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AN ANALYSIS ON $(([r_\alpha, r_\beta], r_\gamma), 2, ([k_\alpha, k_\beta], k_\gamma))$ - REGULAR AND TOTAL REGULAR CUBIC FUZZY GRAPHS

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ABSTRACT. This article introduces the concepts of $(([r_\alpha, r_\beta], r_\gamma), 2, ([k_\alpha, k_\beta], k_\gamma))$ - regular and total $(([r_\alpha, r_\beta], r_\gamma), 2, ([k_\alpha, k_\beta], k_\gamma))$ - regular cubic fuzzy graphs, extending fuzzy graph theory by applying regularity conditions to both vertex and edge membership functions. The study defines and analyzes the vertex d_2 - degree and Total d_2 - degree in cubic fuzzy graphs, including $(([r_\alpha, r_\beta], r_\gamma), 2, ([k_\alpha, k_\beta], k_\gamma))$ - regularity and Total $(([r_\alpha, r_\beta], r_\gamma), 2, ([k_\alpha, k_\beta], k_\gamma))$ - regularity to determine the conditions under which a cubic fuzzy graph is regular. Furthermore, $(([r_\alpha, r_\beta], r_\gamma), 2, ([k_\alpha, k_\beta], k_\gamma))$ - regularity is examined in cyclic cubic fuzzy graphs with specific membership values. Our results formalize new regularity conditions in cubic fuzzy graphs, providing a deeper understanding of their properties. The study further explores the regularity properties of cubic fuzzy graphs, providing new tools for potential applications in fields such as network analysis, decision-making and related fields.

Keywords: Degree of a vertex in Fuzzy Graphs, Total degree of a vertex in Fuzzy Graphs, Regular Fuzzy Graphs, Total Regular Fuzzy Graphs, Cubic Fuzzy Graphs.

AMS Subject Classification: 05C72, 03E72.

1. Introduction

Fuzzy subsets is a mathematical concept that extends the idea of classical sets by allowing degrees of membership rather than binary membership, which simply categorizes the elements as members or non-members. This concept was Introduced by Lotfi A. Zadeh in 1965 [1]. His work on fuzzy sets revolutionized mathematics and computer science by enabling the development of more nuanced and realistic models of complex systems. Zadeh

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extended his work on fuzzy algorithms In 1968 [2]. Fuzzy techniques have been applied in many areas, including artificial intelligence, decision-making, control systems and image processing. In 1971, Zadeh et al. introduced fuzzy operators, similarity relations, fuzzy orderings and fuzzy inference systems [3]. Fuzzy logic emerged as a systematic approach to uncertain reasoning. Later, in 1975, this concept was further enriched by the development of interval-valued fuzzy sets, building on Zadeh's fuzzy set theory to better handle uncertainty and imprecision in real-world applications [5]. While fuzzy sets allowed elements to have single membership values (between 0 and 1), they sometimes failed to adequately represent vagueness. This limitation led researchers to merge fuzzy sets with graph theory to create fuzzy graphs.

Fuzzy graphs are mathematical frameworks that enhance traditional graphs in modeling real-world problems. Numerous researchers have significantly advanced fuzzy graph theory. Notably, Arnold Kaufmann introduced the foundational concept of fuzzy graphs in 1973, utilizing fuzzy relations on fuzzy sets [4]. Building on this work, Azriel Rosenfeld further expanded the field by defining several fuzzy graph parameters in 1975 [6]. Classical Graph Theory with applications by Bondy and Murty [7] and the contribution towards Graph Theory by Balakrishnan and Ranganathan [9], provide essential groundwork in 1991. Early developments in fuzzy Graph theory were driven by Bhattacharya, Moderson and Nair in 2000 [8, 10] . Subsequently, Nagoor Gani and Radha explored regular fuzzy graphs in 2008 [11], with comprehensive methods available in works those of Mathew et al. [17]. Furthermore, Huda Al Mutab introduced additional developments fuzzy graphs in 2019 [19]. Therefore, researchers continue to show strong interest in various areas of research within this field.

In the mathematical field of graph theory, a cubic graph is defined as a 3-regular or trivalent graph, where every vertex has exactly three edges. However, the concept of cubic sets was introduced and investigated by three Korean mathematicians, Jun et al., in 2012 [12]. The development of cubic sets and their applications has been discussed in various studies [23, 25]. They recognized that traditional fuzzy sets and intuitionistic fuzzy sets had limitations in representing complex uncertainties. Their innovation led to the development of cubic fuzzy sets, designed to capture multi-dimensional uncertainty. The concept cubic fuzzy sets was first systematically studied in the late 2000s and early 2010s, with researchers extending classical fuzzy logic applications into this more complex domain.

After the theory of cubic fuzzy graphs was built upon cubic fuzzy sets, researchers discovered that these graphs essentially merge fuzzy graph theory with cubic structures, where each vertex can have varying degrees of membership, but can be specifically applied to structures analogous to traditional cubic graphs. This innovative framework proves highly effective in modeling intricate networks plagued by uncertainty, including social networks, transportation systems and biological networks. The concept of cubic fuzzy graphs emerged as a natural extension of intuitionistic fuzzy graphs and interval-valued fuzzy graphs, enabling enhanced uncertainty modeling. Rashid et al. introduced the concept of cubic fuzzy graphs, where they introduced many new types of graphs and their applications [18]. Furthermore, Muhiuddin et al. proposed a modified definition of cubic fuzzy graphs, introducing key concepts such as strong edges, paths and bridges, as well as path strength and cut vertices [21]. Innovations such as picture fuzzy graphs [28, 35], cubic Pythagorean fuzzy graphs in 2022 [26] and cubic fuzzy graphs with vertex regularity in 2023 [30] have expanded the theoretical landscape. Several studies [20, 27, 32] focused on structural properties and novel applications, including traffic flow [21], airline routing [24], chemical modeling [31], tsunami threat zones [39] and social network analysis [40]. Kosari et al. proposed Topological indices in fuzzy graphs and its applications [37]. Optimization and decision-making techniques using fuzzy graphs have also been investigated [33, 34, 38], with recent contributions to domination integrating sets [42] and Zagreb indices in multi- attribute decision-making in 2025 [43], highlighting the growing practical relevance of fuzzy and cubic fuzzy graph models across disciplines.

Santhi Maheswari and Sekar introduced $(2, k)$-regular fuzzy graph and totally $(2, k)$-regular fuzzy graphs in 2014 [13] and later generalized the concept to $(r, 2, k)$ - regular fuzzy graphs in 2016 [15]. Additionally, Ravi Narayanan and Murugesan introduced the $((r_1, r_2), 2, (c_1, c_2))$ - pseudo regular intuitionistic fuzzy graphs in 2016 [16]. This has motivated us to work on analyzing the regularity in cubic fuzzy graphs. In this article, the concepts $(([r_\alpha, r_\beta], r_\gamma), 2, ([k_\alpha, k_\beta], k_\gamma))$ - regular cubic fuzzy graphs and totally $(([r_\alpha, r_\beta], r_\gamma), 2, ([k_\alpha, k_\beta], k_\gamma))$ - regular cubic fuzzy graphs are newly introduced. This study defines the vertex d_2 - degree and total vertex d_2 - degree in cubic fuzzy graphs and investigates the $(([r_\alpha, r_\beta], r_\gamma), 2, ([k_\alpha, k_\beta], k_\gamma))$ - regularity properties of specific cubic fuzzy graphs, particularly those with underlying cycle structures, using specific membership functions. The results of this paper can be used to model and analyze complex networks with multi-faceted uncertainty.

