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IMPROVING RELIABILITY ALLOCATION FOR COMPLEX NETWORKS USING THE SEA LION ALGORITHM WITH THREE COST FUNCTIONS

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Abstract. This research addresses the issue of improving the reliability of complex networks under multiple cost constraints by employing the Sea Lion Optimization (SLO) algorithm. The problem is framed as a constrained optimization model that aims to maximize system reliability by allocating reliability levels to network elements, considering three main cost functions: The proposed methodology relies on an adaptive search mechanism that dynamically balances the exploration and exploitation phases, enhancing the algorithm's ability to overcome local non-optimal solutions and improve convergence stability. Cost constraints are also incorporated into the update process to ensure that solutions remain within defined economic limits without negatively impacting system performance. The numerical results showed that the proposed model achieves improved overall reliability, along with a more homogeneous reliability distribution among network components, thus reducing critical vulnerabilities and increasing the efficiency of the system as a whole.

Keywords: : Reliability allocation; reliability optimization; sea Lion algorithm.

AMS 2020 Subject Classification: Primary 90C26, 53C20,

1. Introduction

When systems grow bigger, they tend to get trickier to manage. Putting reliability where it matters most helps boost overall function without overspending. Instead of spreading resources thin, focusing on key parts makes a difference. Traditional approaches often struggle when things become too tangled. Finding the best setup then takes much more time and energy. In recent decades, many mathematical and algorithmic methods have been developed and improved to address such problems. Some analytical and linear mathematical methods have been used to solve reliability allocation challenges in relatively simple networks, while mixed-integer dynamic and linear programming algorithms have been used to adapt solutions to large, complex networks. However, these methods often face limitations due to their high computational complexity and the challenges of adapting to changing variables. The reliability of the system can then be derived by

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considering the complementary probabilities i.e., the likelihood of the system remaining operational based on the reliability of the individual components and their interdependencies as captured in the fault tree structure [8, 9]. Furthermore, an effective tool for modeling and analyzing different reliability network properties is provided by signature theory, a crucial component of reliability theory [11,12]. To arrive at such a form, all MCS first have to be evaluated through an application of theory in probability and graphical analysis methodologies. Despite the inherent difficulty of this type of exercise, it is of great importance in formulating a deeper analysis of systems reliability and improving overall performance [13-16]. It belongs to a group of smart computing strategies shaped by random processes found in nature. Watching wherein sea lions hunt in changing seas gave rise to this technique. Quantities that are difficult to obtain analytically permitted to often be approximated using simulation techniques. Lately, figuring out if unusual or tricky structures are safe has gotten much better thanks to progress in structural reliability. A technique to tackle tough engineering challenges - like in what way stuff breaks when stressed over time - is called the Monte Carlo method. Another approach, born from natural patterns, is the Sea Lion Algorithm. Ideas and techniques there right now serve as solid tools for measuring risk without guesswork [14,6]. Because of that hunting mimicry, it handles complicated problems well. It is distinguished by its ability to achieve a good balance between the stages of exploration and exploitation, through movement mechanisms that depend on distance, speed of approach, and group interaction between individuals.

In the field of studying power flow and improving network reliability, several main analytical models have emerged that have proven their effectiveness, the most prominent of which are multipath network power flow model [7,10]. It is a comprehensive framework for simulating power distribution paths in complex networks, which helps in analyzing network resilience and determining optimal power flow paths. network reliability allocation optimization model it aims to achieve a balance between efficiency and cost, by improving system performance while taking into account available resources. Logarithmic model it is highly capable of modeling reliability in systems with decreasing failure rates over time, and is commonly used in systems with gradually improving performance. Exponential behavior model suitable for systems with accelerating or increasing failure patterns, and is used to represent failures whose probability of occurring increases with time or usage. A straight-line approach stands out because it works well without complexity, fitting situations where expenses and performance link in a steady way - ideal when rough predictions matter early on power grids. When approach into play, such become frameworks pieces key grasping structure in shaping design; every and reveals distinct version patterns wherein matching electricity setups various behave.

