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MEASURING JUMP SIZES IN ASSET PRICES WITH AN INDIRECT APPROACH

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Abstract. The aim of this article is to estimate the magnitude of asset price jump sizes using an inverse method applied to historical financial data. Specifically, we adapt a particular form of the Merton jump-diffusion model for this estimation. The model is then discretized using the characteristics of the Poisson process along with the Euler-Maruyama numerical method. Using historical financial data from various assets including global gold ounce prices, Alphabet (Google) stock, and crude oil collected over 2, 6, and 5-year periods, we estimate the price jump size for a short one-week time frame for these assets. This estimation is carried out by minimizing the price jump size inversely, using the discretized function obtained from the Euler-Maruyama numerical method, implemented through simulation in Python software. Finally, the effectiveness of the inverse method in estimating asset price jump sizes is evaluated by comparing the estimated values with the actual observed price jump sizes in the historical data of each asset, taking into account the calculated error.

Keywords: Inverse method, Euler-Maruyama discretization, Merton Jump Diffusion Model, Asset price jump size.

AMS Subject Classification: 91G30, 91G60, 91G15.

1. Introduction

In financial research, the accurate measurement of asset price jump sizes holds significant importance [1].Nevertheless, direct methodologies for quantifying the magnitude of these abrupt movements often present considerable challenges. Consequently, indirect approaches, which typically leverage estimation models such as Levy processes or statistical analyses, are frequently employed to derive approximate measurements [7]. These indirect methods prove particularly valuable in capturing sudden market fluctuations and inherent uncertainties, thereby offering enhanced accuracy within volatile financial environments [3]. Furthermore, recent studies advocate for the integration of indirect techniques with traditional analytical frameworks to improve risk management and refine asset valuation

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strategies [9].This combined approach has seen widespread adoption, particularly in markets characterized by elevated volatility.

In 2013, Neisy et al.[19] introduced a pricing model for European options under a jump-diffusion underlying asset. They then employed appropriate numerical methods to solve this model, which included integral and derivative terms. Also in 2013, Deniz Ilalan [12] demonstrated the destructive effects of rare events on financial stability, utilizing the compound Poisson process to model sudden price movements in financial markets. In 2022, George Jumbe [13] highlighted the challenge of distinguishing between organizations likely to default and those that will not prior to an actual default. He addressed this by modeling default risk using a Poisson jump process driven by a stochastic process.

More recently, in 2020, Gerald H.L. Cheang [6] presented representations of European and American exchange option prices under stochastic volatility jump diffusion. Concurrently in 2020, J. Lars Kirkby [14] developed a novel and efficient transform method for pricing Asian options. This method accommodates very general asset dynamics, including regime-switching Lévy processes, other jump-diffusion models, and stochastic volatility models with jumps. In 2019, Tarik Chakkour [4] showed the inverse problem stability of a continuous-time model designed for public institution finances. Finally, in 2021, Ying Chang et al. [5] argued that approximative fractional Brownian motion is a superior choice to standard Brownian motion for financial studies, using a random derivative technique to derive a semi-analytical pricing formula for call options.

The Merton jump-diffusion model, introduced by Robert C. Merton in 1976, is a mathematical framework designed to capture the dynamic behavior of asset prices [18]. This model integrates two distinct components: a continuous diffusion process, which accounts for gradual and small price fluctuations, and a compound Poisson process, incorporated to represent sudden, discontinuous jumps in price. Further details on the Merton jump-diffusion model and its underlying assumptions are comprehensively provided in Kazuhisa Matsuda's 2004 article [17].

$$\frac{dS_t}{S_t} = (\alpha - \lambda k)dt + \sigma dB_t + (y_t - 1)dN_t \tag{1}$$

Which in the above model:

- The price of the asset at time t : S_t
- The volatility of the asset price : σ
- The drift rate of the asset price : α
- Average number of jumps in a given period of time : λ
- Standard Brownian motion : B_t
- A Poisson process with intensity λ : N_t
- Average of Jump-Size : k
- The asset price jump size : y_t

Merton posits that the magnitude of asset price fluctuations or jumps is a stochastic variable characterized by a normal distribution, specifically[17]:

$$\ln(y_t) \sim N(\mu, \delta^2), \;\; E(y_t) = e^{\left(\mu + \frac{1}{2}\delta^2\right)} \tag{2}$$

