

INVERSE AND CONNECTED DOMINATION IN HYPERTREE NETWORKS

V. SHALINI¹, INDRA RAJASINGH^{1,*}, §

ABSTRACT. A dominating set of a graph $G = (V, E)$ is a subset D of vertices such that every vertex in $V \setminus D$ is adjacent to at least one vertex in D , and the minimum size of such a set is called the domination number denoted by $\gamma(G)$. If D is a minimum dominating set of G and there exists a dominating set D' within $V \setminus D$, then D' is called an inverse dominating set with respect to D . The minimum cardinality of such a set is known as the inverse domination number, denoted by $\gamma'(G)$. A dominating set D is called a connected dominating set if the induced subgraph $\langle D \rangle$ is connected in G . The minimum cardinality of a connected dominating set is called the connected domination number, denoted by $\gamma_c(G)$. In this paper, we have computed the inverse and connected domination numbers for Hypertree Networks.

Keywords: Domination, Inverse Domiantion, Connected Domination, Hypertree Network.

AMS Subject Classification: 05C69

1. INTRODUCTION

Domination problems are extensively studied to identify representative sets for monitoring communication and electrical networks. In land surveying it helps to optimize the number of locations that a surveyor must occupy to take height measurements for an entire region [1]. It also plays a crucial role in parallel processing and supercomputing more significantly applied in advancements of modern science and engineering. In any network, dominating sets serve as central structures for solving routing problems in parallel computing [2]. A non-empty subset $D \subseteq V(G)$ is called a dominating set if every vertex in $V(G) \setminus D$ is adjacent to at least one vertex in D . The cardinality of smallest such set determines the domination number of the graph G denoted by $\gamma(G)$ [3]. Liu (2004) explored domination numbers in 4-regular graphs, showing that $\gamma(G) \leq \frac{4n}{11}$ for a 4-regular graph with n vertices [32]. Hurink and Nieberg (2008) introduced a PTAS for computing minimum independent dominating sets in wireless networks modeled by graphs

¹ Division of Mathematics, School of Advanced Sciences, Vellore Institute of Technology, Chennai Campus, 600127, Tamil Nadu, India.

e-mail: shalu.maths@gmail.com; ORCID: <https://orcid.org/0009-0002-9675-4028>.

e-mail: indrarajasingh@yahoo.com; ORCID: <https://orcid.org/0000-0002-9721-9505>.

* Corresponding author.

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of polynomially bounded growth [33]. Ahangar et al. (2009) provided tight bounds for domination numbers in 4-regular graphs and also analyzed connected, independent, and total domination parameters [34].

Harant (2009) studied domination in bipartite graphs with minimum degree at least 2 and specific cycle restrictions, proving that the domination number is bounded above by $\frac{3n}{8}$ [35]. Desormeaux et al. (2014) introduced the Slater number $s_\ell(T)$ as a lower bound for the domination number of trees, establishing relationships between $\gamma(T)$ and $s_\ell(T)$ [36]. Pandey and Srivastava (2018) focused on domination in subclasses of bipartite graphs, including star-convex and circular-convex bipartite graphs, proving NP-completeness for some cases and providing polynomial-time solutions for others [37]. Determining the domination number $\gamma(G)$ of a graph G is a well-known NP-complete problem, and finding optimal dominating sets in network structures remains computationally challenging.

Let D be a minimum dominating set of a graph G . If the set $V \setminus D$ contains a dominating set denoted as D' , then D' is called an inverse dominating set with respect to D . The inverse domination number, $\gamma'(G)$ represents the size of the smallest inverse dominating set in G [9]. The concept of inverse domination in graphs introduced by Kulli and Sigarkanti in 1991 plays a significant role in ensuring reliable communication in electrical and network systems. If some nodes of a minimum dominating set D in G fail, the inverse dominating set serves as a backup, taking over the role of D . Domke et al. [10] conjectured that the inverse domination number of G is at most its independence number. This conjecture has been proven for several special families of graphs, including claw-free graphs, bipartite graphs, split graphs, very well-covered graphs, chordal graphs, and cactus graphs [11]. Some graphs where the domination number and inverse domination number are equal have been identified by T. TamizhChelvam [12]. The inverse domination number for circulant graphs has been established by V. Cynthiya [13]. Wardani analyzed the inverse domination and inverse total domination numbers of Sierpinski star graphs [20]. Additionally we analyzed the inverse domination numbers for specialized networks such as the X-Tree Network, Sibling Tree Network, Wrapped Butterfly Network, Lollipop Graph, Fly Graph, and Jellyfish Graph have been determined [15, 16, 17]. We had proposed the inverse domination number for Hypertree Networks as a conjecture in [17].