The following abbreviations are used throughout this article

Notations	**Index of symbols**
$\mathcal{G} = (\mathcal{P}, \mathcal{C})$	A graph $\mathcal{G}$ with vertex set $\mathcal{P}$ and edge set $\mathcal{C}$
$\mathcal{C}^* = (\mathcal{L}, \mathcal{M})$	A Cubic fuzzy graph $\mathcal{C}^*$ with vertex set $\mathcal{L}$ and edge set $\mathcal{M}$
$\mathcal{L} = \alpha, \beta$	Interval-valued fuzzy membership value
$\mathcal{M} = \gamma$	Fuzzy membership value
$d_2(\hat{u})$	vertex d_2 - degree
$td_2(\hat{u})$	total vertex d_2 - degree
CFG	Cubic Fuzzy Graph
IVF	Interval-valued fuzzy membership

Table 1. Some abbreviations

2. Basic Definitions

Definition 2.1 (7). *A Graph $\mathcal{G} = (\mathcal{P}, \mathcal{C})$, where $\mathcal{P}$ is a finite set of distinct elements called vertices or points and $\mathcal{C}$ is a collection of unordered pairs $(\hat{u}, \hat{v})$ of distinct elements from $\mathcal{P}$, called edges or connections. $\mathcal{P}$ is denoted by $\mathcal{P}(\mathcal{G})$ and $\mathcal{C}$ by $\mathcal{C}(\mathcal{G})$.*

Definition 2.2 (10). *A fuzzy Graph $\mathcal{G} = (\sigma, \mu)$ is represented as a pair of functions, where $\sigma : \mathcal{P} \to [0, 1]$ is a fuzzy subset defined on a non-empty set $\mathcal{P}$ and $\mu : \mathcal{P} \times \mathcal{P} \to [0, 1]$ is a symmetric fuzzy relation on $\mathcal{P}$, such that forall $\hat{u}, \hat{v} \in \mathcal{P}$, the value of $\mu(\hat{u}, \hat{v}) \leq \sigma(\hat{u}) \wedge \sigma(\hat{v})$.*

Definition 2.3 (20). *An Interval-Valued Fuzzy Set $\mathcal{L}$ on a set $\mathcal{P}$ is formally defined as*

$$L = \{[\alpha(\hat{u}), \beta(\hat{u})]/\hat{u} \in \mathcal{P}\}$$

where α and β are two fuzzy subsets of $\mathcal{P}$, such that $\alpha(\hat{u}) \leq \beta(\hat{u}), \forall \hat{u} \in \mathcal{P}$.

Definition 2.4 (12). *Let $\mathcal{U}$ be a non-empty set. A Cubic Set is a structure of the form*

$$\mathcal{C} = \{(\hat{u}, \mathcal{L}(\hat{u}), \mathcal{M}(\hat{u}))/\hat{u} \in \mathcal{U}\}$$

where $\mathcal{L}(\hat{u})$ represents an interval valued fuzzy subset of $\mathcal{U}$ and $\mathcal{M}(\hat{u})$ denotes a fuzzy subset of $\mathcal{U}$.

Definition 2.5 (12). *A Cubic Fuzzy Set in $\mathcal{P}$ is defined as*

$$\mathcal{L} = \{([\alpha(\hat{u}), \beta(\hat{u})], \gamma(\hat{u}))/\hat{u} \in \mathcal{P}\}$$

where $[\alpha(\hat{u}), \beta(\hat{u})]$ represents the Interval-Valued Fuzzy (IVF) membership value and $\gamma(\hat{u})$ represents the Fuzzy (F) membership value of $\hat{u}$, with $\alpha, \beta, \gamma : \mathcal{P} \to [0,1]$. A Cubic Fuzzy Set is classified as internal if $\gamma(\hat{u}) \in [\alpha(\hat{u}), \beta(\hat{u})]$ and as external if $\gamma(\hat{u}) \notin [\alpha(\hat{u}), \beta(\hat{u})], \forall \hat{u} \in \mathcal{P}$.

Definition 2.6 (21). *A Cubic Fuzzy Graph on $\mathcal{G} = (\mathcal{P}, \mathcal{C})$ consists of paired functions $\mathcal{C}^* = (\mathcal{L}, \mathcal{M})$, where $\mathcal{L} = ([\alpha_1, \beta_1], \gamma_1)$, such that $[\alpha_1, \beta_1] : \mathcal{P} \to [0,1]$ and $\gamma_1 : \mathcal{P} \to [0,1]$ is a Cubic Fuzzy Set on the vertex set $\mathcal{P}$ and $\mathcal{M} = ([\alpha_2, \beta_2], \gamma_2)$ such that $[\alpha_2, \beta_2] : \mathcal{C} \to [0,1]$ and $\gamma_2 : \mathcal{C} \to [0,1]$ is a Cubic Fuzzy Set on the edge set $\mathcal{C}$, satisfying the following conditions:*

$$\alpha_2(\hat{u}, \hat{v}) \le min\{\alpha_1(\hat{u}), \alpha_1(\hat{v})\}, \forall (\hat{u}, \hat{v}) \in \mathcal{C}$$
$$\beta_2(\hat{u}, \hat{v}) \le min\{\beta_1(\hat{u}), \beta_1(\hat{v})\}, \forall (\hat{u}, \hat{v}) \in \mathcal{C}$$
$$\gamma_2(\hat{u}, \hat{v}) \le min\{\gamma_1(\hat{u}), \gamma_1(\hat{v})\}, \forall (\hat{u}, \hat{v}) \in \mathcal{C}$$

Definition 2.7 (10). *In a fuzzy graph $\mathcal{G} = (\sigma, \mu)$ on $\mathcal{G} = (\mathcal{P}, \mathcal{C})$, the connectedness strength between vertices $\hat{u}$ and $\hat{v}$ is defined as*

$$\mu^\infty(\hat{u}, \hat{v}) = sup\{\mu(\hat{u}, \hat{u}_1) \wedge \mu(\hat{u}_1, \hat{u}_2) \wedge ... \wedge \mu(\hat{u}_{n-1}, \hat{v}) : (\hat{u}, \hat{u}_1, \hat{u}_2, ..., \hat{u}_{n-1}, \hat{v})$$

where the supremum is taken over shortest two-length path between $\hat{u}$ and $\hat{v}\}$.

Definition 2.8 (10). *Let $\mathcal{G} = (\sigma, \mu)$ be derived from the crisp graph $\mathcal{G} = (\mathcal{P}, \mathcal{C})$, a vertex degree $\hat{u}$ in $\mathcal{G}$ is denoted by $d(\hat{u})$ and is defined as $d(\hat{u}) = \sum \mu(\hat{u}, \hat{v}), \forall (\hat{u}, \hat{v}) \in \mathcal{C}$ and $d(\hat{u}) = 0, \forall (\hat{u}, \hat{v}) \notin \mathcal{C}$.*

Definition 2.9 (13). *Let $\mathcal{G} = (\sigma, \mu)$ be a fuzzy graph on $\mathcal{G} = (\mathcal{P}, \mathcal{C})$, a vertex d_2 - degree $\hat{u}$ in $\mathcal{G}$ is $d_2(\hat{u}) = \sum \mu^2(\hat{u}, \hat{v})$, where $\mu^2(\hat{u}, \hat{v}) = sup\{\mu(\hat{u}, \hat{u}_1) \wedge \mu(\hat{u}_1, \hat{v}) : (\hat{u}, \hat{u}_1, \hat{v})$ is the shortest two-length path between $\hat{u}$ and $\hat{v}$ \}. Also, $\mu^2(\hat{u}, \hat{v}) = 0, \forall (\hat{u}, \hat{v}) \notin \mathcal{C}$.*

Minimum d_2-degree of $\mathcal{G}$ is $\delta_2(\mathcal{G}) = min\{d_2(\hat{v}) : \hat{v} \in \mathcal{P}\}$

Maximum d_2-degree of $\mathcal{G}$ is $\Delta_2(\mathcal{G}) = max\{d_2(\hat{v}) : \hat{v} \in \mathcal{P}\}$

Definition 2.10 (11). *Let $\mathcal{G} = (\sigma, \mu)$ be a fuzzy graph on $\mathcal{G} = (\mathcal{P}, \mathcal{C})$, a total vertex degree $\tilde{u}$ is denoted by $td(\hat{u})$ and is defined as $td(\hat{u}) = \sum \mu(\hat{u}, \hat{v}) + \sigma(\hat{u}), \forall (\hat{u}, \hat{v}) \in \mathcal{C}$. It can also defined as $td(\hat{u}) = d(\hat{u}) + \sigma(\hat{u})$.*

Definition 2.11 (13). *Let $\mathcal{G} = (\sigma, \mu)$ be a fuzzy graph on $\mathcal{G} = (\mathcal{P}, \mathcal{E})$, a total vertex d_2 - degree $\hat{u} \in \mathcal{P}$ is defined as $td_2(\hat{u}) = \sum \mu^2(\hat{u}, \hat{v}) + \sigma(\hat{u}) = d_2(\hat{u}) + \sigma(\hat{u})$.*

Minimum td_2-degree of $\mathcal{G}$ is $t\delta_2(\mathcal{G}) = min\{td_2(\hat{v}) : \hat{v} \in \mathcal{P}\}$

Maximum td_2-degree of $\mathcal{G}$ is $t\Delta_2(\mathcal{G}) = max\{td_2(\hat{v}) : \hat{v} \in \mathcal{P}\}$