So that:

$$E[(y_t - E(y_t)^2)] = e^{\left(2\mu+\delta^2\right)}(e^{\delta^2} - 1) \tag{3}$$

$y_t - 1$ a log normal distribution with mean and variance:

$$E(y_t - 1) = e^{\left(\mu+\frac{1}{2}\delta^2\right)} - 1 \cong k \tag{4}$$

$$E[(y_t - 1 - E(y_t - 1))^2] = e^{\left(2\mu+\delta^2\right)}(e^{\delta^2} - 1) \tag{5}$$

The expected return proportional to the jump component is as follows:

$$E[(y_t - 1)dN_t] = E[y_t - 1]E[dN_t] = k\lambda dt \tag{6}$$

The total expected return can be expressed as follows:

$$E[\frac{dS_t}{S_t}] = E[(\alpha - \lambda k)dt] + E[\sigma dB_t] + E[(y_t - 1)dN_t] \tag{7}$$

As a result:

$$E[\frac{dS_t}{S_t}] = (\alpha - \lambda k)dt + 0 + k\lambda dt = \alpha dt \tag{8}$$

If, in Equation 1, the asset price does not jump in the time interval, i.e.$dN_t = 0$, the Merton jump-diffusion model reduces to a standard Brownian motion:

$$\frac{dS_t}{S_t} = (\alpha - \lambda k)dt + \sigma dB_t \tag{9}$$

If the asset price experiences a jump at time dt,(i.e.$dN_t = 1$) then:

$$\frac{dS_t}{S_t} = (\alpha - \lambda k)dt + \sigma dB_t + (y_t - 1) \tag{10}$$

The Merton jump-diffusion model describes asset price dynamics as follows [17]:

$$dS_t = (\alpha - \lambda k)S_t dt + \sigma S_t dB_t + (y_t - 1)S_t dN_t \tag{11}$$

This article presents the Merton jump-diffusion model, deriving the pricing formula for assets and exploring options pricing within its framework. To quantify the magnitude of sudden price fluctuations, we utilize the Euler-Maruyama discretization method and the inverse technique.

To clarify the issue regarding the size of an asset price jump, suppose the asset price suddenly jumps from S_t to $y_t S_t$ within a short time frame. In this case, the magnitude of the price jump size is [11]:

$$\frac{dS_t}{S_t} = \frac{y_t S_t - S_t}{S_t} = y_t - 1 \tag{12}$$

The subsequent sections will detail the theoretical underpinnings of our methodology. Specifically, we will elaborate on the properties of the Poisson process, the Euler-Maruyama numerical method, and the inverse method, highlighting their fundamental role in estimating asset price jump sizes within the numerical simulation framework. Following this, the discretized form of the Merton jump-diffusion model equation, obtained via the Euler-Maruyama numerical method, will be presented.

Our numerical simulations will then leverage this discretized equation to estimate jump sizes for a diverse set of financial assets over distinct historical periods:

- Global Gold Spot Price: Data spanning two years, from February 13, 2022, to February 11, 2024.
- Crude Oil Price: Data covering a five-year period, from May 31, 2020, to May 25, 2025.
- Alphabet (Google) Stock Price: Data for a six-year period, from May 26, 2019, to May 18, 2025.

In the results section, the estimated price jump sizes derived from our inverse method will be rigorously compared against the actual values observed in the historical price charts for each asset. This comparison will serve to quantify and demonstrate the efficiency of the inverse method in accurately estimating price jump size magnitudes.

2. Materials and Methods

This section outlines the data sources and computational methods utilized in this study. The financial assets selected for analysis comprise the global gold spot price, crude oil price, and Alphabet (Google) stock price. Historical data for these assets were acquired from reputable financial platforms, specifically Yahoo Finance and Investing websites. All numerical simulations and computational analyses were performed using Python software.

The methodological framework of this article integrates several key components. We will provide detailed explanations of the Euler-Maruyama numerical method, the inherent properties of the Poisson process, and the process of discretizing the Merton jump-diffusion model through the application of the Euler-Maruyama numerical method. Central to our approach is the inverse method, which constitutes our primary technique for estimating asset price jump sizes and is critically employed in the numerical simulation phase.