A dominating set D in a graph G is called a connected dominating set if the induced subgraph $\langle D \rangle$ is connected. The connected domination number of G denoted by $\gamma_c(G)$ represents the size of a minimum connected dominating set [4]. The study of connected domination has significant applications in routing problems and virtual backbone-based routing in wireless networks [5, 6, 7]. Caro and Hansberg [27] linked connected domination to spanning trees with many leaves, establishing bounds that tied domination theory to extremal graph properties. Chen [28] characterized k - γ_c -critical graphs and provided sufficient conditions for equality between total domination $\gamma_t(G)$ and connected domination $\gamma_c(G)$ in block and cactus graphs. Duckworth and Wormald [29] studied small connected dominating sets in d -regular graphs through probabilistic methods, revealing average-case performance trends. Goto et al. [30] derived exact values of $\gamma_c(G)$ in grid graphs, contributing refined methods for computing domination parameters in structured graphs. Mafuta and Mynhardt [31] investigated connected domination in traceable graphs and resolved conjectures related to spanning paths, further deepening the theoretical understanding of $\gamma_c(G)$. Computing a minimum connected dominating set is an NP-hard problem [8].

In this paper, we have obtained exact bounds for the inverse domination and connected domination numbers in Hypertree Networks.

2. PRELIMINARIES

In this section, we give the definitions and results required for our subsequent work.

Definition 2.1. [3] *A non-empty subset $D \subseteq V(G)$ is a dominating set if each vertex in $V(G) \setminus D$ is adjacent to at least one vertex in D . Such a set with minimum cardinality yields the domination number of a graph G , and it is denoted by $\gamma(G)$.*

Definition 2.2. [9] *Let D be a minimum dominating set of G . If $V \setminus D$ contains a dominating set, say D' , of G , then D' is called an inverse dominating set with respect to D . The inverse domination number $\gamma'(G)$ of G is the order of the smallest inverse dominating set in G .*

Definition 2.3. [4] *A dominating set D is said to be a connected dominating set if the induced subgraph $\langle D \rangle$ is connected in G . The minimum of the cardinalities of the connected dominating sets of G is called the connected domination number $\gamma_c(G)$.*

Theorem 2.1. [3] *For any graph G of order p and maximum degree Δ , $\gamma(G) \geq \frac{p}{\Delta+1}$.*

Theorem 2.2. [9] *Let T be a tree such that every non-end vertex is adjacent to at least one end vertex. Then $\gamma(T) + \gamma'(T) = p$.*

Theorem 2.3. [14] *Let G be the hypertree $HT(n)$, $n \geq 1$. Then:*

$$\gamma(G) = \begin{cases} \frac{1}{7}(2^{n+2} + 3), & \text{if } n \equiv 0 \pmod{3}, \\ \frac{1}{7}(2^{n+2} - 1), & \text{if } n \equiv 1 \pmod{3}, \\ \frac{2}{7}(2^{n+1} - 1), & \text{if } n \equiv 2 \pmod{3}. \end{cases}$$

3. INVERSE DOMINATION IN HYPERTREE NETWORKS

Hypertree networks have gained attention in recent years due to their ability to model complex hierarchical and interconnected systems efficiently. Goodman and Hedetniemi [23] were among the early researchers to explore hypertrees, recognizing their capacity to represent intricate relationships within graph structures. Subsequently, [24] Chekuri et al. investigated conjunctive query containment using hypertrees, which significantly improved computational efficiency for database query processing. Subsequent foundational work was carried out by Flum et al. [25], who examined parameterized complexity and hypertree decompositions, showcasing their utility in computational complexity theory and algorithmic design. Gottlob et al. [26] revisited hypertree decompositions, providing improved algorithms for solving constraint satisfaction problems (CSPs) efficiently, contributing to solving NP-hard problems with greater practicality. In practical applications, hypertrees have demonstrated significant potential in various fields. Klamt et al. [21] utilized hypertrees to represent biological and cellular networks effectively, demonstrating their applicability in systems biology. Additionally, recent studies by Zhou et al. [22] extended hypertree network applications into large-scale data analytics and machine learning, noting performance improvements in data mining and clustering tasks. Domination parameters, crucial for understanding the efficiency of resource placement and fault diagnosis in network structures, have also been explored within hypertrees.

