q*-RON point. Then there exists a τ^*_{qn}-open set G^*_{qn} such taht $H^*_{qn} = G^*_{qn}\mid_Y$. Since B^*_{qn} is a base for τ^*_{qn}, so there exists a member $V^*_{qn}\mid_Y$ of $B^*_{qn}\mid_Y$ such that $x_{\mu^*,\pi^*,\nu^*} \in H^*_{qn} \in V^*_{qn}\mid_Y \subseteq H^*_{qn}$. Therefore $H^*_{qn} = \bigcup\{V^*_{qn}\mid_Y : V^*_{qn}\mid_Y \in B^*_{qn}\mid_Y$ and $V^*_{qn}\mid_Y \subseteq H^*_{qn}\}$. Hence $B^*_{qn}\mid_Y$ is a base for $\tau^{*c}_{qn}\mid_Y$ □

4. q*-rung orthopair neutrosophic nodec space

This section q*-RON nodec space are introduced and the properties are discussed

Definition 4.1. *In a* q^**-RONTS* (X, τ^*_{qn})*, a* q^**-RON set* A^*_{qn} *is called* qq^**-RON nowhere dense if the interior of its closure is empty. That is* $int_{qn}(cl_{qn}(A^*_{qn})) = \emptyset_{qn}$*. This means that there exists no non-zero* q^**-RON open set* B^*_{qn} *in* (X, τ^*_{qn}) *such that* $B^*_{qn} \subseteq cl_{qn}(A^*_{qn})$*. The collection of all* q^**-RON nowhere dense sets in* (X, τ^*_{qn}) *is denoted by* $N^*_{qn}(T_{qn})$.

Definition 4.2. *Let* (X, τ^*_{qn}) *be the* q^**-RONTS and* (X, τ^*_{qn}) *is said to be* q^**-RON door space, if every* q^**-RON subset of* X *is either* q^**-RON open or* q^**-RON closed in* (X, τ^*_{qn}).

Example 4.1. *Consider the Example 2.3 and define a* q^**-RON sets* A^*_{qn} *and* B^*_{qn} *as follows:* $A^*_{qn} = \{(a_{qn}, 0.31, 0.31, 0.4), (b_{qn}, 0.31, 0.31, 0.4), (c_{qn}, 0.71, 0.71, 0.21)\}$,
$B^*_{qn} = \{(a_{qn}, 0.31, 0.41, 0.41), (b_{qn}, 0.31, 0.41, 0.41), (c_{qn}, 0.71, 0.51, 0.22)\}$.
Then $int_{qn}(cl_{qn}(A^*_{qn_1})) = A^*_{qn_1}$*,* $int_{qn}(cl_{qn}(A^*_{qn_2})) = A^*_{qn_2}$*,* $int_{qn}(cl_{qn}(A^*_{qn})) = \emptyset_{qn}$*, and* $int_{qn}(cl_{qn}(B^*_{qn})) = \emptyset_{qn}$*. Therefore* $A^*_{qn_1}$ *and* $A^*_{qn_2}$ *are not* q^**-RON nowhere dense sets.* A^*_{qn} *and* B^*_{qn} *are* q^**-RON nowhere dense sets.*

Definition 4.3. *A* q^**-RONTS* (X, τ^*_{qn}) *is termed a* q^**-RON submaximal space if, for every* q^**-RONS* A^*_{qn} *in* (X, τ^*_{qn}) *such that* $cl_{qn}(A^*_{qn}) = 1$*, it follows that* $A^*_{qn} \in \tau^*_{qn}$*. That is* q^**-RON submaximal space if each* q^**-RON dense set in* (X, τ^*_{qn}) *is a* q^**-RONTS open set in* (X, τ^*_{qn}).