2.1. Poisson Process.

Definition 2.1. *if Brownian motion Process serves as a fundamental model for persistent minor noise, the Poisson process operates as a core model for noise that manifests as abrupt events. Let $\lambda > 0$. A random variable X follows a Poisson distribution with parameter λ represented as $Pn(\lambda)$, if it assumes non-integer values $k \geq 0$ with Probabilities [15] :*

$$P(X = k) = e^{-\lambda}\frac{\lambda^k}{k!}$$

The moment generating function of this distribution is represented by $E(e^{uX}) = e^{\lambda(e^u-1)}$

A Poisson process $N(t)$ is a random process characterized by the subsequent attributes:

1. (Independence of increments) $N(t) - N(s)$ is not influenced by prior events, particularly by F_s, the -σ field generated by $N(u), u \leq s$.

2. (Poisson increments) $N(t) - N(s), t > s$, has a Poisson distribution with parameter $\lambda(t - s)$. If $N(0) = 0$, then $N(t)$ has the $Pn(\lambda)t$ distribution.

3. (Step function paths) the paths $N(t), t \geq 0$, are functions that increase with respect to t, altering exclusively by increments of 1.

2.2. **Euler Maruyama numerical method.** In this section, we delve into the Euler-Maruyama (EM) discretization method, a widely used technique for numerically approximating solutions to stochastic differential equations (SDEs). This approach is analogous to approximating definite integrals using the left endpoint rule.

Consider a general SDE of the form:

$$dX_t = \mu(t, X_t)dt + \sigma(t, X_t)dW_t \tag{13}$$

where $\mu(t, X_t)$ is the drift coefficient, $\sigma(t, X_t)$ is the diffusion coefficient, and W_t is a standard Wiener process (Brownian motion).

To discretize this SDE over a time interval $[0, T]$, we divide the interval into N subintervals of length $\Delta t = T/N$, with discrete time points $t_i = i\Delta t$ for $i = 0, 1, \ldots, N$. The integral form of the SDE from t_i to t_{i+1} is:

$$X_{t_{i+1}} = X_{t_i} + \int_{t_i}^{t_{i+1}} \mu(u, X_u)du + \int_{t_i}^{t_{i+1}} \sigma(u, X_u)dW_u \tag{14}$$

The Euler-Maruyama method approximates the integral terms as follows:

- Drift Term Approximation:

The integral of the drift term is approximated by evaluating the drift coefficient at the beginning of the time interval and multiplying by the step size Δt:

$$\int_{t_i}^{t_{i+1}} \mu(u, X_u)du \approx \mu(t_i, X_{t_i})\Delta t \tag{15}$$

- Diffusion Term Approximation:

The stochastic integral of the diffusion term is approximated by evaluating the diffusion coefficient at the beginning of the interval and multiplying by the increment of the Wiener process over that interval.This increment, $\Delta W_i = W_{t_{i+1}} - W_{t_i}$, is a normally distributed random variable with mean 0 and variance Δt, i.e., $\Delta W_i \sim N(0, \Delta t)$. Therefore, ΔW_i can be expressed as $\sqrt{\Delta t}\, z$, where z is a standard normal random variable ($z \sim N(0, 1)$) [16].

$$\int_{t_i}^{t_{i+1}} \sigma(u, X_u)dW_u \approx \sigma(t_i, X_{t_i})(W_{t_{i+1}} - W_{t_i}) = \sigma(t_i, X_{t_i})\sqrt{\Delta t}\, z \tag{16}$$

Combining these approximations, the Euler-Maruyama discretization scheme for the SDE becomes [8]:

$$X_{t_{i+1}} = X_{t_i} + \mu(t_i, X_{t_i})\Delta t + \sigma(t_i, X_{t_i})\sqrt{\Delta t}\, z \tag{17}$$

Here, z represents an independent random variable drawn from a standard normal distribution, and $X_{t_0} = x_0$ is the initial condition. This iterative formula allows for the simulation of sample paths of the SDE.

2.3. **Discretization of the Merton Jump Diffusion Model using the Euler Maruyama Method.** Consider the Merton Jump Diffusion Model as presented in Equation (11), The Euler-Maruyama discretization of this model within the time interval $[t_{i-1}, t_i]$ can be outlined as follows [20]:

$$\int_{t_{i-1}}^{t_i} dS_t = \int_{t_{i-1}}^{t_i} (\alpha - \lambda k) S_t dt + \int_{t_{i-1}}^{t_i} \sigma S_t dB_t + \int_{t_{i-1}}^{t_i} (y_t - 1) S_t dN_t \tag{18}$$