The fundamental skeleton of a hypertree is a complete binary tree T_r of height r . Here the nodes of the tree are numbered as follows: The root node is labeled 1. The root is supposed to be at Level 0. Labels of left and right children are formed by appending 0 and 1, respectively to the labels of the parent node. The decimal and binary labels of the hypertree are given in Figure 1(a). Here the children of the node x are labelled as $2x$ and $2x + 1$. Additional links in a hypertree are horizontal and two nodes are joined in the

same Level i of the tree if their label difference is 2^{i-1} . We denote an r -level hypertree as $HT(r)$. It has $2^{r+1} - 1$ vertices and $3(2^r - 1)$ edges [18]. The root fault hypertree $HT^*(r)$, $r \geq 2$ is a graph obtained from $HT(r)$ by deleting the root vertex at Level 0 [19]. See Figure 2(a).

The domination number of Hypertree Networks has already been obtained in [14]. But this choice of vertices does not help in selecting an inverse dominating set. In this section we have attained a tight bound for inverse domination number in Hypertree Networks.

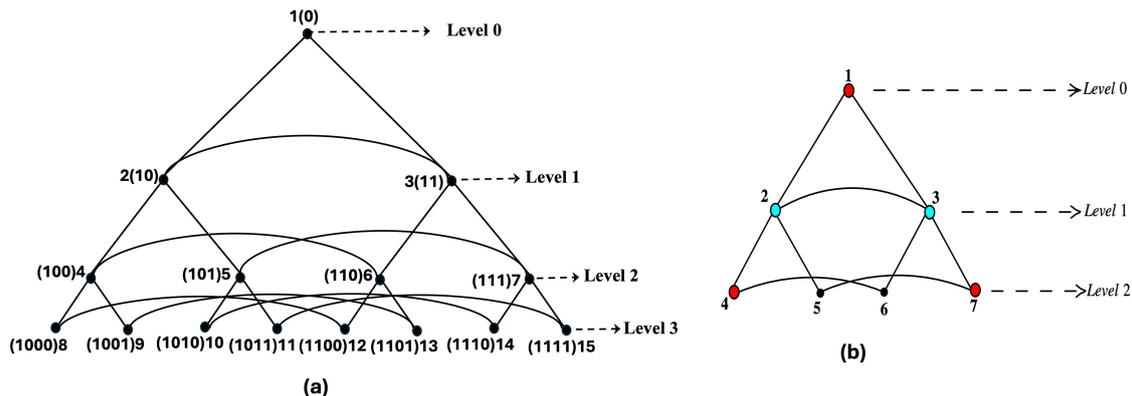


FIGURE 1. (a) $HT(3)$ with decimal and binary labels (b) $HT(2)$ with dominating set marked in blue and inverse dominating set marked in red.

Lemma 3.1. *Let $HT(2)$ be the hypertree of dimension 2. Then the domination number of $HT(2)$ is 2 and the inverse domination number of $HT(2)$ is 3. In other words, $\gamma(HT(2)) = 2$ and $\gamma'(HT(2)) = 3$.*

Proof. No vertex in $HT(2)$ is of degree 6. Therefore $\gamma(HT(2)) \geq 2$. It is easy to check that the two 4 degree vertices labeled 2 and 3 in $HT(2)$ dominate $HT(2)$, See Fig.1(b). Further $D = \{2, 3\}$ is the only minimum dominating set of $HT(2)$. Hence $\gamma(HT(2)) = 2$. The complement of D contains 5 vertices and no two of them can dominate $HT(2)$. Therefore $\gamma'(HT(2)) \geq 3$. It is easy to verify that $D' = \{1, 4, 7\}$ which is a subset of $V \setminus D$, is a dominating set of $HT(2)$. This implies $\gamma'(HT(2)) = 3$. \square

Lemma 3.2. *Let $HT^*(3)$ be the root fault hypertree of dimension 3. Then $\gamma(HT^*(3)) = \gamma'(HT^*(3)) = 4$.*