Definition 4.4. *Let* (X, τ^*_{qn}) *be* q^**-RONTS. The space* (X, τ^*_{qn}) *is called* q^**-RON nodec space if every* q^**-RON nowhere dense set is closed.*

Example 4.2. *Let* $X = \{a_{qn}, b_{qn}, c_{qn}\}$ *for all* $k \in \{1, 2, 3\}$ *,* A_{qn_k} *be a q-RONS:*
$A_{qn_1} = \{(a_{qn}, 0.7, 0.5, 0.1), (b_{qn}, 0.5, 0.5, 0.3), (c_{qn}, 0.4, 0.6, 0.4)\}$,
$A_{qn_2} = \{(a_{qn}, 0.6, 0.5, 0.7), (b_{qn}, 0.3, 0.4, 0.5), (c_{qn}, 0.3, 0.2, 0.8)\}$
$A_{qn_3} = \{(a_{qn}, 0.7, 0.5, 0.6), (b_{qn}, 0.5, 0.5, 0.4), (c_{qn}, 0.4, 0.2, 0.4)\}$,
where for all $x \in X, 0 \le \mu^q_{A_{qn_k}}(x_{qn}) + \nu^q_{A_{qn_k}}(x_{qn}) \le 1$, $q \ge 1$ *and*
$0 \le \mu^q_{A_{qn_k}}(x_{qn}) + \pi^q_{A_{qn_k}}(x_{qn}) + \nu^q_{A_{qn_k}}(x_{qn}) \le 2$.
Then $A^*_{qn_1} = \{(a_{qn}, 0.5, 0.3, 0.1), (b_{qn}, 0.5, 0.5, 0.3), (c_{qn}, 0.4, 0.6, 0.4)\}$,
$A^*_{qn_2} = \{(a_{qn}, 0.3, 0.3, 0.4), (b_{qn}, 0.3, 0.4, 0.5), (c_{qn}, 0.2, 0.2, 0.7)\}$
$A^*_{qn_3} = \{(a_{qn}, 0.4, 0.3, 0.3), (b_{qn}, 0.5, 0.5, 0.4), (c_{qn}, 0.4, 0.2, 0.4)\}$,
Clearly $\tau^*_{qn} = \{\emptyset^*_{qn}, X^*_{qn}, A^*_{qn_1}, A^*_{qn_2}, A^*_{qn_3}\}$ *is a* q^**-RONT. Now, define a* q^**-RON set* A^*_{qn}

as follows: $A^*_{qn} = \{(a_{qn}, 0.305, 0.4, 0.091), (b_{qn}, 0.202, 0.6, 0.20), (c_{qn}, 0.101, 0.701, 0.302)\}$.
Then $cl(A^*_{qn}) = \{(a_{qn}, 0.305, 0.4, 0.091), (b_{qn}, 0.202, 0.6, 0.20), (c_{qn}, 0.101, 0.701, 0.302)\}$ *and* $int_{qn}(cl_{qn}(A^*_{qn})) = \emptyset_{qn}$. *Therefore,* A^*_{qn} *is* q^**-RON nowhere dense set.*
Since $cl_{qn}(A^*_{qn}) = A^*_{qn}$, *the nowhere dense set* A^*_{qn} *is closed.*
Hence, the space (X_{qn}, τ^*_{qn}) *is called the* q^**-RON nodec space.*

Theorem 4.1. *For a* q^**-RONTS* (X, τ^*_{qn}), *the following conditions are equivalent:*

(i) (X, τ^*_{qn}) *is a* q^**-RON nodec space.*
(ii) *Every* q^**-RON nowhere dense subset of* (X, τ^*_{qn}) *is both* q^**-RON closed and* q^**-RON discrete.*
(iii) *Every* q^**-RON subset of* (X, τ^*_{qn}) *containing a* q^**-RON dense open set is* q^**-RON open.*

