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PROPOSED MATHEMATICAL EQUATION FOR EVALUATING THE RESIDUAL PUNCHING SHEAR CAPACITY OF HIGH-PERFORMANCE CONCRETE SLABS AFTER BEING SUBJECTED TO IMPACT LOADING

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Abstract. The research investigates the validity of high-performance concrete slabs under punching shear after they are exposed to impact load. The importance of this study lies in opening new avenues for researchers regarding slabs subjected to impact loading, specifically addressing whether they can be repaired and strengthened or are more likely to require reconstruction. The experimental program consists of eight slabs with different parameters tested under the same impact loading, and then the punching shear capacity is evaluated. One of these slabs was reinforced by traditional reinforcement, while the other specimens discussed different techniques designed to find alternative modified methods to resist punching shear. Finally, the suggested empirical formula predicts the value of punching shear for high-performance concrete slabs that suffer from impact load. The accuracy of the suggested equation reached 90.5% from experimental results.

Keywords: residual capacity, punching shear, impact load, high-performance concrete, empirical equation. .

AMS 2020 Subject Classification: 74C99

1. Introduction

Present-day constructions and protective structural engineering have been highly recommended to use high-performance concrete slabs due to mechanical strength, durability, and resistance to static and impact loads [1]. Experimental demonstrations have shown that the effect of impact load results in highly stressed waves, which cause dangerous deterioration in concrete and a loss of bond between the concrete and reinforcement [2, 3]. Logically, this leads to reducing the punching shear capacity [4]. Main parameters that affect the reduction of punching shear are impact force, slab thickness, reinforcement, and the type of concrete used [5, 6]. Previous analytical models developed for static or blast-induced punching shear have been found inadequate for predicting post-impact residual

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capacity, as they generally neglect the complex damage evolution and strain-rate effects caused by high-velocity loading [7]. Strengthening methods using fiber-reinforced polymers (FRP) or ultra-high-performance concrete (UHPC) overlays have shown partial recovery of shear strength but rarely restore it to pre-damage levels [8]. Therefore, results from the experimental approach are essential to investigate the residual punching shear capacity and to calibrate numerical models for post-impact performance assessment [9]. The residual punching shear capacity following such events is an essential factor for evaluating post-impact serviceability and structural safety; however, its behavior remains inadequately comprehended [10]. According to the existing literature, no analytical or empirical expression exists to predict the residual punching shear capacity of high-performance concrete slabs after exposure to impact loading, highlighting a significant gap in design and assessment methodologies. In this study, eight high-performance concrete (HPC) slabs were examined under impact loading conditions. Subsequently, the same specimens were tested under punching shear loads without any strengthening in order to investigate an important question: after being subjected to an impact load, do the slabs retain their capacity to withstand static loads, or do they completely lose their load-bearing ability? Several parameters were considered, including the influence of reinforcement ratio and slab thickness, which varied from 40 mm to 60 mm and 80 mm. In addition, the effect of steel fiber content was studied at three volumetric ratios: 0.51%, and 2%. Another proposed parameter was the introduction of slab camber at the mid-span, with values of 10 mm and 20 mm. Finally, a predictive equation was proposed to estimate the residual punching shear capacity of slabs after subjected to impact loading. This equation can be regarded as an initial step seed for future researchers to refine and develop into a comprehensive predictive model.

2. Material and method

One of the essential key factors that affect achieving high-performance concrete is the proper selection of ingredients and using steel fiber [11]. The used steel fibers were manufactured with a brass-coated surface to improve adhesion with the surrounding matrix and to enhance corrosion resistance. The tensile strength is greater than 2350 MPa. A straight shape of steel fiber was used with the average length of 13 mm and the diameter of 0.2 mm, which provided a satisfactory aspect ratio for resistance to crack propagation. The effect of steel fibers becomes significant starting from 1% to 2%; below these ratios, there is no considerable effect [12]. Therefore, three fiber volume fractions were considered: 0.5%, 1%, and 2%.

Eight slabs (800X800 mm) were considered in this research; one of these was a control, and it is reinforced by a 6 mm diameter steel bar. The rest of the seven slabs used high-performance concrete with steel fiber rather than traditional reinforcement. The water-cementitious (cement and silica fume) ratio was about 25%, the percent of silica fume equal to 15% of the cement amount, and the Glenium 54 used in 4% of the cementitious materials. Different parameters were taken into consideration: slab thickness, ratio of steel fiber, and using camber in the middle of slabs. The mechanical properties of the high-performance concrete were evaluated by testing the compressive strength of three cubes of each considered type. All details are clarified in Table 1.