Proof. Consider $HT^*(3)$ labelled as in Fig.2(a). Let D be a minimum dominating set of $HT^*(3)$. If $\{1, 2\} \subseteq D$, then remaining vertices to be dominated induce 4 independent edges. This requires 4 more vertices to be included in D , yielding a dominating set of cardinality 6. If $\{1\} \in D$ and $\{2\} \notin D$, then vertices 5 and 6 must be in D . This leaves out 4 isolated vertices to be dominated. Again cardinality of D is at least 6. Suppose both 1 and 2 are not in D , then the remaining vertices induce 2 disjoint copies of $HT^*(2)$. See Fig.2(b). A singleton vertex in $HT^*(2)$ is not sufficient to dominate all vertices of $HT^*(2)$. Therefore $\gamma(HT^*(2)) \geq 2$. Then $\gamma(HT^*(3)) \geq 4$. It can be easily verified that $D = \{3, 5, 9, 14\}$ and $D' = \{4, 6, 7, 12\}$ are minimum dominating sets of $HT^*(3)$ with $D \cap D' \neq \emptyset$. Therefore $\gamma(HT^*(3)) = 4$ and $\gamma'(HT^*(3)) = 4$. \square

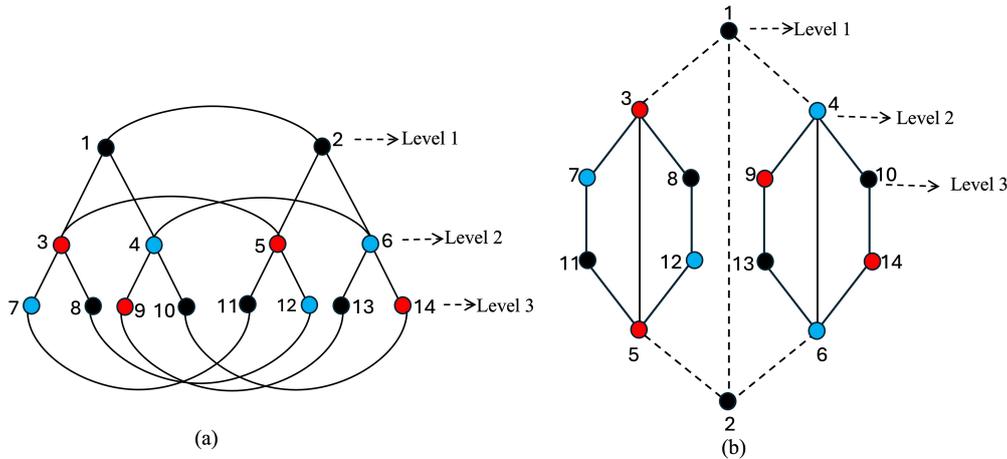


FIGURE 2. (a) $HT^*(3)$ with dominating set marked in red and inverse dominating set marked in blue (b) Two disjoint copies of $HT^*(2)$ in $HT^*(3)$ with dominating set marked in red and inverse dominating set marked in blue.

Theorem 3.1. *Let G be the hypertree $HT(r)$ of dimension $r \geq 0$. Then*

$$\gamma'(HT(r)) = \gamma(HT(r)) = \begin{cases} \frac{1}{7}(2^{r+2} + 3); & r \equiv 0 \pmod{3} \\ \frac{1}{7}(2^{r+2} - 1); & r \equiv 1 \pmod{3} \end{cases}$$

and

for $r \equiv 2 \pmod{3}$, $\gamma(HT(r)) = \frac{1}{7}(2^{r+2} - 2)$ and $\gamma'(HT(r)) = \frac{1}{7}(2^{r+2} + 5)$.