Also, with respect to the numerical approximation of the integral $\int_a^b f(t)dt = f(a)(b-a)$, Therefore:

$$\begin{aligned} S_{t_i} - S_{t_{i-1}} =& (\alpha - \lambda k) S_{t_{i-1}} + \sigma S_{t_{i-1}} (B_{t_i} - B_{t_{i-1}}) \\ &+ (y_{t_{i-1}} - 1) S_{t_{i-1}} (N_{t_i} - N_{t_{i-1}}) \end{aligned} \tag{19}$$

As result:

$$S_{t_i} = S_{t_{i-1}} + S_{t_{i-1}} \Big((\alpha - \lambda k) + \sigma (B_{t_i} - B_{t_{i-1}}) + (y_{t_{i-1}} - 1)(N_{t_i} - N_{t_{i-1}}) \Big) \tag{20}$$

Such that:

$$(B_{t_i} - B_{t_{i-1}}) \sim N(0, t_i - t_{i-1}) \tag{21}$$

by using Poisson distribution Definition 2.1, we have:

$$(N_{t_i} - N_{t_{i-1}}) \sim Poisson(\lambda(t_i - t_{i-1})) \tag{22}$$

Consequently, applying Equations (20) , (21), (22) yields:

$$\begin{aligned} S_{t_i} = S_{t_{i-1}} + S_{t_{i-1}} \Big(& (\alpha - \lambda k) + \sigma N(0, t_i - t_{i-1}) \\ & + (y_{t_{i-1}} - 1) Poisson(\lambda(t_i - t_{i-1})) \Big) \end{aligned} \tag{23}$$

2.4. **Inverse Problems.**

2.4.1. *The Forward Problem.* In the context of problem classification, a forward problem is characterized by having the cause known, with the objective being to predict its corresponding effect. Such problems are typically considered well-posed. Mathematically, let x denote the cause (e.g., input parameters or inherent system properties) and y represent the effect (e.g., observed data or system output). If F is a mathematical model or operator that describes the functional relationship between x and y, then the forward problem is formally expressed as:

$$y = F(x)$$

The primary goal is to determine y, given the specified x and the functional operator F.

2.4.2. *The Inverse Problem.* Conversely, an inverse problem is defined by having the effect (observations) at hand, with the objective of reconstructing or inferring the cause (e.g., unknown parameters or internal system properties). These problems constitute the fundamental basis for numerous practical applications across diverse fields, including medicine (e.g., MRI and CT imaging), geophysics (e.g., seismology), astronomy, materials science, and finance (e.g., model calibration). Mathematically, building upon the forward model $y = F(x)$, the primary goal in an inverse problem is to determine x, given the observed data y and the functional operator F [2].

2.4.3. *Inverse Problem for Obtaining Asset Price Jump Size.* This section details the application of the inverse method for estimating the asset price jump size. As derived in the previous section, the discretized form of the Merton jump-diffusion model Equation 23 involves several parameters: α, σ, k, λ, S_{t_i}, and $S_{t_{i-1}}$. Our primary objective is to estimate y_t, which represents the asset price jump size, using the inverse method. For this purpose, we assume that the other model parameters (α, σ, k, λ) are constant and known. The estimation of y_t is then achieved by minimizing an objective function derived from Equation 23 through numerical simulation in Python. Specifically, given the model's output, S_{t_i} (the asset's price at time t_i), and its previous state, $S_{t_{i-1}}$ (the price at time t_{i-1}), along with the fixed values of the other parameters, we infer the model input, y_t over short time intervals.

The subsequent numerical simulation section will elaborate on how the constant parameters of Equation 23 are determined using historical financial data for global gold prices, crude oil prices, and Alphabet (Google) stock prices. Subsequently, we will detail the process of obtaining y_t, the central parameter of our problem, via the inverse method [10].

3. Numerical Simulation

Here, we apply Equation 23 using real-world asset data. As an example, we obtain weekly gold price data from the Yahoo Finance website for the period February 13, 2022, to February 11, 2024.