Definition 2.12 (15). *If every vertex of $\mathcal{G}$ has an equal degree r and a d_2 - degee k, then $\mathcal{G}$ is said to be a $(r, 2, k)$ - Regular Fuzzy Graph.*

Definition 2.13 (15). *If every vertex in $\mathcal{G}$ possesses an identical total degree r and total d_2 - degree k, then $\mathcal{G}$ is said to be a Totally $(r, 2, k)$ - Regular Fuzzy Graph.*

Definition 2.14 (7). *The distance $d(\hat{u}, \hat{v})$ between two vertices $\hat{u}$ and $\hat{v}$ is the length of a shortest $(\hat{u}, \hat{v})$- path.*

Definition 2.15 (7). *A graph $\mathcal{G}$ contains a cycle C_n of length n, defined as a vertex sequence $(\hat{u}_0, \hat{u}_1, \hat{u}_2, ..., \hat{u}_{n-1}, \hat{u}_0)$, such that for $1 \le i \le n$ - 2, the vertices $\hat{u}_i$ and $\hat{u}_{i+1}$ are adjacent; $\hat{u}_{n-1}$ and $\hat{u}_0$ are also adjacent and $\hat{u}_0, \hat{u}_1, \hat{u}_2, ..., \hat{u}_{n-1}$ are distinct.*

A cycle C_n is classified as an even cycle if n is even and an odd cycle if n is odd.

3. A vertex's d_2 - degree and Total d_2 - degree in a Cubic Fuzzy Graph

Definition 3.1. *In a Cubic Fuzzy Graph $\mathcal{C}^* : (\mathcal{L}, \mathcal{M})$ on a graph $\mathcal{G} : (\mathcal{P}, \mathcal{C})$, a vertex degree $\hat{u}$ in $\mathcal{C}^*$ is an interval-valued fuzzy membership number, such that $d_{[\alpha,\beta]}(\hat{u}) = \sum[\alpha_1, \beta_1](\hat{u}, \hat{v}), \forall(\hat{u}, \hat{v}) \in \mathcal{C}$ and also $d_{[\alpha,\beta]}(\hat{u}) = 0, \forall(\hat{u}, \hat{v}) \notin \mathcal{C}$. Similarly, a vertex degree $\hat{u}$ in $\mathcal{C}^*$ is a fuzzy membership number, such that $d_{[\gamma]}(\hat{u}) = \sum[\gamma_1](\hat{u}, \hat{v}), \forall(\hat{u}, \hat{v}) \in \mathcal{C}$ and also $d_{[\gamma]}(\hat{u}) = 0, \forall(\hat{u}, \hat{v}) \notin \mathcal{C}$. Therefore, a vertex degree in a Cubic Fuzzy Graph $\mathcal{C}^*$ is defined as $d(\hat{u}) = (d_{[\alpha,\beta]}(\hat{u}), d_{[\gamma]}(\hat{u}))$.*

Definition 3.2. *In a Cubic Fuzzy Graph $\mathcal{C}^* : (\mathcal{L}, \mathcal{M})$ on a graph $\mathcal{G} : (\mathcal{P}, \mathcal{C})$, a vertex d_2 - degree $\hat{u}$ in $\mathcal{C}^*$ is an interval-valued fuzzy membership number, such that $d_{2[\alpha,\beta]}(\hat{u}) = \sum[\alpha_1, \beta_1]^2(\hat{u}, \hat{v})$, where $[\alpha_1, \beta_1]^2(\hat{u}, \hat{v}) = sup\{\alpha_1, \beta_1(\hat{u}, \hat{u}_1) \wedge \alpha_1, \beta_1(\hat{u}_1, \hat{v}) : (\hat{u}, \hat{u}_1, \hat{v})$ is the shortest two-length path between $\hat{u}$ and $\hat{v}\}$ and also $d_{2[\alpha,\beta]}(\hat{u}) = 0, \forall(\hat{u}, \hat{v}) \notin \mathcal{C}$. Alternatively, a vertex d_2 - degree $\hat{u}$ in $\mathcal{C}^*$ is a fuzzy membership number, such that $d_{2[\gamma]}(\hat{u}) = \sum[\gamma_1]^2(\hat{u}, \hat{v})$, where $[\gamma_1]^2(\hat{u}, \hat{v}) = sup\{\gamma_1(\hat{u}, \hat{u}_1) \wedge \gamma_1(\hat{u}_1, \hat{v}) : (\hat{u}, \hat{u}_1, \hat{v})$ is the shortest two-length path between $\hat{u}$ and $\hat{v}\}$ and also $d_{2[\gamma]}(\hat{u}) = 0, \forall(\hat{u}, \hat{v}) \notin \mathcal{C}$. Thus, a vertex d_2 - degree in a Cubic Fuzzy Graph $\mathcal{C}^*$ is defined as $d_2(\hat{u}) = (d_{2[\alpha,\beta]}(\hat{u}), d_{2[\gamma]}(\hat{u}))$.*

Minimum d_2-degree of a CFG $\mathcal{C}^*$ is $\delta_2(\mathcal{C}^*) = min\{d_{2([\alpha,\beta],\gamma)}(\hat{u}) : \hat{u} \in \mathcal{P}\}$

Maximum d_2-degree of a CFG $\mathcal{C}^*$ is $\Delta_2(\mathcal{C}^*) = max\{d_{2([\alpha,\beta],\gamma)}(\hat{u}) : \hat{u} \in \mathcal{P}\}$

Example 3.1. *Consider a Fuzzy Graph $\mathcal{G} : (\mathcal{P}, \mathcal{C})$ with $\mathcal{P} = \{\hat{u}, \hat{v}, \hat{w}, \hat{x}, \hat{y}, \hat{z}\}$ and $\mathcal{C} = \{\hat{u}\hat{v}, \hat{v}\hat{w}, \hat{w}\hat{x}, \hat{w}\hat{z}, \hat{x}\hat{y}, \hat{y}\hat{u}\}$ is defined a Cubic Fuzzy Graph $\mathcal{C}^* : (\mathcal{L}, \mathcal{M})$. $\mathcal{L}(\hat{u}) = ([0.4, 0.5], 0.7)$, $\mathcal{L}(\hat{v}) = ([0.4, 0.6], 0.8), \mathcal{L}(\hat{w}) = ([0.4, 0.6], 0.7), \mathcal{L}(\hat{x}) = ([0.4, 0.6], 0.8), \mathcal{L}(\hat{y}) = ([0.4, 0.5], 0.4)$, $\mathcal{L}(\hat{z}) = ([0.3, 0.6], 0.5)$ and $\mathcal{M}(\hat{u}\hat{v}) = ([0.1, 0.4], 0.6), \mathcal{M}(\hat{w}) = ([0.3, 0.5], 0.6), \mathcal{M}(\hat{w}\hat{x}) = ([0.4, 0.6], 0.7), \mathcal{M}(\hat{w}\hat{z}) = ([0.2, 0.5], 0.4), \mathcal{M}(\hat{x}\hat{y}) = ([0.2, 0.3], 0.1), \mathcal{M}(\hat{y}\hat{u}) = ([0.2, 0.5], 0.5)$*

$d(\hat{u}) = ([0.2, 0.5], 0.4) + ([0.1, 0.4], 0.6) = ([0.3, 0.9], 1.0)$.
Similarly, $d(\hat{v}) = ([0.7, 1.2], 1.4), d(\hat{w}) = ([0.9, 1.6], 1.7), d(\hat{x}) = ([0.6, 0.9], 0.8), d(\hat{y}) = ([0.7, 1.1], 0.7)$ and $d(\hat{z}) = ([0.2, 0.5], 0.4)$

$$d_2(\hat{u}) = ([0.1 \wedge 0.3 + 0.2 \wedge 0.2, 0.4 \wedge 0.5 + 0.5 \wedge 0.3], 0.6 \wedge 0.6 + 0.4 \wedge 0.1)$$
$$= ([0.1 + 0.2, 0.4 + 0.3], 0.6 + 0.1) = ([0.3, 0.7], 0.7)$$

Similarly, $d_2(\hat{v}) = ([0.5, 1.0], 1.0), d_2(\hat{w}) = ([0.4, 0.7], 0.8), d_2(\hat{x}) = ([0.7, 1.3], 1.1), d_2(\hat{y}) = ([0.3, 0.3], 0.2)$ and $d_2(\hat{z}) = ([0.4, 1.0], 0.8)$.