Proof. We partition the levels $L_0, L_1, L_2, L_3, \dots, L_r$ of $HT(r)$ into maximum number of disjoint 3-levels, beginning from L_r . Clearly the left out levels will be L_0 when $r \pmod{3} = 0$ inducing a singleton vertex, L_0 and L_1 when $r \pmod{3} = 1$ inducing a 3-cycle and L_0, L_1 and L_2 when $r \pmod{3} = 2$ inducing $HT(2)$. In other words, we have the partitioning set P of the levels of $HT(r)$ as follows:

(i) When $r \pmod{3} = 0$, $P = \{L_0\} \cup \bigcup_{i=0}^{\lfloor \frac{r}{3} \rfloor - 1} \{L_{r-3i-2}, L_{r-3i-1}, L_{r-3i}\}$;

(ii) When $r \pmod{3} = 1$, $P = \{L_0, L_1\} \cup \bigcup_{i=0}^{\lfloor \frac{r}{3} \rfloor - 1} \{L_{r-3i-2}, L_{r-3i-1}, L_{r-3i}\}$;

(iii) When $r \pmod{3} = 2$, $P = \{L_0, L_1, L_2\} \cup \bigcup_{i=0}^{\lfloor \frac{r}{3} \rfloor} \{L_{r-3i-2}, L_{r-3i-1}, L_{r-3i}\}$.

We note that L_{r-3i-2} has 2^{r-3i-2} vertices for any i , $0 \leq i \leq \lfloor \frac{r}{3} \rfloor - 1$.

Hence $L_{r-3i-2}, L_{r-3i-1}, L_{r-3i}$ together induce 2^{r-3i-3} disjoint copies of $HT^*(3)$, $0 \leq i \leq \lfloor \frac{r}{3} \rfloor - 1$. Thus the number α of vertex disjoint copies of $HT^*(3)$ in $HT(r)$, $r \geq 4$, is

$$\alpha = \sum_{i=0}^{\lfloor \frac{r}{3} \rfloor - 1} 2^{r-3i-3} = \frac{2^r}{7} \left(\frac{2^{3\lfloor \frac{r}{3} \rfloor} - 1}{2^{\lfloor \frac{r}{3} \rfloor}} \right) = \frac{1}{7}(2^r - 2^{r \pmod{3}})$$

By Lemma 3.2, each $HT^*(3)$ contains two vertex disjoint sets of vertices of cardinality 4 that dominate $HT^*(3)$. We call them irregular D -Quadruple Twins.

We construct two minimum dominating sets D and D' of $HT(r)$, $r \geq 4$ as follows. Include one pair of irregular D -Quadruple Twin vertices in D and the other pair of irregular D -Quadruple Twin vertices in D' from each of the α copies of $HT^*(3)$. The vertices of $HT(r)$ that are not covered by these α copies of $HT^*(3)$ induce $HT(0)$ or $HT(1)$ according as $r \equiv 0$ or $1 \pmod{3}$. Hence one more vertex is to be included in any minimum dominating set to dominate the left out vertices. This is done by including the left child of the root

vertex in D and the corresponding right child in D' . When $r \equiv 2 \pmod 3$, the left out vertices in Levels L_0, L_1 and L_2 induce the subgraph isomorphic to $HT(2)$. By Lemma 3.1 $\gamma(HT(2)) = 2$ and $\gamma'(HT(2)) = 3$. Hence include two more dominating vertices of $HT(2)$ in D and 3 other dominating vertices of $HT(2)$ in D' . Thus D and D' are dominating sets of the same cardinality $4\alpha + 1$ when $r \equiv 0, 1 \pmod 3$ and when $r \equiv 2 \pmod 3$, D is of cardinality $4\alpha + 2$ and D' is of cardinality $4\alpha + 3$.

We compute $4\alpha + 1$ when $r \equiv 0, 1 \pmod 3$ and $4\alpha + 2, 4\alpha + 3$ when $r \equiv 2 \pmod 3$ as follows where $\alpha = \frac{1}{7}(2^r - 2^{r \pmod 3})$:

when $r \equiv 0 \pmod 3$

$$4\alpha + 1 = \frac{4}{7}(2^r - 1) + 1 = \frac{1}{7}(2^{r+2} + 3)$$

when $r \equiv 1 \pmod 3$

$$4\alpha + 1 = \frac{4}{7}(2^r - 2) + 1 = \frac{1}{7}(2^{r+2} - 1)$$

and

when $r \equiv 2 \pmod 3$

$$4\alpha + 2 = \frac{4}{7}(2^r - 4) + 2 = \frac{1}{7}(2^{r+2} - 2)$$

$$4\alpha + 3 = \frac{1}{7}(2^{r+2} - 2) + 1 = \frac{1}{7}(2^{r+2} + 5)$$

Thus,

$$\gamma'(HT(r)) = \gamma(HT(r)) = \begin{cases} \frac{1}{7}(2^{r+2} + 3); & r \equiv 0 \pmod 3 \\ \frac{1}{7}(2^{r+2} - 1); & r \equiv 1 \pmod 3 \end{cases}$$

and for $r \equiv 2 \pmod 3$, $\gamma(HT(r)) = \frac{1}{7}(2^{r+2} - 2)$ and $\gamma'(HT(r)) = \frac{1}{7}(2^{r+2} + 5)$. □