Figure 1 illustrates the weekly gold price chart. We aim to identify the significant price jump size that occurred between March 5, 2023, and March 12, 2023, where the price increased from 1867.83 to 1987.93$, representing a 6.43 percent increase. Table 1 presents the weekly gold closing prices and their corresponding returns. Our analysis of the gold price data yields an average return and standard deviation as follows. Specifically, the average return and standard deviation of the gold price are:

- Average return -(α) = 0.00412
- Standard Deviation - (σ) = 0.0191

Based on the data in Table 1, we count the number of weeks where the price change exceeded 1 percent as the number of price jumps within the 106-week period. Additionally, we calculate the average price change for weeks with a change greater than 1 percent as the average price jump size. The resulting values for these parameters are as follows:

- Average number of jumps (λ) =0.557
- Average of Jump-Size (k) = 0.0223

We aim to determine the magnitude of the price jump size between March 5, 2023, and March 12, 2023. To do this, according to Equation 23, we consider the gold price on March 5, 2023, as follows:

Table 1. Weekly Gold Closing Prices

Row number	Date	Close Price	Return
1	13/02/2022	1897.87	-0.0054
2	20/02/2022	1887.56	0.0429
3	27/02/2022	1968.45	0.0086
4	06/03/2022	1985.29	-0.0323
5	13/03/2022	1921.09	0.0189
6	20/03/2022	1957.4	-0.0169
7	27/03/2022	1924.3	0.0112
8	03/04/2022	1945.85	0.0147
9	10/04/2022	1974.54	-0.0227
10	17/04/2022	1929.73	-0.0173
11	24/04/2022	1896.4	-0.0071
12	01/05/2022	1882.96	-0.0381
...	...	...	...
...	...	...	...
56	26/02/2023	1854.97	0.0069
57	05/03/2023	1867.83	0.0643
58	12/03/2023	1987.93	-0.0054
...	...	...	...
...	...	...	...
101	07/01/2024	2048.72	-0.0096
102	14/01/2024	2029.09	-0.0053
103	21/01/2024	2018.34	0.01
104	28/01/2024	2038.59	-0.0071
105	04/02/2024	2024.16	-0.0055
106	11/02/2024	2013.1	

Figure 1. Weekly Gold Price Chart

$$S_{t_{i-1}} = 1867.83$$

Additionally, the gold price on March 12, 2023, is:

$$S_{t_i} = 1987.93$$

By plugging in the obtained parameters into Equation (23) ,also by applying the indirect technique and minimizing the discretized Equation (23) through Python simulations, we estimate the average price jump size for ten different random scenarios. Therefore, based on the results in Table 2, the overall average of the estimated jump sizes is:

TABLE 2. The results obtained for different random numbers.

Row number	Random number	Result
1	0.30263394	1.0748
2	0.358658	1.0621
3	0.32897355	1.0688
4	0.3571733	1.0625
5	0.34865893	1.0644
6	0.36085619	1.0616
7	0.39007166	1.0550
8	0.34900297	1.0643
9	0.37235585	1.0590
10	0.31656362	1.0716

- Mean Result= 1.0644

Therefore, according to Equation (12), the calculated price jump size is:

$$\begin{aligned} \hat{y} &= y - 1 \\ &= 1.0644 - 1 = 0.0644 \end{aligned} \tag{24}$$

According to Table 1, the actual price jump size between March 5, 2023, and March 12, 2023, is 0.0643. Additionally, based on Equation (24), we have:

- Real Jump Size=0.0643
- Predicted Jump Size=0.0644

Subsequently, we apply Equation 23 to other assets, beginning with weekly crude oil price data. This data was obtained from the Investing.com website and covers a five-year period, specifically from May 31, 2020, to May 25, 2025.

Figure 2 displays the weekly crude oil price chart. Within this dataset, we specifically identified a significant price jump that occurred between February 20, 2022, and February 27, 2022. During this period, the price notably increased from 91.59$ to 115.68$, representing a substantial 26.3 percent rise. Table 3 subsequently provides the weekly crude oil closing prices and their corresponding returns. Following the identical methodology and steps previously applied to the gold price data, we proceeded to calculate the relevant parameters for the crude oil price data, as detailed below:

- Average return -(α) = -0.00493

Table 3. Weekly Oil crude Closing Prices

Row number	Date	Close Price	Return
1	31/05/2020	39.55	-0.0831
2	07/06/2020	36.26	0.0962
3	14/06/2020	39.75	-0.0317
4	21/06/2020	38.49	0.0561
5	28/06/2020	40.65	-0.0024
6	05/07/2020	40.55	0.00098
...	...	...	...
...	...	...	...
170	20/02/2022	91.59	0.2630
171	27/02/2022	115.68	-0.0548
172	06/03/2022	109.33	-0.0423
...	...	...	...
...	...	...	...
256	20/04/2025	62.33	-0.0648
257	27/04/2025	58.29	0.0468
258	04/05/2025	61.02	0.0240
259	11/05/2025	62.49	-0.0153
260	18/05/2025	61.53	0.0043
261	25/05/2025	61.8	