Definition 3.3. *In a Cubic Fuzzy Graph $\mathcal{C}^* : (\mathcal{L}, \mathcal{M})$ on a graph $\mathcal{G} : (\mathcal{P}, \mathcal{C})$, a total vertex degree $\hat{u} \in \mathcal{P}$ is defined as $td(\hat{u}) = (td_{[\alpha,\beta]}(\hat{u}), td_{[\gamma]}(\hat{u}))$, where $td_{[\alpha,\beta]}(\hat{u}) = \sum[\alpha_1, \beta_1](\hat{u}, \hat{v}) + [\alpha, \beta](\hat{u}), \forall(\hat{u}, \hat{v}) \in \mathcal{C}$ and $td_{[\gamma]}(\hat{u}) = \sum[\gamma_1](\hat{u}, \hat{v}) + [\gamma](\hat{u}), \forall(\hat{u}, \hat{v}) \in \mathcal{C}$. Alternatively, a total vertex degree can also be expressed as $td(\hat{u}) = d(\hat{u}) + \mathcal{L}(\hat{u})$, where $\mathcal{L}(\hat{u}) = ([\alpha, \beta](\hat{u}), [\gamma](\hat{u}))$.*

Definition 3.4. *In a Cubic Fuzzy Graph $\mathcal{C}^* : (\mathcal{L}, \mathcal{M})$ on a graph $\mathcal{G} : (\mathcal{P}, \mathcal{C})$, a total vertex d_2 - degree $\hat{u} \in \mathcal{P}$ is defined as $td_2(\hat{u}) = (td_{2[\alpha,\beta]}(\hat{u}), td_{2[\gamma]}(\hat{u}))$, where $td_{2[\alpha,\beta]}(\hat{u}) = \sum[\alpha_1, \beta_1]^2(\hat{u}, \hat{v}) + [\alpha, \beta](\hat{u})$ and $td_{2[\gamma]}(\hat{u}) = \sum[\gamma_1]^2(\hat{u}, \hat{v}) + [\gamma](\hat{u})$. Alternatively, a total vertex d_2 - degree can also be expressed as $td_2(\hat{u}) = d_2(\hat{u}) + \mathcal{L}(\hat{u})$, where $\mathcal{L}(\hat{u}) = ([\alpha, \beta](\hat{u}), [\gamma](\hat{u}))$.*

Minimum td_2-degree of a CFG $\mathcal{C}^*$ is $t\delta_2(\mathcal{C}^*) = min\{td_{2([\alpha,\beta],\gamma)}(\hat{u}) : \hat{u} \in \mathcal{P}\}$

Maximum td_2-degree of a CFG $\mathcal{C}^*$ is $t\Delta_2(\mathcal{C}^*) = max\{td_{2([\alpha,\beta],\gamma)}(\hat{u}) : \hat{u} \in \mathcal{P}\}$

Example 3.2. *Consider a Fuzzy Graph $\mathcal{G} : (\mathcal{P}, \mathcal{C})$ with $\mathcal{P} = \{\hat{u}, \hat{v}, \hat{w}, \hat{x}\}$ and $\mathcal{C} = \{\hat{u}\hat{v}, \hat{v}\hat{w}, \hat{w}\hat{x}, \hat{x}\hat{u}\}$ is defined a Cubic Fuzzy Graph $\mathcal{C}^* : (\mathcal{L}, \mathcal{M})$. $\mathcal{L}(\hat{u}) = ([0.4, 0.5], 0.6), \mathcal{L}(\hat{v}) = ([0.5, 0.6], 0.7), \mathcal{L}(\hat{w}) = ([0.6, 0.8], 0.6), \mathcal{L}(\hat{x}) = ([0.5, 0.5], 0.9)$ and $\mathcal{M}(\hat{u}\hat{v}) = ([0.4, 0.5], 0.5)$, $\mathcal{M}(\hat{v}\hat{w}) = ([0.2, 0.3], 0.5), \mathcal{M}(\hat{w}\hat{x}) = ([0.3, 0.4], 0.6), \mathcal{M}(\hat{x}\hat{u}) = ([0.3, 0.3], 0.6)$*

$d(\hat{u}) = ([0.7, 0.8], 1.1), d(\hat{v}) = ([0.6, 0.8], 1.0), d(\hat{w}) = ([0.5, 0.7], 1.1)$ *and* $d(\hat{x}) = ([0.6, 0.7], 1.2)$
Then, $td(\hat{u}) = ([0.7, 0.8], 1.1) + ([0.4, 0.5], 0.6) = ([1.1, 1.3], 1.7)$
Similarly, $td(\hat{v}) = ([1.1, 1.4], 1.7), td(\hat{w}) = ([1.1, 1.5], 1.7)$ *and* $td(\hat{x}) = ([1.1, 1.2], 2.1)$
$d_{2\alpha}(\hat{u}) = sup\{0.4 \wedge 0.2, 0.3 \wedge 0.3\} = sup\{0.2, 0.3\} = 0.3$
$d_{2\beta}(\hat{u}) = sup\{0.5 \wedge 0.3, 0.3 \wedge 0.4\} = sup\{0.3, 0.3\} = 0.3$
$d_{2\gamma}(\hat{u}) = sup\{0.5 \wedge 0.5, 0.6 \wedge 0.6\} = sup\{0.5, 0.6\} = 0.6$
So, $d_2(\hat{u}) = ([0.3, 0.3], 0.6)$
Similarly, $d_2(\hat{v}) = ([0.3, 0.3], 0.5), d_2(\hat{w}) = ([0.3, 0.3], 0.6),$ $d_2(\hat{x}) = ([0.3, 0.3], 0.5)$ *and* $td_2(\hat{u}) = ([0.7, 0.8], 1.2), td_2(\hat{v}) = ([0.8, 0.9], 1.2), td_2(\hat{w}) = ([0.9, 1.1], 1.2),$ $td_2(\hat{x}) = ([0.8, 0.8], 1.4)$

4. $(([r_\alpha, r_\beta], r_\gamma), 2, ([k_\alpha, k_\beta], k_\gamma))$ - Regular Cubic Fuzzy Graphs

Definition 4.1. *In a Cubic Fuzzy Graph* $\mathcal{C}^* : (\mathcal{L}, \mathcal{M})$ *on a graph* $\mathcal{G} : (\mathcal{P}, \mathcal{C})$*, if every vertex* $\hat{u}$ *in* $\mathcal{P}$ *has an equal degree* $([r_\alpha, r_\beta], r_\gamma)$ *and a* d_2 *- degree* $([k_\alpha, k_\beta], k_\gamma)$*, then* $\mathcal{C}^*$ *is termed a* $(([r_\alpha, r_\beta], r_\gamma), 2, ([k_\alpha, k_\beta], k_\gamma))$ *- Regular Cubic Fuzzy Graph.*

Example 4.1. *Consider a graph* $\mathcal{G} : (\mathcal{P}, \mathcal{C})$ *with* $\mathcal{P} = \{\hat{u}_1, \hat{u}_2, \hat{u}_3, \hat{u}_4, \hat{u}_5, \hat{u}_6, \hat{u}_7, \hat{u}_8\}$ *and* $\mathcal{C} = \{\hat{u}_1\hat{u}_2, \hat{u}_2\hat{u}_3, \hat{u}_3\hat{u}_4, \hat{u}_4\hat{u}_5, \hat{u}_5\hat{u}_6, \hat{u}_6\hat{u}_7, \hat{u}_7\hat{u}_8, \hat{u}_8\hat{u}_1\}$ *is defined a Cubic Fuzzy Graph* $\mathcal{C}^* : (\mathcal{L}, \mathcal{M})$.
$\mathcal{L}(\hat{u}_1) = ([0.3, 0.7], 0.8), \mathcal{L}(\hat{u}_2) = ([0.4, 0.7], 0.9), \mathcal{L}(\hat{u}_3) = ([0.5, 0.6], 1.0), \mathcal{L}(\hat{u}_4) = ([0.3, 0.7], 0.2), \mathcal{L}(\hat{u}_5) = ([0.4, 0.6], 0.4), \mathcal{L}(\hat{u}_6) = ([0.5, 0.7], 0.5), \mathcal{L}(\hat{u}_7) = ([0.4, 0.6], 0.8), \mathcal{L}(\hat{u}_8) = ([0.3, 0.7], 1.0)$ *and* $\mathcal{M}(\hat{u}_1\hat{u}_2) = ([0.2, 0.6], 0.4), \mathcal{M}(\hat{u}_2\hat{u}_3) = ([0.3, 0.5], 0.7), \mathcal{M}(\hat{u}_3\hat{u}_4) = ([0.2, 0.6], 0.4), \mathcal{M}(\hat{u}_4\hat{u}_5) = ([0.3, 0.5], 0.7), \mathcal{M}(\hat{u}_5\hat{u}_6) = ([0.2, 0.6], 0.4), \mathcal{M}(\hat{u}_6\hat{u}_7) = ([0.3, 0.5], 0.7), \mathcal{M}(\hat{u}_7\hat{u}_8) = ([0.2, 0.6], 0.4), \mathcal{M}(\hat{u}_8\hat{u}_1) = ([0.3, 0.5], 0.7)$