4. CONNECTED DOMINATION IN HYPERTREE NETWORKS

We begin with a few observations on the structure of a hypertree. $HT(r)$ consists of a complete binary tree T_r of level r as a spanning subgraph. Let it be rooted at v . Further, it contains two vertex disjoint induced subgraphs, each isomorphic to T_{r-1} , with the children x and y of v as the root vertices respectively. We name the left subtree as LT_{r-1} and the right subtree as RT_{r-1} . See Figure 3.

Theorem 4.1. *Let G be the hypertree $HT(r)$ of dimension $r \geq 0$. Then $\gamma_c(G) = 3 \cdot 2^{r-2} - 1$.*

Proof. In any minimum dominating set D of G , there are a total of at least 2^{r-1} vertices from Level r and Level $(r - 1)$. We select all the 2^{r-1} vertices in Level $(r - 1)$ of $H(T(r))$ in D . Consider any two vertices a, b in D from Level $(r - 1)$ of LT_{r-1} . If D is connected then there is a path between a and b , all whose vertices are in D . For D to be minimum the path selected between a and b should be a shortest path. Such a shortest path travels the edges of the binary tree LT_{r-2} rooted at x . Since a and b are arbitrary, we conclude that every vertex in the tree LT_{r-2} rooted at x must be in D .

We claim that D is a dominating set. The vertices in Level $(r - 1)$ of $HT(r)$ dominate all the vertices of Level $(r - 2)$ and Level r in $HT(r)$. The end vertices in LT_{r-1} of all the horizontal edges dominate all vertices in RT_{r-4} . x dominates the root v of $HT(r)$. Thus D is a minimum connected dominating set and

$$|D| = 2^{r-1} + \frac{1}{2}(2^{r-2} + 2^{r-3} + \dots + 2) = 2^{r-1} + (2^{r-3} + 2^{r-4} + \dots + 1) = 3 \cdot 2^{r-2} - 1. \quad \square$$

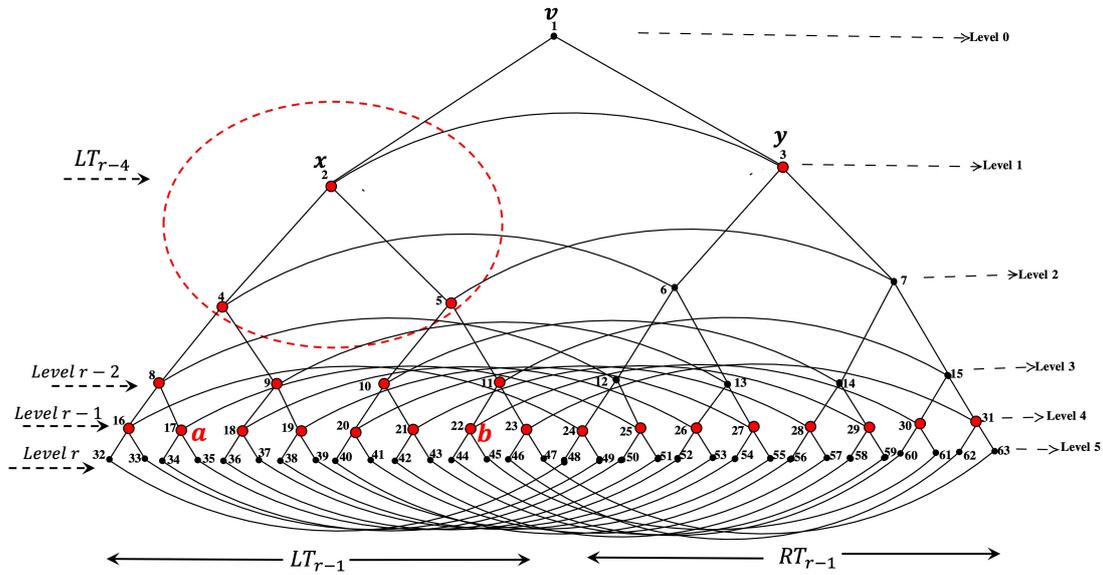


FIGURE 3. $HT(5)$ with connected dominating set marked in red.