Proof. $(i) \Longrightarrow (ii)$ Assume that (X, τ^*_{qn}) is a q*-RON nodec space. Let A^*_{qn} be a q*-RON nowhere dense subset of (X, τ^*_{qn}). By definition 4.1 $int_{qn}(cl_{qn}(A^*_{qn})) = \emptyset_{qn}$. Hence, A^*_{qn} is q*-RON closed. $(ii) \Longrightarrow (iii)$ Now, Assume Every q*-RON nowhere dense subset of (X, τ^*_{qn}) is both q*-RON closed and q*-RON discrete. Let B^*_{qn} be a q*-RON subset of (X, τ^*_{qn}) containing a q*-RON dense open set O^*_{qn}. Since $O^*_{qn} = int_{qn}(O^*_{qn}) \subseteq int_{qn}(B^*_{qn})$. Therfore, B^*_{qn} must itself be open,because any set containing a dense subset cannot be a closed, nowhere dense set. Thus B is q*-RON open. $(iii) \Longrightarrow (i)$ Assume that every q*-RON subset of (X, τ^*_{qn}) containing a q*-RON dense open set is q*-RON open. Let C^*_{qn} be a q*-RON nowhere dense subset of (X, τ^*_{qn}). Then, $int_{qn}(cl_{qn}(C^*_{qn})) = \emptyset$ implies $(X, \tau^*_{qn}) - int_{qn}(cl_{qn}(C^*_{qn})) = (X, \tau^*_{qn})$ implies $cl_{qn}[(X, \tau^*_{qn}) - cl_{qn}(C^*_{qn})] = (X_{qn}, \tau^*_{qn})$ implies $cl_{qn}[int_{qn}((X_{qn}, \tau^*_{qn}) - B^*_{qn})] = (X, \tau^*_{qn})$. This means that $int_{qn}((X, \tau^*_{qn}) - B^*_{qn})$ is a q*-RON dense open set, and $int_{qn}((X, \tau^*_{qn}) - C^*_{qn}) \subseteq (X, \tau^*_{qn}) - C^*_{qn}$. Hence ,$(X, \tau^*_{qn}) - C^*_{qn}$ is q*-RON open. Thus C^*_{qn} is q*-RON closed. Therefore (X, τ^*_{qn}) is q*-RON nodec space. □

Theorem 4.2. *For a* q^**-RONTS* (X, τ^*_{qn}), *the following conditions are equivalent:*

(i) (X, τ^*_{qn}) *is a* q^**-RON nodec space.*
(ii) $cl_{qn}(A^*_{qn}) = A^*_{qn} \bigcup cl_{qn}(int_{qn}(cl_{qn}(A^*_{qn})))$ *for each* $A^*_{qn} \subseteq (X, \tau^*_{qn})$.

Proof. $(i) \Longrightarrow (ii)$ Let (X, τ^*_{qn}) be a q*-RON nodec space and $A^*_{qn} \subseteq (X, \tau^*_{qn})$. Therefore, $cl_{qn}(A^*_{qn}) = A^*_{qn} \bigcup cl_{qn}(int_{qn}(cl_{qn}(A^*_{qn})))$. $(ii) \Longrightarrow (i)$ If $A^*_{qn} \subseteq (X, \tau^*_{qn})$ is q*-RON nowhere dense, then $cl_{qn}(A^*_{qn}) = A^*_{qn} \bigcup cl_{qn}(int_{qn}(cl_{qn}(A^*_{qn}))) = A^*_{qn} \bigcup \emptyset = A^*_{qn}$. Therefore, A^*_{qn} is closed. Thus (X, τ^*_{qn}) is a q*-RON nodec space. □

Theorem 4.3. *For a* q^**-RON topological space* (X, τ^*_{qn}), *the following conditions are equivalent:*

(i) (X, τ^*_{qn}) *is a* q^**-RON nodec space.*
(ii) *For every* $A^*_{qn} \subseteq (X, \tau^*_{qn})$, *if* $cl_{qn}(int_{qn}(cl_{qn}(A^*_{qn}))) \subseteq A^*_{qn}$, *then* B^*_{qn} *is a* q^**-RON closed.*
(iii) $int_{qn}(A^*_{qn}) = A^*_{qn} \bigcap int_{qn}(cl_{qn}(int_{qn}(A^*_{qn})))$ *for every* $A^*_{qn} \subseteq (X, \tau^*_{qn})$.