$d_2(\hat{u}_1) = ([0.4, 1.0], 0.8), \forall \hat{u}_1 \in \mathcal{P}$ *and* $d(\hat{u}_1) = ([0.5, 1.1], 1.1), \forall \hat{u}_1 \in \mathcal{P}$. *Thus* $\mathcal{C}^*$ *is a* $(([0.5, 1.1], 1.1), 2, ([0.4, 1.0], 0.8))$ *- Regular Cubic Fuzzy Graph.*

Example 4.2. *Non-Regular Cubic Fuzzy Graphs can satisfy the condition of being* $(2, ([k_\alpha, k_\beta], k_\gamma))$ *- Regular Cubic Fuzzy Graphs.*

Let a Cubic Fuzzy Graph $\mathcal{C}^* : (\mathcal{L}, \mathcal{M})$ on a cyclic graph $\mathcal{G} : (\mathcal{P}, \mathcal{C})$ with an odd cycle length. Let the edges be indexed as $e_1, e_2, e_3, ..., e_{2n}, e_{2n+1}$. If alternate edges possess equal IVF membership numbers and fuzzy membership numbers, then $\alpha_1(e_i) = \begin{cases} c_\alpha & \text{for odd } i \\ c_{\alpha_1} & \text{for even } i \end{cases}$, $\beta_1(e_i) = \begin{cases} c_\beta & \text{for odd } i \\ c_{\beta_1} & \text{for even } i \end{cases}$ and $\gamma_1(e_i) = \begin{cases} c_\gamma & \text{for odd } i \\ c_{\gamma_1} & \text{for even } i \end{cases}$

$$\begin{aligned} d_2(\hat{u}_1) &= ([\alpha_1(e_1) \wedge \alpha_1(e_2), \beta_1(e_1) \wedge \beta_2(e_2)], \gamma_1(e_1) \wedge \gamma_1(e_2)) + ([\alpha_1(e_{2n}) \wedge \alpha_1(e_{2n+1}), \beta_1(e_{2n}) \wedge \\ &\quad \beta_1(e_{2n+1})], \gamma_1(e_{2n}) \wedge \gamma_1(e_{2n+1})) \\ &= ([c_\alpha \wedge c_{\alpha_1}, c_\beta \wedge c_{\beta_1}], c_\gamma \wedge c_{\gamma_1}) + ([c_{\alpha_1} \wedge c_\alpha, c_{\beta_1} \wedge c_\beta], c_{\gamma_1} \wedge c_\gamma) \\ &= ([c_\alpha, c_\beta], c_\gamma) + ([c_\alpha, c_\beta], c_\gamma) = ([2c_\alpha, 2c_\beta], 2c_\gamma) \end{aligned}$$

$$\begin{aligned} d_2(\hat{u}_2) &= ([\alpha_1(e_2) \wedge \alpha_1(e_3), \beta_1(e_2) \wedge \beta_1(e_3)], \gamma_1(e_2) \wedge \gamma_1(e_3)) + ([\alpha_1(e_1) \wedge \alpha_1(e_{2n+1}), \\ &\quad \beta_1(e_1) \wedge \beta_1(e_{2n+1})], \gamma_1(e_1) \wedge \gamma_1(e_{2n+1})) \\ &= ([c_{\alpha_1} \wedge c_\alpha, c_{\beta_1} \wedge c_\beta], c_{\gamma_1} \wedge c_\gamma) + ([c_\alpha \wedge c_\alpha, c_\beta \wedge c_\beta], c_\gamma \wedge c_{\gamma;}) \\ &= ([c_\alpha, c_\beta], c_\gamma) + ([c_\alpha, c_\beta], c_\gamma) = ([2c_\alpha, 2c_\beta], 2c_\gamma) \end{aligned}$$

Proceeding similarly, we get $d_2(\hat{u}_n) = d_2(\hat{u}_i) = ([2c_\alpha, 2c_\beta], 2c_\gamma)$, where $2c_\alpha = 2k_\alpha$, $2c_\beta = 2k_\beta$, $2c_\gamma = 2k_\gamma, \forall \hat{u} \in \mathcal{P}$. Hence, $\mathcal{C}^*$ is a $(2, ([k_\alpha, k_\beta], k_\gamma))$ - Regular Cubic Fuzzy Graph.

$$\begin{aligned} \text{Now, } d(\hat{u}_1) &= ([\alpha_2(e_1), \beta_2(e_1)], \gamma_2(e_1)) + ([\alpha_2(e_{2n+1}), \beta_2(e_{2n+1})], \gamma_2(e_{2n+1})) \\ &= ([c_\alpha, c_\beta], c_\gamma) + ([c_\alpha, c_\beta], c_\gamma) = ([2c_\alpha, 2c_\beta], 2c_\gamma) \end{aligned}$$

$$\begin{aligned} d(\hat{u}_2) &= ([\alpha_2(e_2), \beta_2(e_2)], \gamma_2(e_2)) + ([\alpha_2(e_1), \beta_2(e_1)], \gamma_2(e_1)) \\ &= ([c_{\alpha_1}, c_{\beta_1}], c_{\gamma_1}) + ([c_\alpha, c_\beta], c_\gamma) = ([c_{\alpha_1} + c_\alpha, c_{\beta_1} + c_\beta], c_{\gamma_1} + c_\gamma), \text{ where } c_{\alpha_1} + c_\alpha \end{aligned}$$

$= k_{\alpha_1} + k_\alpha$, $c_{\beta_1} + c_\beta = k_{\beta_1} + k_\beta$, $c_\gamma + c_{\gamma_1} = k_\gamma + k_{\gamma_1}, \forall \hat{u} \in \mathcal{P}$. So, $d(\hat{u}_1) \neq d(\hat{u}_2)$. Hence, $\mathcal{C}^*$ is a non-regular Cubic Fuzzy Graph that satisfies the conditions of a $(2, ([k_\alpha, k_\beta], k_\gamma))$ - Regular Cubic Fuzzy Graph.

Example 4.3. *Let a Cubic Fuzzy Graph $\mathcal{C}^* : (\mathcal{L}, \mathcal{M})$ on an odd cyclic graph $\mathcal{G} : (\mathcal{P}, \mathcal{C})$ with length ≥ 5. Let the membership functions on edges e_i be defined as*
$\alpha_1(e_i) = \begin{cases} c_\alpha & \text{for odd } i \\ x \geq c_\alpha & \text{for even } i \end{cases}$, $\beta_1(e_i) = \begin{cases} c_\beta & \text{for odd } i \\ y \geq c_\beta & \text{for even } i \end{cases}$ *and*
$\gamma_1(e_i) = \begin{cases} c_\gamma & \text{for odd } i \\ z \geq c_\gamma & \text{for even } i \end{cases}$, *where x, y, z are non-constant functions.*

Let a Cubic Fuzzy Graph $\mathcal{C}^* : (\mathcal{L}, \mathcal{M})$ on a cyclic graph $\mathcal{G} : (\mathcal{P}, \mathcal{C})$ with an odd cycle length ≥ 5. Then $d_2(\hat{u}) = ([(c_\alpha \wedge x), c_\beta \wedge y], c_\gamma \wedge z) + ([x \wedge c_\alpha, y \wedge c_\beta], z \wedge c_\gamma) = ([c_\alpha, c_\beta], c_\gamma) + ([c_\alpha, c_\beta], c_\gamma) = ([2c_\alpha, 2c_\beta], 2c_\gamma), \forall \hat{u} \in \mathcal{P}$, where $k_\alpha = 2c_\alpha, k_\beta = 2c_\beta, k_\gamma = 2c_\gamma$. Thus, $d_2(\hat{u}) = ([k_\alpha, k_\beta], k_\gamma), \forall \hat{u} \in \mathcal{P}$. Hence, $\mathcal{C}^*$ is a $(2, ([k_\alpha, k_\beta], k_\gamma))$ - Regular Cubic Fuzzy Graph.
Then, $d(\hat{u}_1) = ([c_\alpha, c_\beta], c_\gamma) + ([c_\alpha, c_\beta], c_\gamma) = ([2c_\alpha, 2c_\beta], 2c_\gamma)$ and
$d(\hat{u}_i) = ([a + c_\alpha, b + c_\beta], c + c_\gamma), \forall i = 2, 3, ..., 5$. So, $d(\hat{u}_1) \neq d(\hat{u}_i)$. Hence, $\mathcal{C}^*$ is a non - regular $(2, ([k_\alpha, k_\beta], k_\gamma))$ - Regular Cubic Fuzzy Graph.