5. INVERSE DOMINATION IN BENES NETWORKS

In this section we add a partial result on the inverse domination of Benes Networks. The r -dimensional butterfly network, denoted by $BF(r)$, has the vertex set: $V = \{(x, i) : x \in V(Q_r), 0 \leq i \leq n\}$ where $V(Q_r)$ represents the vertex set of the n -dimensional hypercube Q_r . Two vertices (x, i) and (y, j) are connected by an edge in $BF(r)$ if and only if $j = i + 1$ and either $x = y$, or x differs from y in precisely the j -th bit [38].

An r -dimensional Benes network has $2r + 1$ levels, each with 2^r nodes. The nodes from level 0 to level r in the network form an r -dimensional butterfly. The middle level of the Benes network is shared by these butterflies. An r -dimensional Benes network is denoted by $B(r)$ [39]. Figure 4 shows a $B(3)$ network.

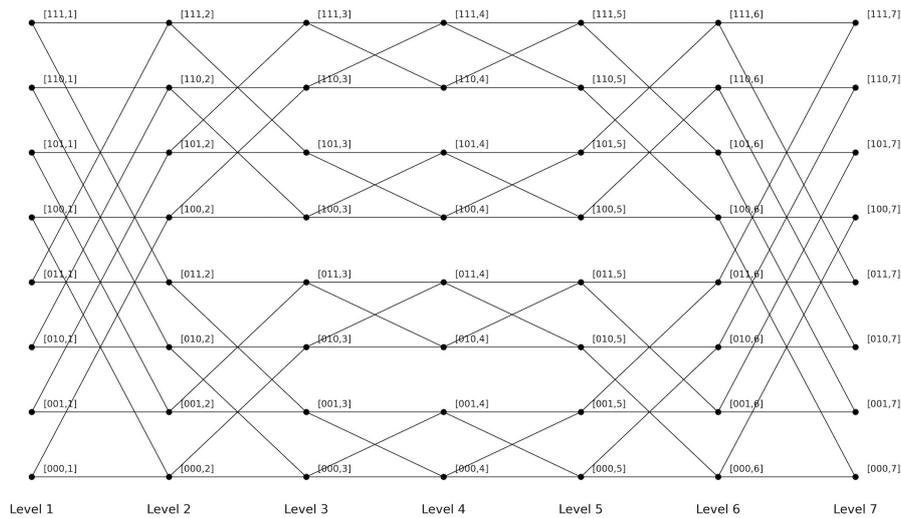


FIGURE 4. $B(3)$ with 8 processor and 56 switching nodes.

Proposition 5.1. *Let $B(r)$ be the Benes Network of dimension r , $r \geq 2$. Then $\gamma'(B(r)) \leq (r - 1)2^r$.*

Proof. Benes network $B(r)$ of dimension r has $2r + 1$ Levels with 2^r vertices in each level. Further two consecutive levels constitute 2^{r-1} vertex disjoint 4-cycles. Choosing 2^{r-1} vertices, one from each 4-cycle in each Level except in Level 1, Level r and Level $2r + 1$ constitute a dominating set of $B(r)$. Interestingly, the left out vertices in each of these levels also form a dominating set. Hence $\gamma'(B(r)) \leq (r - 1)2^r$. See Figure 5. \square

We propose an open problem that $\gamma'(B(r)) = (2r + 1)2^{r-2}$ for $r \geq 4$.