Proof. $(i) \Longrightarrow (ii)$ Let (X, τ^*_{qn}) be a q*-RON nodec space. In a q*-RON nodec space, no dense set exists that is not open. Now, consider a set A^*_{qn} such that $cl_{qn}(int_{qn}(cl_{qn}(A^*_{qn}))) \subseteq A^*_{qn}$ implies $cl_{qn}(int_{qn}(cl_{qn}(A^*_{qn}))) \subseteq A^*_{qn}$ implies A^*_{qn} is closed set . Thus, A^*_{qn} is q*-RON closed. $(ii) \Longrightarrow (iii)$ Assume that for every $A^*_{qn} \subseteq (X, \tau^*_{qn})$, if $cl_{qn}(int_{qn}(cl_{qn}(A^*_{qn}))) \subseteq A^*_{qn}$, then B^*_{qn} is a q*-RON closed. To prove $int_{qn}(A^*_{qn}) = A^*_{qn} \bigcap int_{qn}(cl_{qn}(int_{qn}(A^*_{qn})))$ for every $A^*_{qn} \subseteq (X, \tau^*_{qn})$. Let $A^*_{qn} \subseteq (X, \tau^*_{qn})$ and we know that $cl_{qn}(int_{qn}(A^*_{qn})) \subseteq int_{qn}(A^*_{qn})$ implies $int_{qn}(A^*_{qn}) = A^*_{qn} \bigcap int_{qn}(cl_{qn}(int_{qn}(A^*_{qn})))$. $(iii) \Longrightarrow (i)$ Let $A^*_{qn} \subseteq (X, \tau^*_{qn})$ be a q*-RON nowhere dense set. Since $int_{qn}(A^*_{qn}) = A^*_{qn} \bigcap int_{qn}(cl_{qn}(int_{qn}(A^*_{qn})))$, then $int_{qn}((A^*_{qn})^c) = (A^*_{qn})^c \bigcap int_{qn}(cl_{qn}(int_{qn}((A^*_{qn})^c)))$, and is equivalent to $cl_{qn}(A^*_{qn}) = A^*_{qn} \bigcup cl_{qn}(int_{qn}(cl_{qn}(A^*_{qn})))$. Hence $cl_{qn}(A^*_{qn}) = A^*_{qn} \bigcup cl_{qn}(int_{qn}(cl_{qn}(A^*_{qn}))) = A^*_{qn} \bigcup \emptyset_{qn}$ $= A^*_{qn}$. Therefore, A^*_{qn} is q*-RON closed. Thus (X, τ^*_{qn}) is a q*-RON nodec space. □

Definition 4.5. *A q*-RONTS (X, τ^*_{qn}) is called q*-RON submaximal if every q*-RON dense subset of (X, τ^*_{qn}) is q*-RON open.*

Theorem 4.4. *Every q*-RON submaximal space is q*-RON nodec space.*

Proof. Let (X, τ^*_{qn}) be a q*-RON submaximal space. Let $A^*_{qn} \in N^*_{qn}(T_{qn})$. If A_{qn} is q*-RON closed then q*-RON nodec space.
Suppose that A^*_{qn} is not q*-RON closed. Then, ${A_{qn}}^{*c}$ is not q*-RON open and, by q*-RON submaximality of (X, τ^*_{qn}), ${A_{qn}}^{*c}$ is not q*-RON dense in (X, τ^*_{qn}). Therefore $cl_{qn}(A^*_{qn})^c \neq X$. This implies that there exists $x_{qn} \in P^*_{qn}$ such that $x_{qn} < cl(A^{*c})$; moreover, there exists a q*-RON open set B^*_{qn} that contains x_{qn} and $B^*_{qn} \bigcap (A_{qn})^{*c} = \emptyset$. Hence $B^*_{qn} \subseteq A^*_{qn}$, which means $int_{qn}(A^*_{qn}) \neq \emptyset$. This contradicts the assumption that $A^*_{qn} \in N^*_{qn}(T_{qn})$. Thus, A^*_{qn} must be q*-RON closed. Therefore, (X, τ^*_{qn}) is q*-RON nodec space. □