Example 4.4. *Let a Cubic Fuzzy Graph $\mathcal{C}^* : (\mathcal{L}, \mathcal{M})$ on a path graph $\mathcal{G} : (\mathcal{P}, \mathcal{C})$ of length n vertices. If the function $\mathcal{M}$ is constant, then $\mathcal{C}^*$ is a non- regular $(2, ([k_\alpha, k_\beta], k_\gamma)))$ - Regular Cubic Fuzzy Graph.*

Proof. If the function $\mathcal{M}$ is constant and $\mathcal{M}(\hat{u}\hat{v}) = ([k_\alpha, k_\beta], k_\gamma)), \forall \hat{u}\hat{v} \in \mathcal{C}$, then $d_2(\hat{u}) = ([k_\alpha, k_\beta], k_\gamma)), \forall \hat{u} \in \mathcal{P}$. Thus $\mathcal{C}^*$ is a $(2, ([k_\alpha, k_\beta], k_\gamma))$ - Regular Cubic Fuzzy Graph. However, $d(\hat{u}_i) = ([k_\alpha, k_\beta], k_\gamma))$ for external vertices and $d(\hat{u}_i) = ([2k_\alpha, 2k_\beta], 2k_\gamma))$ for internal vertices. This implies $\mathcal{C}^*$ is not a Regular Cubic Fuzzy Graphs. Therefore $\mathcal{C}^*$ is a non - regular $(2, ([k_\alpha, k_\beta], k_\gamma))$ - Regular Cubic Fuzzy Graph.

5. Total $(([r_\alpha, r_\beta], r_\gamma), 2, ([k_\alpha, k_\beta], k_\gamma))$ - Regular Cubic Fuzzy Graphs

Definition 5.1. *In a Cubic Fuzzy Graph $\mathcal{C}^* : (\mathcal{L}, \mathcal{M})$ on a graph $\mathcal{G} : (\mathcal{P}, \mathcal{C})$, if every vertex in $\mathcal{C}^*$ possesses an identical Total degree $([r_\alpha, r_\beta], r_\gamma)$ and td_2 - degree $([k_\alpha, k_\beta], k_\gamma)$, then $\mathcal{C}^*$ is defined as a Total $(([r_\alpha, r_\beta], r_\gamma), 2, ([k_\alpha, k_\beta], k_\gamma))$ - Regular Cubic Fuzzy Graph.*

Example 5.1. *Consider a Fuzzy Graph $\mathcal{G} : (\mathcal{P}, \mathcal{C})$ with $\mathcal{P} = \{\hat{u}, \hat{v}, \hat{w}, \hat{x}, \hat{y}\}$ and $\mathcal{C} = \{\hat{u}\hat{v}, \hat{v}\hat{w}, \hat{w}\hat{x}, \hat{x}\hat{y}, \hat{y}\hat{u}\}$ is defined a Cubic Fuzzy Graph $\mathcal{C}^*$: $(\mathcal{L}, \mathcal{M})$.* $\mathcal{L}(\hat{u}) = ([0.4, 0.7], 0.3)$, $\mathcal{L}(\hat{v}) = ([0.4, 0.7], 0.3), \mathcal{L}(\hat{w}) = ([0.4, 0.7], 0.3), \mathcal{L}(\hat{x}) = ([0.4, 0.7], 0.3), \mathcal{L}(\hat{y}) = ([0.4, 0.7], 0.3)$ *and* $\mathcal{M}(\hat{u}\hat{v}) = ([0.2, 0.6], 0.1), \mathcal{M}(\hat{v}\hat{w}) = ([0.2, 0.6], 0.1), \mathcal{M}(\hat{w}\hat{x}) = ([0.2, 0.6], 0.1), \mathcal{M}(\hat{x}\hat{y}) = ([0.2, 0.6], 0.1), \mathcal{M}(\hat{y}\hat{u}) = ([0.2, 0.6], 0.1)$

$d_2(\hat{u}) = ([0.4, 1.2], 0.2)$ *and* $d(\hat{u}) = ([0.4, 1.2], 0.2), \forall \hat{u} \in P$. *This implies that $\mathcal{C}^*$ is a* $(([0.4, 1.2], 0.2), 2, ([0.4, 1.2], 0.2))$ *- Regular Cubic Fuzzy Graph. Furthermore,* $td_2(\hat{u}) = ([0.8, 1.9], 0.5)$ *and* $td(\hat{u}) = ([0.8, 1.9], 0.5), \forall \hat{u} \in \mathcal{P}$. *Thus, $\mathcal{C}^*$ is also a Total* $(([0.8, 1.9], 0.5), 2, ([0.8, 1.9], 0.5))$ *- Regular Cubic Fuzzy Graph.*

Remark 5.1. *A $(([r_\alpha, r_\beta], r_\gamma), 2, ([k_\alpha, k_\beta], k_\gamma))$ - Regular Cubic Fuzzy Graph, that is also a Total $(([r_\alpha, r_\beta], r_\gamma), 2, ([k_\alpha, k_\beta], k_\gamma))$ - Regular Cubic Fuzzy Graph.*

Example 5.2. *Consider a graph $\mathcal{G} : (\mathcal{P}, \mathcal{C})$ with $\mathcal{P} = \{\hat{u}_1, \hat{u}_2, \hat{u}_3, \hat{u}_4, \hat{u}_5, \hat{u}_6, \hat{u}_7, \hat{u}_8\}$ and $\mathcal{C} = \{\hat{u}_1\hat{u}_2, \hat{u}_2\hat{u}_3, \hat{u}_3\hat{u}_4, \hat{u}_4\hat{u}_5, \hat{u}_5\hat{u}_6, \hat{u}_6\hat{u}_7, \hat{u}_7\hat{u}_8, \hat{u}_8\hat{u}_1\}$ is defined a Cubic Fuzzy Graph $\mathcal{C}^* : (\mathcal{L}, \mathcal{M})$.* $\mathcal{L}(\hat{u}_1) = ([0.3, 0.7], 0.8), \mathcal{L}(\hat{u}_2) = ([0.4, 0.7], 0.9), \mathcal{L}(\hat{u}_3) = ([0.5, 0.6], 1.0), \mathcal{L}(\hat{u}_4) = ([0.3, 0.7],$

$0.2), \mathcal{L}(\hat{u}_5) = ([0.4, 0.6], 0.4), \mathcal{L}(\hat{u}_6) = ([0.5, 0.7], 0.5), \mathcal{L}(\hat{u}_7) = ([0.4, 0.6], 0.8), \mathcal{L}(\hat{u}_8) = ([0.3, 0.7], 1.0)$ *and* $\mathcal{M}(\hat{u}_1\hat{u}_2) = ([0.2, 0.6], 0.4), \mathcal{M}(\hat{u}_2\hat{u}_3) = ([0.3, 0.5], 0.7), \mathcal{M}(\hat{u}_3\hat{u}_4) = ([0.2, 0.6], 0.4), \mathcal{M}(\hat{u}_4\hat{u}_5) = ([0.3, 0.5], 0.7), \mathcal{M}(\hat{u}_5\hat{u}_6) = ([0.2, 0.6], 0.4), \mathcal{M}(\hat{u}_6\hat{u}_7) = ([0.3, 0.5], 0.7), \mathcal{M}(\hat{u}_7\hat{u}_8) = ([0.2, 0.6], 0.4), \mathcal{M}(\hat{u}_8\hat{u}_1) = ([0.3, 0.5], 0.7)$

Here, $d_2(\hat{u}) = ([0.4, 1.0], 0.8)$*and* $d(\hat{u}) = ([0.5, 1.1], 1.1), \forall \hat{u} \in \mathcal{V}$. *This implies that* $\mathcal{C}^*$ *is a* $(([0.5, 1.1], 1.1), 2, ([0.4, 1.0], 0.8))$ *- Regular Cubic Fuzzy Graph. However,* $td_2(\hat{u}) \neq td_2(\hat{x})$ *and* $td(\hat{u}) \neq td(\hat{x})$*. Therefore,* $\mathcal{C}^*$ *is not a Total* $(([r_\alpha, r_\beta], r_\gamma), 2, ([k_\alpha, k_\beta], k_\gamma))$ *- Regular Cubic Fuzzy Graph.*