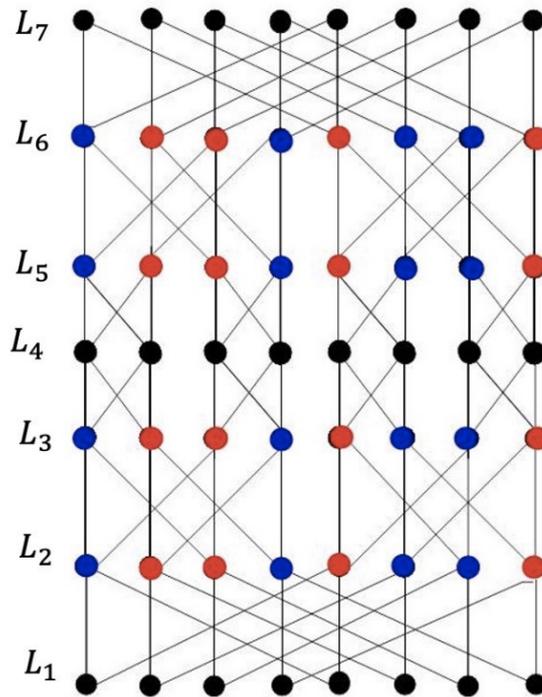


FIGURE 5. Red vertices dominate $B(3)$; Blue vertices also dominate $B(3)$ constituting an inverse dominating set.

6. CONCLUSION

In this paper, we have derived the inverse and connected domination numbers in Hypertree Networks. Additionally, the domination and inverse domination numbers are equal in the Hypertree Network $HT(r)$ when $r \equiv 0, 1 \pmod{3}$. However, when $r \equiv 2 \pmod{3}$, the inverse domination number of $HT(r)$ exceeds the domination number by one.

This result has meaningful implications in practical scenarios involving hierarchical and fault-tolerant systems. In hypertree-modeled environments such as distributed data centers, sensor networks, or parallel processing architectures, the domination number represents the minimum number of controller or monitoring nodes needed for full system oversight. The inverse domination number, on the other hand, reflects the minimum set of backup or validation nodes required to ensure every controller is covered. When $\gamma(HT(r)) = \gamma'(HT(r))$, as in the cases where $r \equiv 0, 1 \pmod{3}$, no additional resources

are needed for redundancy. However, when $r \equiv 2 \pmod{3}$, the inverse domination number exceeds the domination number by one, indicating a structural requirement for at least one additional backup node. This insight is especially relevant in optimizing resource allocation, redundancy planning, and fault resilience in hierarchical system designs.

Furthermore, for the Benes Network $B(r)$ of dimension $r \geq 2$, we establish that the inverse domination number is bounded above by $(r - 1) \cdot 2^r$. This result provides a useful guideline for determining the minimal number of backup nodes required in Benes-based interconnection networks, reinforcing their efficiency and scalability for robust communication infrastructures.

7. FUTURE WORK

Studying the domination and inverse domination numbers of architectures like Hyper-Butterfly Networks is crucial for optimizing performance, fault tolerance, and resource allocation in parallel computing and network systems. Similarly, tree-like structures such as Christmas Trees and Slim Trees play a significant role in routing, clustering, and biological networks, making their domination parameters essential for understanding resilience and efficiency. Investigating these properties can lead to advancements in network robustness, resource management, and fault-tolerant designs. Future research can focus on deriving tight bounds, efficient algorithms, and analyzing the impact of node failures, contributing to improved wireless communication, distributed computing and cybersecurity.

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V. Shalini is a Research Scholar in the Department of Mathematics at Vellore Institute of Technology, Chennai, under the guidance of Dr. Indra Rajasingh. She has graduated with an M.Phil. in Mathematics and has teaching experience of three years. Her research focuses on domination and inverse domination in interconnection networks, especially in Wrapped Butterfly and X-tree Networks. She has publications in SCOPUS and ESCI-indexed journals and has presented research papers in national and international conferences. Her academic interests include graph theory, combinatorics, and approximation algorithms. She is proficient in LaTeX, SageMath, MATLAB, and R.



Dr. Indra Rajasingh is a Retired Professor and Dean at the School of Advanced Sciences, VIT, Chennai, with over 44 years of teaching experience and extensive contributions to mathematical research. She got her Ph.D. from IIT Madras in 1984, M.Sc. from Bombay University and M.Phil. from Madurai Kamaraj University. Her primary research interests lie in interconnection networks, domination theory, embedding problems, and combinatorics. She has guided over 25 Ph.D. scholars and served as Principal Investigator on three DST-funded projects. Dr. Rajasingh has published more than 150 research papers and has served as editor and convenor for multiple international conferences. She has also been actively involved in academic governance, serving on boards of studies, university committees, and international review panels. Her collaborative work spans institutions across India and abroad, establishing her as a prominent contributor in mathematical and computational research.