Table 1. Slab details

Slab ID	Reinforcement	Steel fiber	Thickness of slab mm	Using camber mm	Average compressive strength MPa
SC	Φ6@100 mm both direction	/	40	/	92.11
S1	/	0.5%	40	/	88.42
S2	/	0.5%	60	/	90.35
S3	/	0.5%	80	/	87.31
S4	/	1.0 %	40	/	84.46
S5	/	2.0%	40	/	104.09
S6	/	0.5%	40	10	89.31
S7	/	0.5%	40	20	85.28

All slabs were tested under impact load produced from a dropped free steel ball from a distance of 1200 mm, then tested under concentrated load to evaluate the residual punching shear capacity as shown in Figure 1.

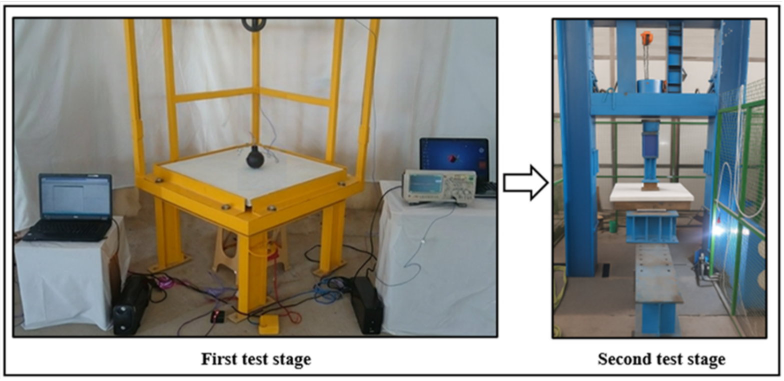

Figure 1.

It is important to point out that the acceleration of motion from the initial contact between the steel ball and the top surface of the slabs was recorded by a firmly attached accelerometer with the impactor. Immediately after impact, the velocity rapidly decreases to zero, resulting in a sharp deceleration. The oscilloscope trace clearly illustrated the full motion of the impactor, including the first strike, rebound, and subsequent minor movements on the slab. Although the steel ball and drop height are the same, the impact force is not identical for all slabs because each slab absorbs the impact energy in a different way. Stiffer slabs stop the impactor body in a shorter time, resulting in higher force. So, the variation in slab properties directly affects the magnitude of the impact load.

3. Results of experiments

3.1. **Impact test.** It can be observed that the response of a concrete member to impact loading exhibits different actions, which are generally classified as either global effects or local effects. Figure 2 demonstrates the influence of impact loading on concrete elements.

The resulting deformation modes—such as penetration, punching, spalling, scabbing, perforation, slab deflection, and the formation of radial cracks—are governed by several factors. These include the mechanical properties of both the impacting body and the target structure, as well as the kinetic energy associated with the impactor[13].

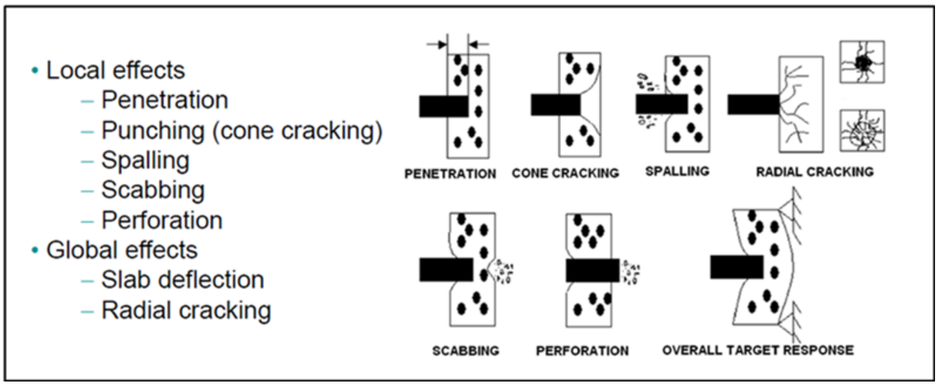

Figure 2.

The weight of the steel ball and the dropping height were adjusted to ensure that the impact produced only global effects, as verified through additional specimen testing. All specimens were subjected to the same kinetic energy prior to impact, calculated as 58.8 joules, while neglecting air resistance and assuming a gravitational acceleration of 9.8 m/s^2. The results of the impact loading tests are summarized in Table 2.