Remark 5.2. *A* $(([r_\alpha, r_\beta], r_\gamma), 2, ([k_\alpha, k_\beta], k_\gamma))$ *- Regular Cubic Fuzzy Graph does not necessarily possess a Total* $(([r_\alpha, r_\beta], r_\gamma), 2, ([k_\alpha, k_\beta], k_\gamma))$ *- Regular Cubic Fuzzy Graph.*

Theorem 5.1. *Consider a Cubic Fuzzy Graph* $\mathcal{C}^* : (\mathcal{L}, \mathcal{M})$ *on a graph* $\mathcal{G} : (\mathcal{P}, \mathcal{C})$*. Then, the function* $\mathcal{L}$ *is constant if and only certain conditions are met, which can be stated as follows:*
(i) $\mathcal{C}^*$ *is a* $(([r_\alpha, r_\beta], r_\gamma), 2, ([k_\alpha, k_\beta], k_\gamma))$ *- Regular Cubic Fuzzy Graph.*
(ii) $\mathcal{C}^*$ *is a Total* $(([r_\alpha + c_\alpha, r_\beta + c_\beta], r_\gamma + c_\gamma), 2, ([k_\alpha + c_\alpha, k_\beta + c_\beta], k_\gamma + c_\gamma))$ *- Regular Cubic Fuzzy Graph.*

Proof. Consider $\mathcal{L}(\hat{u}) = ([c_\alpha, c_\beta], c_\gamma), \forall \hat{u} \in \mathcal{P}$. Assume that $\mathcal{C}^*$ is a $(([r_\alpha, r_\beta], r_\gamma), 2, ([k_\alpha, k_\beta], k_\gamma))$ - Regular Cubic Fuzzy Graph. Then $d_2(\hat{u}) = ([k_\alpha, k_\beta], k_\gamma)$ and $d(\hat{u}) = ([r_\alpha, r_\beta], r_\gamma)$, $\forall \hat{u} \in \mathcal{P}$. So, $td_2(\hat{u}) = d_2(\hat{u}) + \mathcal{L}(\hat{u})$ and $td(\hat{u}) = d(\hat{u}) + \mathcal{L}(\hat{u}), \forall \hat{u} \in \mathcal{P}$.
$\Rightarrow td_2(\hat{u}) = ([k_\alpha, k_\beta], k_\gamma) + ([c_\alpha, c_\beta], c_\gamma)$ and $td(\hat{u}) = ([r_\alpha, r_\beta], r_\gamma) + ([c_\alpha, c_\beta], c_\gamma), \forall \hat{u} \in \mathcal{P}$.
$\Rightarrow td_2(\hat{u}) = ([k_\alpha + c_\alpha, k_\beta + c_\beta], k_\gamma + c_\gamma)$ and $td(\hat{u}) = ([r_\alpha + c_\alpha, r_\beta + c_\beta], r_\gamma + c_\gamma), \forall \hat{u} \in \mathcal{P}$.
Hence, $\mathcal{C}^*$ is a Total $(([r_\alpha + c_\alpha, r_\beta + c_\beta], r_\gamma + c_\gamma), 2, ([k_\alpha + c_\alpha, k_\beta + c_\beta], k_\gamma + c_\gamma))$ - Regular Cubic Fuzzy Graph. This establishes the implication (i)$\Longrightarrow$ (ii). Furthermore, let's assume $\mathcal{C}^*$ is a Total $(([r_\alpha + c_\alpha, r_\beta + c_\beta], r_\gamma + c_\gamma), 2, ([k_\alpha + c_\alpha, k_\beta + c_\beta], k_\gamma + c_\gamma))$ - Regular Cubic Fuzzy Graph.
Then, $td_2(\hat{u}) = ([k_\alpha + c_\alpha, k_\beta + c_\beta], k_\gamma + c_\gamma)$ and $td(\hat{u}) = ([r_\alpha + c_\alpha, r_\beta + c_\beta], r_\gamma + c_\gamma), \forall \hat{u} \in \mathcal{P}$. This implies $d_2(\hat{u}) + \mathcal{L}(\hat{u}) = ([k_\alpha + c_\alpha, k_\beta + c_\beta], k_\gamma + c_\gamma)$ and $d(\hat{u}) + \mathcal{L}(\hat{u}) = ([r_\alpha + c_\alpha, r_\beta + c_\beta], r_\gamma + c_\gamma), \forall \hat{u} \in \mathcal{P}$. Therefore, $d_2(\hat{u}) + ([c_\alpha, c_\beta], c_\gamma) = ([k_\alpha, k_\beta], k_\gamma) + ([c_\alpha, c_\beta], c_\gamma)$ and $d(\hat{u}) + ([c_\alpha, c_\beta], c_\gamma) = ([r_\alpha, r_\beta], r_\gamma) + ([c_\alpha, c_\beta], c_\gamma), \forall \hat{u} \in \mathcal{P}$. Hence, $d_2(\hat{u}) = ([k_\alpha, k_\beta], k_\gamma)$ and $d(\hat{u}) = ([r_\alpha, r_\beta], r_\gamma), \forall \hat{u} \in \mathcal{P}$
Thus, $\mathcal{C}^*$ is a $(([r_\alpha, r_\beta], r_\gamma), 2, ([k_\alpha, k_\beta], k_\gamma))$ - Regular Cubic Fuzzy Graph. Therefore, (ii) $\Longrightarrow$ (i) is proved. Conversely, let's establish the equivalence between (i) and (ii). Assuming the function $\mathcal{L}(\hat{u})$ is non-constant, there exists at least one pair $\hat{u}, \hat{w} \in \mathcal{P}$ where $\mathcal{L}(\hat{u}) \neq \mathcal{L}(\hat{w})$, resulting in $td_2(\hat{u}) \neq td_2(\hat{w})$ and $td(\hat{u}) \neq td(\hat{w})$. Let $\mathcal{C}^*$ be a $(([r_\alpha, r_\beta], r_\gamma), 2, ([k_\alpha, k_\beta], k_\gamma))$ - Regular Cubic Fuzzy Graph and Total $(([r_\alpha + c_\alpha, r_\beta + c_\beta], r_\gamma + c_\gamma), 2, ([k_\alpha + c_\alpha, k_\beta + c_\beta], k_\gamma + c_\gamma))$ - Regular Cubic Fuzzy Graph. Then, $d_2(\hat{u}) = d_2(\hat{w}) = ([k_\alpha, k_\beta], k_\gamma)$ and $d(\hat{u}) = d(\hat{w}) = ([r_\alpha, r_\beta], r_\gamma)$.
So, $td_2(\hat{u}) = d_2(\hat{u}) + \mathcal{L}(\hat{u}), td_2(\hat{w}) = d_2(\hat{w}) + \mathcal{L}(\hat{w})$ and $td(\hat{u}) = d(\hat{u}) + \mathcal{L}(\hat{u}), td(\hat{w}) = d(\hat{w}) + \mathcal{L}(\hat{w})$
$\Rightarrow td_2(\hat{u}) = ([k_\alpha, k_\beta], k_\gamma) + \mathcal{L}(\hat{u}), td_2(\hat{w}) = ([k_\alpha, k_\beta], k_\gamma) + \mathcal{L}(\hat{w})$ and $td(\hat{u}) = ([r_\alpha, r_\beta], r_\gamma) + \mathcal{L}(\hat{u}), td(\hat{w}) = ([r_\alpha, r_\beta], r_\gamma) + \mathcal{L}(\hat{w})$
Since $\mathcal{L}(\hat{u}) \neq \mathcal{L}(\hat{w})$ i.e., $td_2(\hat{u}) \neq td_2(\hat{w}) = ([k_\alpha, k_\beta], k_\gamma) + \mathcal{L}(\hat{u}) \neq ([k_\alpha, k_\beta], k_\gamma) + \mathcal{L}(\hat{w})$ and $td(\hat{u}) \neq td(\hat{w}) = ([r_\alpha, r_\beta], r_\gamma) + \mathcal{L}(\hat{u}) \neq ([r_\alpha, r_\beta], r_\gamma) + \mathcal{L}(\hat{w})$.
$\Rightarrow td_2(\hat{u}) \neq td_2(\hat{w})$ and $td(\hat{u}) \neq td(\hat{w})$
So, $\mathcal{C}^*$ is not Total $(([r_\alpha + c_\alpha, r_\beta + c_\beta], r_\gamma + c_\gamma), 2, ([k_\alpha + c_\alpha, k_\beta + c_\beta], k_\gamma + c_\gamma))$ - Regular Cubic Fuzzy Graph. This conclusion contradicts our assumption. Now, let's assume that $\mathcal{C}^*$ is a Total $(([r_\alpha + c_\alpha, r_\beta + c_\beta], r_\gamma + c_\gamma), 2, ([k_\alpha + c_\alpha, k_\beta + c_\beta], k_\gamma + c_\gamma))$ - Regular Cubic