2. Multipath network model for power flow

Performance-aspects — stable output + reliable delivery — are much easier to evaluate when routes branch out into alternative branches. Multiple pathways allows supply to be redirected safely in the event of a line failure. These systems describe how energy travels from production sites to the local networks. Configurations of this kind help demonstrate how authentic blue framework devices work a minute ago. The route includes equipment like converters, substations and monitoring hubs. Sometimes engineers will represent these as directional diagrams. Multipath configurations allow one to visualize how electricity can travel through power grids. Each arrow is also tagged with usage regulations and live status when in use. In those drawings, dots represent plants or neighborhoods that received power. The arrows between them act as wires capable of passing current, until a certain point.

2.1. **Components of the model:** Nodes: $V_1, V_2, ..., V_8$. Represent intermediate points or modules (such as processors, switches, or components). Paths/Edges $Arc\, \varkappa_1, \varkappa_2, ..., \varkappa_{10}$.Represent links, channels, or power lines. $S \longrightarrow K$: the path from the source (S) to the target (K) via multiple paths.

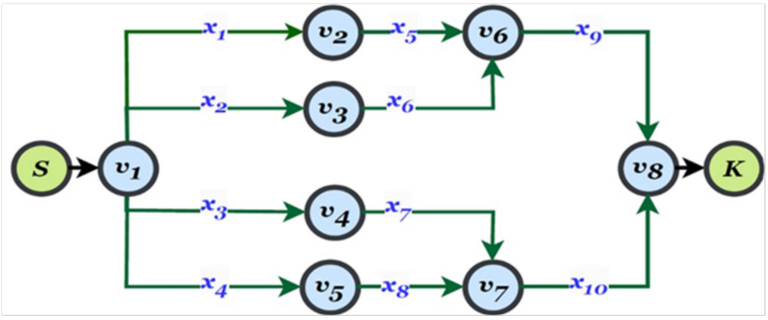

Figure 1. Complex Network

3. Optimization of reliability allocation for the network

Take a network of non-separable components ([1-3]). Now, depend on the following: R_i is greater than or equal to zero or less than or equal to one. Component reliability i; $C_i(R_i)$ costs and individual components i, $C\,(R_1, ..., R_n) = i = 1$ na $c_i\,(Ri)$ where parts' prices are $a_i > 0$. The system's effectiveness is determined by R_S, and the reliability of the devices' goal is represented by R_G. Each system component has a specific function, and there are many variations. Each device module has its a special set of features. Different levels of security are available for the same features. The goal is to ensure that every component of the device is reliable. The problem is a nonlinear threshold with a function and an evaluable cost [5,15]. Search

$$\min\,(R_1, ..., R_n) = \sum_{i=1}^{n} a_i C_i(R_i), \quad a_i > 0 \tag{1}$$

such that

$$R_G \le R_S, \qquad 0 \le R_i \le 1, \quad i = 1, ..., n.$$

Suppose that the function of cost is adequate. $C_i(R_i)$ meets all the requirements mentioned above. [2, 17]. The target of the previous technique was to achieve a comprehensive cost basis [17, 18]. The system stability limit was lowered. However, this falls within the scope of R_G. So, we can extract all minimal path sets of Fig. 1.

$MP_{S_1} = \{x_{1,2}x_{2,6}x_{6,8}\}, MP_{S_2} = \{x_{1,3}x_{3,6}x_{6,8}\}, MP_{S_3} = \{x_{1,4}x_{4,7}x_{7,8}\}, MP_{S_4} = \{x_{1,5}x_{5,7}x_{7,8}\}$

Take Figure 1, by applying (PTM), we get the reliability polynomial of the complex network as:

$$\begin{aligned} R_{SYSTEM} \;\; = \;\; & \varkappa_1\varkappa_5\varkappa_9 + \varkappa_2\varkappa_6\varkappa_9 - \varkappa_1\varkappa_2\varkappa_5\varkappa_6\varkappa_9 + \varkappa_3\varkappa_7\varkappa_{10} + \varkappa_4\varkappa_8\varkappa_{10} - \varkappa_3\varkappa_4\varkappa_7\varkappa_8\varkappa_{10} \\ & -\varkappa_1\varkappa_3\varkappa_5\varkappa_7\varkappa_9\varkappa_{10} - \varkappa_2\varkappa_3\varkappa_6\varkappa_7\varkappa_9\varkappa_{10} + \varkappa_1\varkappa_2\varkappa_3\varkappa_5\varkappa_6\varkappa_7\varkappa_9\varkappa_{10} - \varkappa_1\varkappa_4\varkappa_5\varkappa_8\varkappa_9\varkappa_{10} \\ & -\varkappa_2\varkappa_4\varkappa_6\varkappa_8\varkappa_9\varkappa_{10} + \varkappa_1\varkappa_2\varkappa_4\varkappa_5\varkappa_6\varkappa_8\varkappa_9\varkappa_{10} + \varkappa_1\varkappa_3\varkappa_4\varkappa_5\varkappa_7\varkappa_8\varkappa_9\varkappa_{10} \\ & +\varkappa_2\varkappa_3\varkappa_4\varkappa_6\varkappa_7\varkappa_8\varkappa_9\varkappa_{10} - \varkappa_1\varkappa_2\varkappa_3\varkappa_4\varkappa_5\varkappa_6\varkappa_7\varkappa_8\varkappa_9\varkappa_{10} \end{aligned} \tag{2}$$

For i.i.d reliability components, it becomes:

$$R_{SYSTEM} = 4\kappa^3 - 2\kappa^5 - 4\kappa^6 + 4\kappa^8 - \kappa^{10} \tag{3}$$

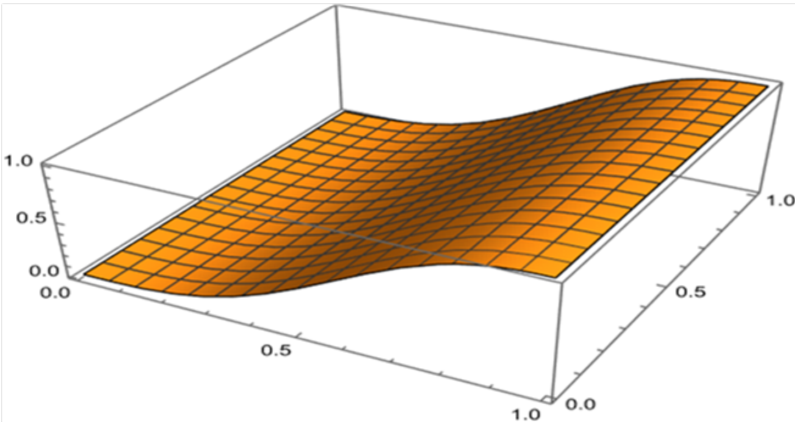

FIGURE 2. Reliability function

4. THE COST FUNCTIONS

4.1. **The Logarithmic Model.** The logarithmic model is a powerful tool in analyzing the reliability of software systems, especially those that undergo iterative development and improvement. It outperforms traditional models in representing a decreasing failure rate over time, but it is not suitable for systems with constant or increasing failure characteristics, or those that include physical components that are not repaired during operation.

R_i is greater than or equal to zero or less than one, and a_i $(i = 1, 2, ..., n)$ are constants. Considering that. [16, 19] proposed it in the following format.

$$C_i(R_i) = a_i \ln\left(\frac{1}{1 - R_i}\right), \quad a_i > 0, \quad (i = 1, \ldots, n.)$$

The optimization problem would become:

$$\text{Minimizing } C(R_1, \ldots, R_n) = \sum_{i=1}^{n} a_i \ln\left(\frac{1}{1 - R_i}\right), \quad (i = 1, \ldots, n.)$$

$$\text{Subject to: } R_s \geq R_G, \quad 0 \leq R_i < 1, \quad (i = 1, \ldots, n.)$$

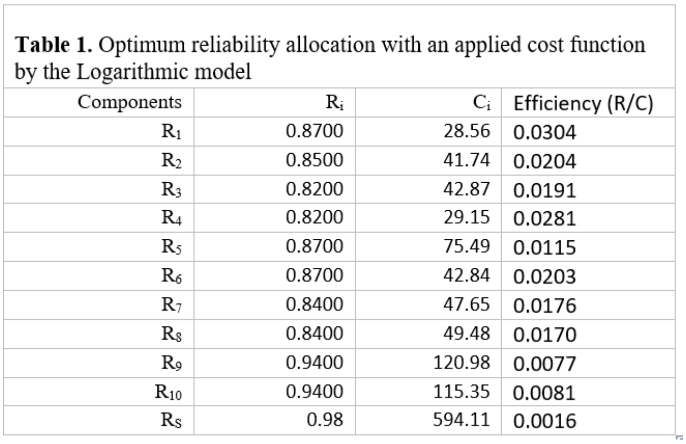

Table 1. Optimum reliability allocation with an applied cost function by the Logarithmic model