3.2. **Punching shear test.** The slabs were loaded progressively using a universal testing machine (see Figure 1, second test stage) that subjects a solid square steel cube measuring 15×15 cm to induce punching shear in the two-way slabs. In addition, an electronic dial gauge was positioned at the center of the slab's bottom surface to record deflections until failure. The load deflection curve for specimens is clarified in Figure 3.

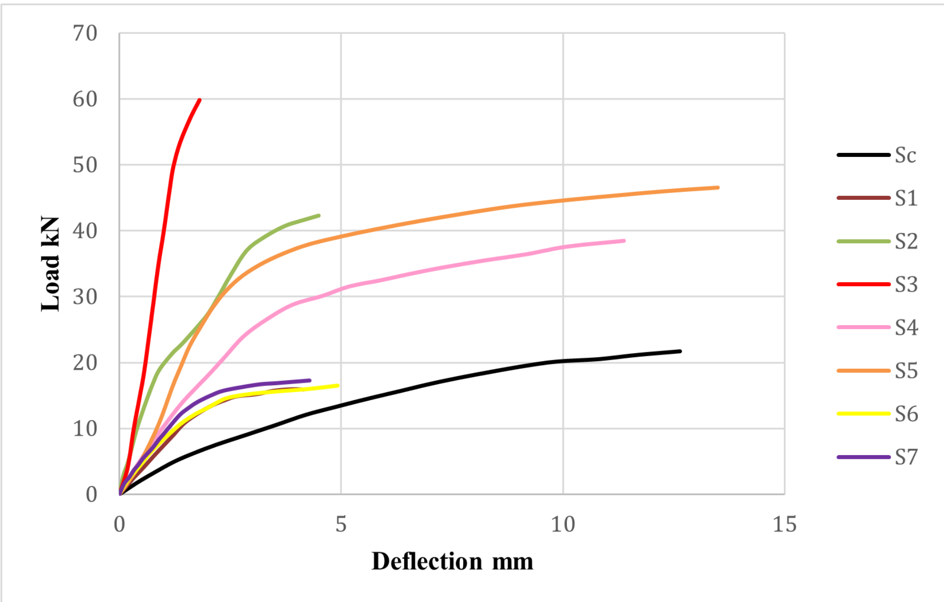

Figure 3. Load deflection curve for specimens

4. Discussion

Toughness is a measure of a material's ability to absorb energy and plastically deform prior to fracture. It is often defined as the amount of energy per unit volume that a material can sustain before failure occurs. A widely used method for evaluating toughness involves calculating the area under the load-deflection curve derived from experimental observations [14]. The product value of impact load with maximum displacement, which the slab reached, refers to the significant indicator for the dynamic response for the concrete slabs under impact load. This value represents the absorbed energy of the structural system during the vibration time, which can be called the work done by the impact load. Toughness and absorbed impact energy are computed in Table 3. This reflects the ability of the structural member to resist the sudden collapse and dispel a good part of the impact energy, so the relation between the maximum impact load and maximum displacement is considered an effective way to evaluate the toughness of slabs and efficiency of energy absorption. This is directly related to punching shear after being subjected to impact load. TABLE 3.

Table 3. Toughness and absorbed impact energy

Slab No.	Slab ID	Toughness kN.m	Absorbed Impact Energy kN.m
1	SC	0.0902	0.0303
2	S1	0.0233	0.0283
3	S2	0.0642	0.0225
4	S3	0.0300	0.0163
5	S4	0.1619	0.0214
6	S5	0.2510	0.0158
7	S6	0.0303	0.0242
8	S7	0.0275	0.0212

We are currently examining the relationship between punching shear strength and toughness, and the results indicate a strong correlation between these two parameters, with a correlation coefficient of 0.9997. Similarly, a strong relationship was observed between punching shear strength and the absorbed impact energy, with a correlation coefficient of 0.9999, as illustrated in Figure 4 and Figure 5. These findings provide a solid basis for proposing an empirical equation capable of predicting the shear strength of specimens after being subjected to impact loading.