Remark 4.1. *Example* 4.3 *demonstrates that the converse of the Theorem* 4.4 *need not be true.*

Example 4.3. *Let $X = \{a_{qn}\}$ and $A_{qn} = \{a_{qn_1}, a_{qn_2}\}$. Consider the q*-RON indiscrete topology $\tau^*_{qn} = \{X_{qn}, \emptyset_{qn}\}$ on X. Clearly (X, τ^*_{qn}) is a q*-RON nodec space, but not q*-RON submaximal.*

Definition 4.6. *A q*-RONTS (X, τ^*_{qn}) is called q*-RON door if every q*-RON subset of (X, τ^*_{qn}) is either q*-RON open or q*-RON closed.*

Remark 4.2. *q*-RON door space $\Longrightarrow$ q*-RON submaximal space $\Longrightarrow$ q*-RON nodec space.*

Definition 4.7. *A q*-RONTS (X, τ^*_{qn}) is called strongly q*-RON nodec if each q*-RON nowhere dense set $A^*_{qn} \subseteq (X, \tau^*_{qn})$ is finite and q*-RON closed.*

Theorem 4.5. *Let $f_{qn} : (X, \tau^*_{qn}) \longrightarrow (Y, \tau^*_{qn})$ be a continuous function between two q*-RON nodec spaces. If $A_{qn} \subseteq X$ is q*-RON nowhere dense, then $f_{qn}(A^*_{qn})$ is q*-RON nowhere dense in Y.*

Proof. Let $f_{qn} : (X, \tau^*_{qn}) \longrightarrow (Y, \tau^*_{qn})$ be a continuous function between two q*-RON nodec spaces. Therefore $int_{qn}(cl_{qn}(A^*_{qn})) = \emptyset^*_{qn}$. Since f_{qn} is continuous , the image of the closure of A^*_{qn}. That is $f_{qn}(cl_{qn}(A^*_{qn})) \subseteq cl_{qn}(f_{qn}(A^*_{qn}))$. Thus, we have $int_{qn}(f_{qn}(cl_{qn}(A^*_{qn}))) \subseteq int_{qn}(cl_{qn}(f_{qn}(A^*_{qn})))$. Since A^*_{qn} is q*-RON nowhere dense in X. Therefore $int_{qn}(cl_{qn}(A^*_{qn})) = \emptyset \Longrightarrow int_{qn}(f_{qn}(cl_{qn}(A^*_{qn}))) = \emptyset^*_{qn} \Longrightarrow int_{qn}(cl_{qn}(f_{qn}(A^*_{qn})))$ $= \emptyset^*_{qn} \Longrightarrow f_{qn}(A^*_{qn})$ is q*-RON nowhere dense in Y. □

Theorem 4.6. *For a continuous function $f_{qn} : (X, \tau^*_{qn}) \longrightarrow (Y, \tau^*_{qn})$, if A^*_{qn} is a q*-RON dense set in Y, then the preimage $f^{-1}(A^*_{qn})$is also q∗-RON dense in X.*