- Standard Deviation - $(\sigma) = 0.03866$

Also, we have:

- Average number of jumps (λ) =0.815
- Average of Jump-Size $(k) = 0.04554$

Figure 2. Weekly Oil crude Price Chart

Our objective is to determine the magnitude of the price jump size that occurred between February 20, 2022, and February 27, 2022. In accordance with Equation 23, we designate the crude oil price at February 20, 2022, as the initial price for our calculation:

$$S_{t_{i-1}} = 91.59$$

Additionally, the crude oil price on February 27, 2022, is:

$$S_{t_i} = 115.68$$

Utilizing the previously obtained parameters and applying our inverse method, we proceeded to estimate the average price jump size.This was achieved by minimizing an objective function derived from the discretized Equation 23, through repeated numerical simulations conducted in Python across ten different random scenarios. Consequently, the estimated average price jump size is:

- Mean Result= 1.2569

Therefore, according to Equation 12, the calculated price jump size is:

$$1.2569 - 1 = 0.2569$$

Furthermore, as per Table 3, the real price jump size observed between February 20, 2022, and February 27, 2022, is 0.2630.

Next, we apply Equation 23 to the Alphabet (Google) equity price data. This data was obtained from the Investing.com website and covers a six-year period, specifically from May 26, 2019, to May 18, 2025, as follows:

TABLE 4. Weekly Alphabet(Google) Closing Prices

Row number	Date	Close Price	Return
1	26/05/2019	55.33	-0.0345
2	02/06/2019	53.42	0.0166
3	09/06/2019	54.31	0.0360
4	16/06/2019	56.27	-0.0378
5	23/06/2019	54.14	0.0459
6	30/06/2019	56.63	0.0113
...	...	...	...
...	...	...	...
226	17/01/2021	94.63	-0.0344
227	24/01/2021	91.37	0.1430
228	31/01/2021	104.44	0.0029
...	...	...	...
...	...	...	...
308	13/04/2025	151.16	0.0714
309	20/04/2025	161.96	0.0127
310	27/04/2025	164.03	-0.0687
311	04/05/2025	152.75	0.0879
312	11/05/2025	166.19	0.0137
313	18/05/2025	168.47	

Figure 3 illustrates the weekly Alphabet (Google) equity price chart. Within this dataset, we identified a significant price jump size that occurred between January 24,