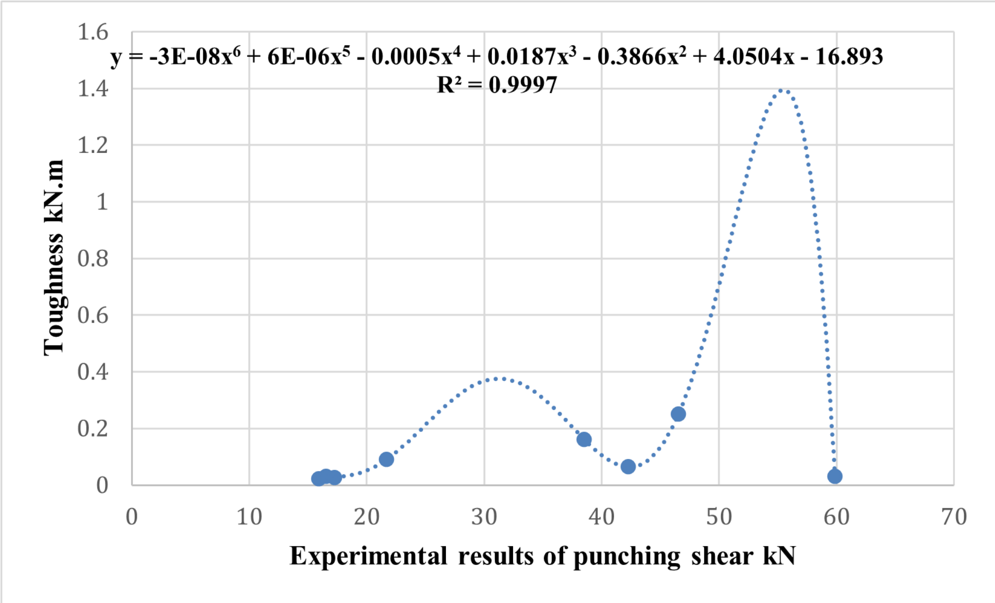

Figure 4. Correlation between experimental punching shear results and toughness

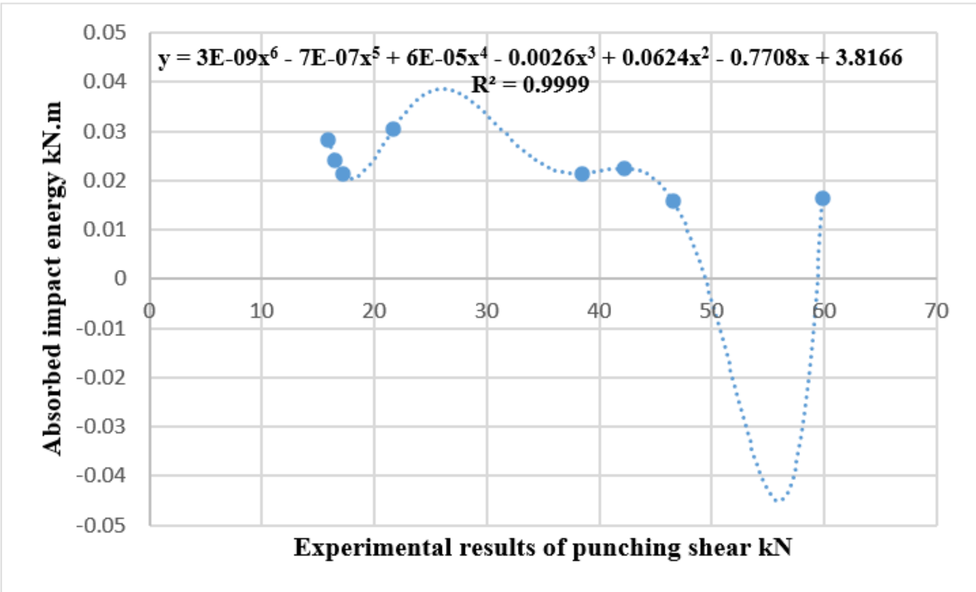

Figure 5. Correlation between experimental punching shear results and absorbed impact energy

5. Suggested empirical formula

Depending on the experimental results and statistical analysis, the following empirical closed form is suggested to predict the value of punching shear for the two-way high-performance concrete slabs suffering from impact load. Because there are not enough specimens to depend on to produce a formula from the first line-stage, it should depend on the previous one and then modify it. There are two main reasons why the starting form of the ACI code is not valid: The compressive strength's square root should not exceed 5.8 [15]. The second reason is that the code's formula does not consider the reduction effect of impact load. In fact, there is no available empirical formula that considers this effect. Ammash et al. (2012) suggested a formula to predict the punching shear capacity of normal- and high-strength concrete cite16. This formula was experimentally evaluated using 58 specimens tested by different researchers from various countries. Therefore, it can be considered a reasonable mathematical expression that may serve as a starting point for further modification to make it applicable for evaluating the punching shear capacity of high-performance concrete slabs subjected to impact loading. The original equation:

$Vc = \alpha \left(fc^{\diagdown}\right)^{\gamma} \beta_0 d,$

$\gamma = 0.5$ for compressive strength less than 80 MPa or equal to [0.71-0.0028 $fc^{\diagdown}$] for compressive strength equal to or greater than 80 MPa putting into consideration γ greater than 0.45,

$\alpha = 0.8 \left[\frac{(15+350\rho)\rho\alpha}{\beta_0 d} + 0.425\right],$

$a =$ diameter of circular column or longest side length for rectangular column,

$fc^{\diagdown} =$ concrete compressive strength,

$d =$ effective depth of critical zone of slabs,

$\rho =$ tension reinforcement ratio of slab at the critical section,

$\beta_0 =$ the critical section perimete,

The value of γ will be considered as it is because it takes the effect of increasing the value of concrete compressive strength, while the value of α will be modified according to the experimental results; the suggested value will be

$\varphi = IR \left[\alpha * \rho + \frac{sf*d}{7.5-sf}\right]$

Where

$IR =$ impact reduction factor and will be equal to [impact force * maximum deflection] in $kN.m$

$a =$ longest side length for rectangular column

$\rho =$ the tension reinforcement ratio of slab

$sf =$ steel fiber ratio used in concrete

The final suggested formula will be

$Vc = \varphi \left(fc'\right)^{\gamma} \beta_0 d$

Table 4. A comparison of the experimental results with the suggested empirical formula

IR	[illegible]	[illegible]	*fc'*	*d*	*Sf*	*Suggested formula* ([illegible])	*Experimental results*	*% difference*
0.03031	**0.452**	**760**	**92.3**	**40**	**0**	**21.34**	**21.68**	**1.58**
0.02832	**0.462**	**760**	**88.6**	**40**	**0.5**	**19.52**	**15.92**	**22.62**
0.02248	**0.457**	**840**	**90.5**	**60**	**0.5**	**37.99**	**42.25**	**10.08**
0.01626	**0.465**	**920**	**87.5**	**80**	**0.5**	**54.72**	**59.84**	**8.56**
0.02138	**0.473**	**760**	**84.6**	**40**	**1**	**32.66**	**38.51**	**15.19**
0.01578	**0.418**	**760**	**104.2**	**40**	**2**	**48.73**	**46.54**	**4.71**
0.02415	**0.459**	**760**	**89.5**	**40**	**0.5**	**16.54**	**16.51**	**0.18**
0.02124	**0.471**	**760**	**85.4**	**40**	**0.5**	**14.98**	**17.25**	**13.16**
					Average			**9.51**

Where
fc' = concrete compressive strength,
γ = [0.71-0.0028 fc']
β_0 = the critical section perimeter
d = effective depth of critical zone of slabs.

The average deviation between the results obtained from the proposed equation and the experimental results is 9.5%. Some of the predicted values were very close to the experimental outcomes, reaching a confidence level of approximately 99%, while others were around 78%. In general, certain results could be excluded based on the standard deviation of the model outcomes; however, due to the limited number of models, this was not feasible. Moreover, since this study aims to lay the groundwork for developing an initial equation capable of predicting the punching shear resistance of slabs subjected to impact loading, it is recommended that future researchers consider incorporating additional variables to refine this equation and ultimately establish a reliable model that can be adopted by design codes concerned with the rehabilitation and strengthening of structural members.

6. Conclusion

An examination of the residual punching shear capacity after being subjected to impact loading reveals that slab thickness plays a significant role in enhancing the slab's ability to withstand impact forces, making it suitable for rehabilitation rather than demolition. Increasing the thickness by 1.5 times resulted in a 62% improvement, while doubling the thickness led to a 73% increase in residual punching shear strength. Additionally, doubling the steel fiber content improved the slab performance by approximately 59%, and increasing it to twice that amount yielded a 66% enhancement. These observations indicate that slab thickness has a more pronounced effect on performance than the steel fiber ratio. It is also worth noting that introducing the mid-span camber into the slab geometry did not produce noticeable improvement, as its performance was comparable to that of the flat slab. When comparing a conventionally reinforced slab with another identical slab containing 0.5% steel fibers, the conventionally reinforced specimen exhibited a 27% higher capacity. However, increasing the steel fiber content to 1% resulted in a 44% higher capacity compared to the conventionally reinforced slab, and further increasing the fiber content to 2% raised the difference to 53%. Finally, the proposed equation for predicting the residual punching shear capacity after impact loading can be considered a preliminary step toward developing a more comprehensive model. It serves as a foundation for future research aimed at refining and validating such predictive equations to be adopted in design and rehabilitation codes.

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