Components	R_i	C_i	Efficiency (R/C)
R_1	0.8700	28.56	0.0304
R_2	0.8500	41.74	0.0204
R_3	0.8200	42.87	0.0191
R_4	0.8200	29.15	0.0281
R_5	0.8700	75.49	0.0115
R_6	0.8700	42.84	0.0203
R_7	0.8400	47.65	0.0176
R_8	0.8400	49.48	0.0170
R_9	0.9400	120.98	0.0077
R_{10}	0.9400	115.35	0.0081
R_S	0.98	594.11	0.0016

R_1 and R_4 represent a good economic choice in terms of the balance between reliability and cost R_9 and R_{10} are excellent for critical systems that require the highest reliability, whatever the price. Good system reliability (0.98) can be achieved by using a smart mix of medium and high reliability components. Some components offer good reliability but without obvious economic efficiency compared to others in Figure 3.

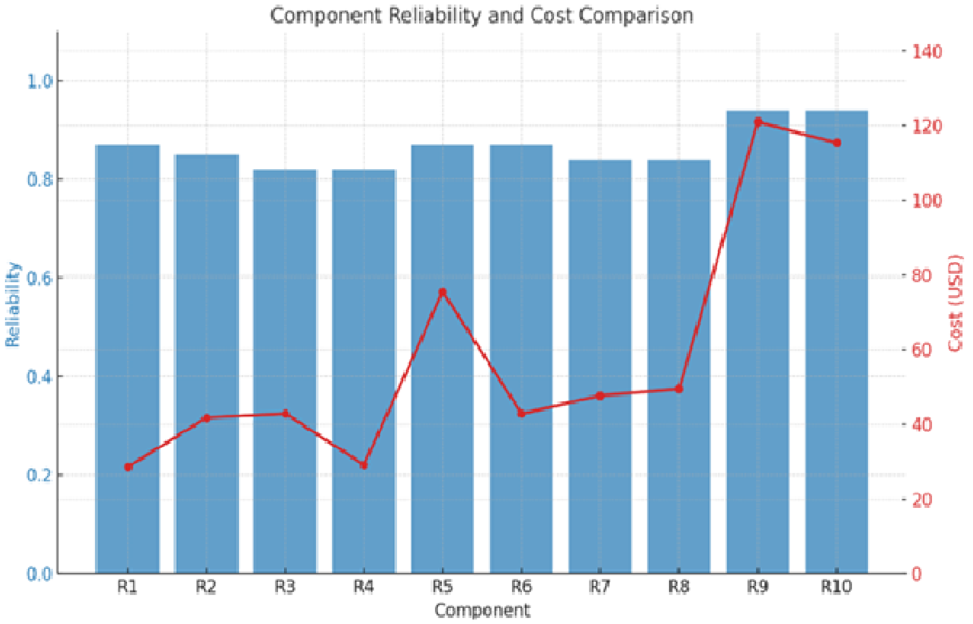

Figure 3. The reliability of each component and the cost by Logarithmic model behavior

4.2. **The Model of Exponential Behavior.** The exponential behavior model is the cornerstone of reliability analysis, suitable for stable systems, but too simplistic for systems that are subject to variations in performance or wear and tear, and is therefore only recommended for use in cases where the failure rate stability condition is met.