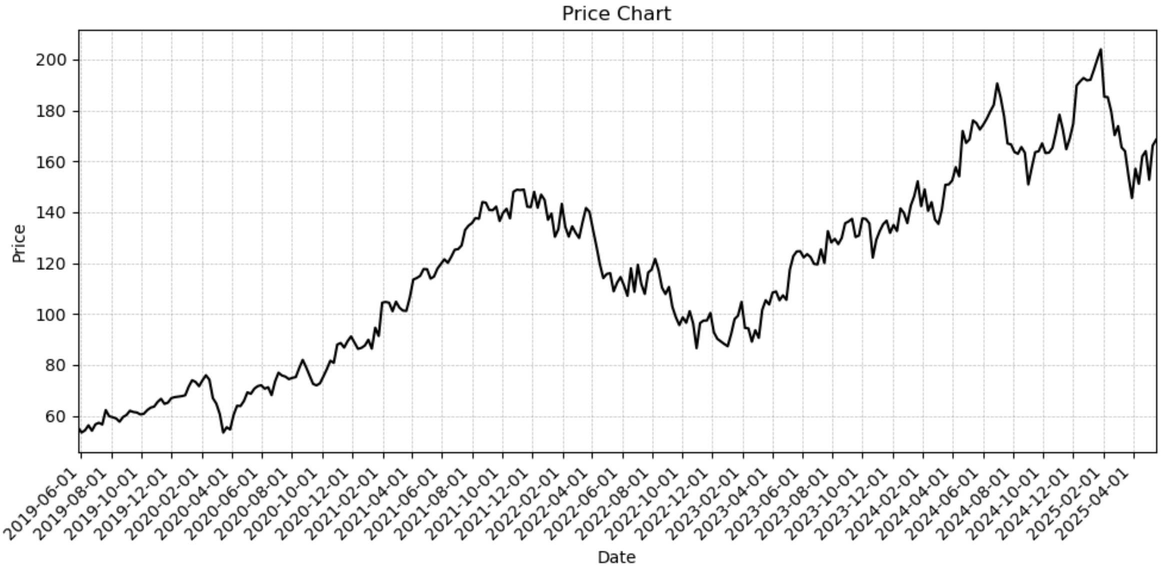

FIGURE 3. Weekly Alphabet(Google) Price Chart

2021, and January 31, 2021. During this period, the price increased notably from 91.37\$ to 104.44\$ representing a 14.3 percent increase. Table 4 subsequently presents the weekly Alphabet (Google) equity closing prices and their corresponding returns. Following the identical methodology and steps previously applied to the gold price data, we proceeded to calculate the relevant parameters for the Alphabet (Google) equity price data, as detailed below:

- Average return -(α) = 0.0044
- Standard Deviation - (σ) = 0.04189

Also, we have:

- Average number of jumps (λ) =0.7948
- Average of Jump-Size (k) = 0.039

Our objective is to determine the magnitude of the price jump size that occurred between January 24, 2021, and January 31, 2021. In accordance with Equation 23, we designate the Alphabet (Google) equity price on January 24, 2021, as the initial price for our calculation:

$$S_{t_{i-1}} = 91.37$$

Additionally, the Alphabet (Google) equity price on January 31, 2021, is:

$$S_{t_i} = 104.44$$

Utilizing the previously obtained parameters and applying our inverse method, we proceeded to estimate the average price jump size.This was achieved by minimizing an objective function derived from the discretized Equation 23, through repeated numerical simulations conducted in Python across ten different random scenarios. Consequently, the average of the estimated price jump sizes is:

- Mean Result = 1.14215

Therefore, according to Equation 12, the estimated price jump size is:

- 1.14215 -1= 0.14215

Furthermore, as per Table 4, the actual price jump size observed between January 24, 2021, and January 31, 2021, is 0.014304.

4. Result

Table 5. Comparison of Real and Estimated Price Jump Sizes for Various Assets.

Symbol	Period(year)	Real jump size	Estimated price jump size	Error
Gold	2	0.0643	0.0644	0.0001
Oil crude	5	0.2630	0.2569	0.0064
Alphabet(Google)	6	0.14304	0.14215	0.00085

This section details the empirical validation of our inverse method for estimating asset price jump sizes. We applied the method to historical data for three distinct financial assets: Gold, Crude Oil, and Alphabet (Google), spanning various timeframes. Table 5 below comprehensively presents the comparison between the real (or true) jump sizes and the values estimated by our model, alongside the calculated estimation errors.

Upon examining Table 5, it is evident that our proposed inverse method demonstrates a high degree of accuracy in estimating the price jump sizes. For Gold, observed over a 2-year period, the estimated jump size of 0.0644 was exceptionally close to the real jump size of 0.0643, resulting in a minimal estimation error of just 0.0001. This indicates excellent precision for assets with potentially lower volatility or clearer jump characteristics.

Similarly, for Alphabet (Google), analyzed over a 5-year period, the method yielded an estimated jump size of 0.14215, which is very close to the real value of 0.14304, leading to a small error of 0.00085.This highlights the method's effectiveness even for individual equities that can exhibit more complex jump behaviors.

While the estimation for Crude Oil(over 4 years) showed a slightly higher error of 0.0064 (estimated at 0.2569 versus a real jump of 0.2630), this value remains relatively small in the context of financial modeling where data is often noisy and subject to significant real-world fluctuations.The overall consistency of the small estimation errors across diverse asset classes and varying observation periods strongly supports the accuracy, robustness, and practical applicability of our inverse method for accurately identifying and quantifying price jump events in financial markets. These results affirm the method's potential as a valuable tool for risk management.

5. Conclusion

Our research demonstrates the successful application of the inverse method in measuring the jump size of asset prices. The low error observed in Table 5, in estimating the jump size of financial assets such as the global gold price, crude oil price, and Alphabet (Google) stock price with different time periods, compared to the actual jump size of these data, shows the efficiency of the inverse method in estimating the jump size of assets in a short time frame using historical market data.

6. Discussion

Future research could leverage these findings by expanding its application to diverse asset classes like cryptocurrencies and fixed income, and conducting comparative analyses with advanced machine learning algorithms. Further studies could also explore the impact of fundamental factors on jump magnitudes and investigate integrating these jump estimates into sophisticated risk and volatility models to enhance financial forecasting and risk management.

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