Proof. Let $f_{qn} : (X, \tau^*_{qn}) \longrightarrow (Y, \tau^*_{qn})$ be a continuous function between two q*-RON nodec spaces. Since $A^*_{qn} \subseteq Y$ then $cl_{qn}(A^*_{qn}) = Y$. Since f_{qn} is continuous, for any subset $A^*_{qn} \subseteq Y$, therefore $f^{-1}_{qn}(cl_{qn}(A^*_{qn})) = cl_{qn}(f^{-1}_{qn}(A^*_{qn}))$implies$f^{-1}_{qn}(Y_{qn}) = cl_{qn}(f^{-1}_{qn}(A^*_{qn}))$ implies $X = cl_{qn}(f^{-1}_{qn}(A^*_{qn}))$. Therefore $f^{-1}_{qn}(A)$ also q∗-RON dense in X. □

Theorem 4.7. *If $f : (X, \tau^*_{qn}) \longrightarrow (Y, \tau^*_{qn})$ is a surjective continuous function and (X_{qn}, τ^*_{qn}) is a q^*-RON nodec space, then (Y, τ^*_{qn}) is also a q^*-RON nodec space.*

Proof. Let $f : (X, \tau^*_{qn}) \longrightarrow (Y, \tau^*_{qn})$ be a continuous and surjective function between two q*-RONTS, and assume that (X, τ^*_{qn}) is a q*-RON nodec space. Therfore X is both q*-RON closed and q*-RON discrete. Let $A^*_{qn} \subseteq Y$ be a q*-RON nowhere dense set in Y, therefore $int_{qn}(cl_{qn}(A^*_{qn})) = \emptyset^*_{qn}$. Since f_{qn} is surjective, for any set $A^*_{qn} \subseteq Y_{qn}$, therefore $f^{-1}_{qn}(A^*_{qn}) \subseteq X$. Since f_{qn} is continuous,therefore $f^{-1}_{qn}(A^*_{qn})$ is a q*-RON nowhere dense set in X. Since (X, τ^*_{qn}) is q*-RON nodec space, therefore every q*-RON nowhere dense set in X is q*-RON closed and q*-RON discrete. Thus $f^{-1}_{qn}(A^*_{qn})$ is both q*-RON closed and q*-RON discrete. Since f_{qn} is surjective , therefore $A^*_{qn} = f_{qn}(f^{-1}_{qn}(A^*_{qn}))$ implies A^*_{qn} is both q*-RON closed and q*-RON discrete in Y. Thus (Y, τ^*_{qn}) is a q*-RON nodec space. □

Theorem 4.8. *If A^*_{qn} is q^*-RON dense in Y, then $f^{-1}_{qn}(A^*_{qn})$ is q^*-RON dense in X.*

Proof. Let $A^*_{qn} \subseteq Y$ be a q*-RON dense set in Y, therefore $cl_{qn}(A^*_{qn}) = Y$. To prove that $cl_{qn}(f^{-1}_{qn}(A^*_{qn})) = X$. Since f_{qn} is continuous function, therefore $f^{-1}_{qn}(cl_{qn}(A^*_{qn}))$ is closed in X. Since A^*_{qn} is q*-RON dense in Y, therefore $f^{-1}(cl(A^*_{qn})) = f^{-1}(Y) = X$ implies $cl_{qn}(f^{-1}_{qn}(A^*_{qn})) = X$ implies $f^{-1}_{qn}(A^*_{qn})$ is q*-RON dense in X. □

5. Conclusions and Future Work

This paper extends standard topological structures to account for indeterminacy and uncertainty by introducing and exploring the notion of q*-rung orthopair neutrosophic subspaces and nodec spaces. Important features are established, such as the fact that any nowhere dense subset in such spaces is closed, which is consistent with but extends conventional topological conclusions via the parameter q. The work mostly concentrated on theoretical features and particular specifics of q*-RON spaces, even with its positive outcomes. There are several real-time applications of q*-RONT spaces in fields such as image processing, decision-making, and data analysis. In decision-making, these spaces can effectively handle vague or imprecise information, making them suitable for multi-criteria decision-making frameworks. They have the potential to address complex problems, such as evaluating medical risk conditions. Such applications represent a valuable direction for future research involving q*-RONT spaces.

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