Let $0 < f_i < 1$ be the feasibility factor, $(R_{i,\min}, R_{i,\max})$ be lower and upper reliability intervals, respectively [3,4]. Exponential behavior is another important cost function

$$C_i\left(R_i\right) = \exp\left[\left(1-f_i\right)\frac{R_i - R_{i,\min}}{R_{i,\max} - R_i}\right], \qquad i = 1, 2, \ldots n.$$

Then the issue of optimization becomes

$$\min C\left(R_1, \ldots, R_n\right) = \sum_{i=1}^{n} a_i \exp\left[\left(1-f_i\right)\frac{R_i - R_{i,\min}}{R_{i,\max} - R_i}\right], \qquad i = 1, 2, \ldots n.$$

subject to $R_S \geq R_G$, $R_{i,\min} \leq R_i < R_{i,\max}$ $\qquad i = 1, 2, \ldots n.$

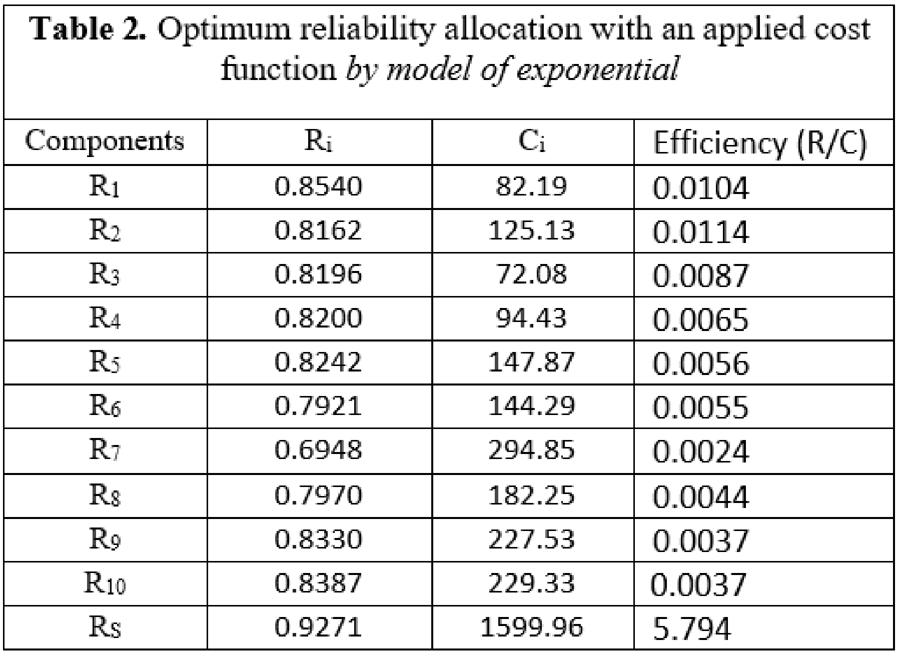

Table 2. Optimum reliability allocation with an applied cost function *by model of exponential*

Components	R_i	C_i	Efficiency (R/C)
R_1	0.8540	82.19	0.0104
R_2	0.8162	125.13	0.0114
R_3	0.8196	72.08	0.0087
R_4	0.8200	94.43	0.0065
R_5	0.8242	147.87	0.0056
R_6	0.7921	144.29	0.0055
R_7	0.6948	294.85	0.0024
R_8	0.7970	182.25	0.0044
R_9	0.8330	227.53	0.0037
R_{10}	0.8387	229.33	0.0037
R_S	0.9271	1599.96	5.794

The analysis shows that R_1 and R_2 offer the highest reliability at a relatively low cost, making them the optimal choice in systems seeking to balance performance versus cost. R_7, despite its high cost, has the lowest reliability, making it an inefficient choice in budget-conscious systems. System performance can be further optimized by selecting components with higher efficiency (R/C), without negatively impacting overall system reliability in Figure 4.

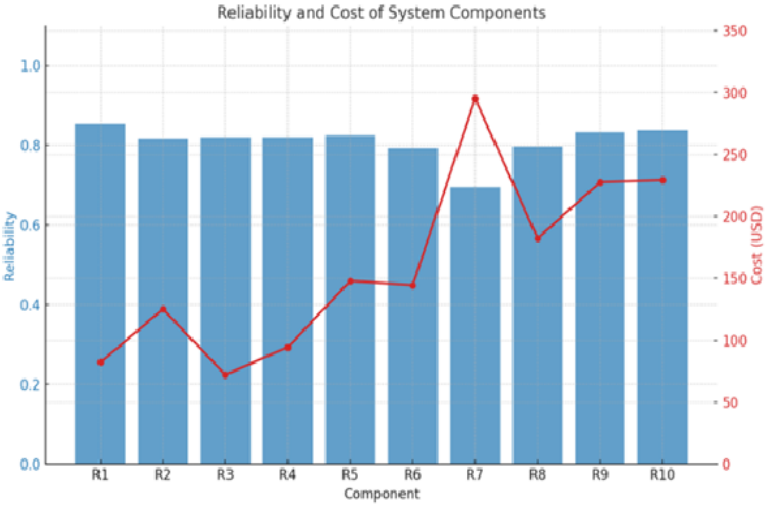

FIGURE 4. The reliability of each component and cost by model of exponential behavior

4.3. **The Linear Model.** Suppose that R_i is greater than or equal to zero or less than one of the $(R_{i,\min}, R_{i,\max})$ [3,7]. The linear model can be expressed in the following form:

$$C_i(R_i) = a_i R_i,\ a_i > 0, \qquad i = 1, 2, ...n.$$

The optimization problem then becomes:

$$\min C_i\left(R_1, ..., R_n\right) = \sum_{i=1}^{n} a_i R_i \qquad i = 1, 2, ...n,$$

subjet to

$$R_S \geq R_G, R_{i,\min} \leq R_i < R_{i,\max} \qquad i = 1, 2, ...n.$$

Table 3. Optimum reliability allocation with an applied cost function *by Linear Model*

Components	R_i	C_i	Efficiency (R/C)
R_1	0.8700	14.79	0.0588
R_2	0.8500	12.75	0.0667
R_3	0.8200	13.94	0.0588
R_4	0.8200	13.12	0.0625
R_5	0.8700	28.71	0.0303
R_6	0.8700	15.66	0.0556
R_7	0.8400	19.32	0.0435
R_8	0.8400	12.60	0.0667
R_9	0.9400	15.98	0.0588
R_{10}	0.9400	25.38	0.0370
R_S	0.98	172.25	0.0056

Table 3 shows that the the highest reliability was for the ninth and tenth components, and the least reliable was at the third and fourth, from which we obtained the best reliability of the system with this assignment, which was 0.98, with a total cost of 172.25 in Figure 5.

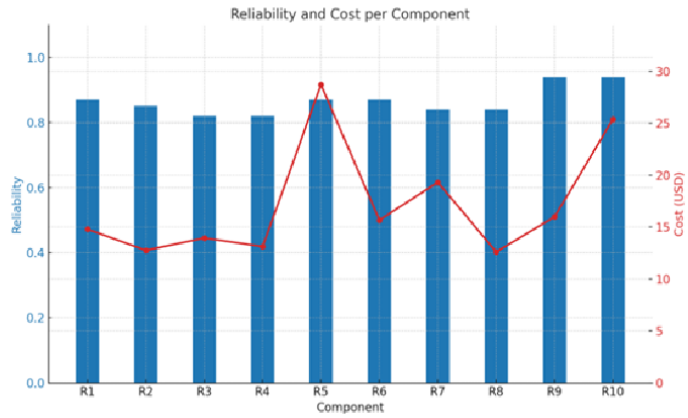

Figure 5. The reliability of each component and cost by Linear Model

5. CONCLUSION

This study concluded with an integrated analytical framework aimed at improving the reliability of complex electrical power networks through a multipath power flow model, based on the sea lion optimization algorithm and under the application of three different cost functions: logarithmic, exponential, and linear. Not far into the test, energy spread smoothly through several channels, cutting waste while allowing room to adapt during operation. When tested with a log-based formula, outcomes hit near-perfect dependability - 0.980 - with spending sitting around 594.11. Switching to an exponent-style method dropped reliability down to 0.927, yet price jumped sharply past 1500, fitting only where failure isn't tolerated and funds are loose. A straight-line approach matched the log version's trustworthiness, again scoring 0.980, though this time costing just over 172 - the cheapest by far, ideal when money is tight and tasks stay basic. Picking the right math rule clearly depends on what the system actually needs. Mixing smart search methods with multi-route designs turns out to be a solid path toward stronger, longer-lasting power setups.

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