Fuzzy Graph. Then, $td_2(\hat{u}) = td_2(\hat{w})$ and $td(\hat{u}) = td(\hat{w})$.
$\Rightarrow d_2(\hat{u}) + \mathcal{L}(\hat{u}) = d_2(\hat{w}) + \mathcal{L}(\hat{w})$ and $d(\hat{u}) + \mathcal{L}(\hat{u}) = d(\hat{w}) + \mathcal{L}(\hat{w})$
$\Rightarrow d_2(\hat{u}) \neq d_2(\hat{w})$ and $d(\hat{u}) \neq d(\hat{w})$. So $\mathcal{C}^*$ is not $(([r_\alpha, r_\beta], r_\gamma), 2, ([k_\alpha, k_\beta], k_\gamma))$ - Regular Cubic Fuzzy Graph. This leads to a contradiction, implying that our initial assumption was incorrect. Consequently, the function $\mathcal{L}$ must be constant □

Theorem 5.2. *If a Cubic Fuzzy Graph on $\mathcal{C}^* : (L, M)$ is both $(([r_\alpha, r_\beta], r_\gamma), 2, ([k_\alpha, k_\beta], k_\gamma))$ - Regular and Total $(([r_{\alpha_1}, r_{\beta_1}], r_{\gamma_1}), 2, ([k_{\alpha_1}, k_{\beta_1}], k_{\gamma_1}))$ - Regular Cubic Fuzzy Graph, then the function $\mathcal{L}$ must be constant.*

Proof. Let $\mathcal{C}^*$ be a $\mathcal{C}^* : (L, M)$ is both $(([r_\alpha, r_\beta], r_\gamma), 2, ([k_\alpha, k_\beta], k_\gamma))$ - Regular and Total $(([r_{\alpha_1}, r_{\beta_1}], r_{\gamma_1}), 2, ([k_{\alpha_1}, k_{\beta_1}], k_{\gamma_1}))$- Regular Cubic Fuzzy Graph. Then, $d_2(\hat{u}) = ([k_\alpha, k_\beta], k_\gamma)$ and $td_2(\hat{u}) = ([k_{\alpha_1}, k_{\beta_1}], k_{\gamma_1}), d(\hat{u}) = ([r_\alpha, r_\beta], r_\gamma)$ and $td(\hat{u}) = ([r_{\alpha_1}, r_{\beta_1}], r_{\gamma_1}), \forall \hat{u} \in \mathcal{P}$.
Now, $td_2(\hat{u}) = ([k_{\alpha_1}, k_{\beta_1}], k_{\gamma_1})$ and $td(\hat{u}) = ([r_{\alpha_1}, r_{\beta_1}], r_{\gamma_1}), \forall \hat{u} \in \mathcal{P}$
$\Rightarrow d_2(\hat{u}) + \mathcal{L}(\hat{u}) = ([k_{\alpha_1}, k_{\beta_1}], k_{\gamma_1})$ and $d(\hat{u}) + \mathcal{L}(\hat{u}) = ([r_{\alpha_1}, r_{\beta_1}], r_{\gamma_1}), \forall \hat{u} \in \mathcal{P}$
$\Rightarrow ([k_\alpha, k_\beta], k_\gamma) + \mathcal{L}(\hat{u}) = ([k_{\alpha_1}, k_{\beta_1}], k_{\gamma_1})$ and $([r_\alpha, r_\beta], r_\gamma) + \mathcal{L}(\hat{u}) = ([r_{\alpha_1}, r_{\beta_1}], r_{\gamma_1}), \forall \hat{u} \in \mathcal{P}$
$\Rightarrow \mathcal{L}(\hat{u}) = ([k_{\alpha_1}, k_{\beta_1}], k_{\gamma_1}) - ([k_\alpha, k_\beta], k_\gamma)$ and $\mathcal{L}(\hat{u}) = ([r_{\alpha_1}, r_{\beta_1}], r_{\gamma_1}) - ([r_\alpha, r_\beta], r_\gamma), \forall \hat{u} \in \mathcal{P}$
$\Rightarrow \mathcal{L}(\hat{u}) = ([k_{\alpha_1} - k_\alpha, k_{\beta_1} - k_\beta], k_{\gamma_1} - k_\gamma)$ and $([r_{\alpha_1} - r_\alpha, r_{\beta_1} - r_\beta], r_{\gamma_1} - r_\gamma), \forall \hat{u} \in \mathcal{P}$. This implies that function $\mathcal{L}$ is constant. □

6. $(([r_\alpha, r_\beta], r_\gamma), 2, ([k_\alpha, k_\beta], k_\gamma))$ - Regular cyclic Cubic Fuzzy Graphs with specific membership values

Theorem 6.1. *Let a Cubic Fuzzy Graph $\mathcal{C}^* : (\mathcal{L}, \mathcal{M})$ on a cyclic graph $\mathcal{G} : (\mathcal{P}, \mathcal{C})$ with an odd cycle length ≥ 5. If function $\mathcal{M}$ is constant , then $\mathcal{C}^*$ satisfies $(([r_\alpha, r_\beta], r_\gamma), 2, ([k_\alpha, k_\beta], k_\gamma))$ - Regular Cubic Fuzzy Graph, where $([r_\alpha, r_\beta], r_\gamma) = 2\mathcal{M}(\hat{u}\hat{v})$ and $([k_\alpha, k_\beta], k_\gamma) = 2\mathcal{M}(\hat{u}\hat{v})$.*

Proof. If function $\mathcal{M}$ is constant and $\mathcal{M}(\hat{u}\hat{v}) = ([k_\alpha, k_\beta], k_\gamma), \forall \hat{u} \in \mathcal{P}$. Then $d_2\alpha_1(\hat{u}) = \{k_\alpha \wedge k_\alpha\} + \{k_\alpha \wedge k_\alpha\} = k_\alpha + k_\alpha = 2k_\alpha, d_2\beta_1(u) = \{k_\beta \wedge k_\beta\} + \{k_\beta \wedge k_\beta\} = k_\beta + k_\beta = 2k_\beta$ and $d_2\gamma_1(\hat{u}) = \{c_\gamma \wedge c_\gamma\} + \{c_\gamma \wedge c_\gamma\} = c_\gamma + c_\gamma = 2c_\gamma, \forall \hat{u} \in \mathcal{P}$. So, $d_2(\hat{u}) = ([k_\alpha, k_\beta], k_\gamma), \forall \hat{u} \in \mathcal{P}$. Also, $d(\hat{u}) = ([k_\alpha, k_\beta], k_\gamma) + ([k_\alpha, k_\beta], k_\gamma) = ([2k_\alpha, 2k_\beta], 2k_\gamma)$. Hence $\mathcal{C}^*$ is a $(([r_\alpha, r_\beta], r_\gamma), 2, ([k_\alpha, k_\beta], k_\gamma))$ - Regular Cubic Fuzzy Graph. □

Remark 6.1. *The converse of theorem 6.1. is not necessarily to be true.*

Example 6.1. *Consider a graph $\mathcal{G} : (\mathcal{P}, \mathcal{C})$ with $\mathcal{P} = \{\hat{u}_1, \hat{u}_2, \hat{u}_3, \hat{u}_4, \hat{u}_5, \hat{u}_6, \hat{u}_7, \hat{u}_8\}$ and $\mathcal{C} = \{\hat{u}_1\hat{u}_2, \hat{u}_2\hat{u}_3, \hat{u}_3\hat{u}_4, \hat{u}_4\hat{u}_5, \hat{u}_5\hat{u}_6, \hat{u}_6\hat{u}_7, \hat{u}_7\hat{u}_8, \hat{u}_8\hat{u}_1\}$ is defined a Cubic Fuzzy Graph $\mathcal{C}^* : (\mathcal{L}, \mathcal{M})$. $\mathcal{L}(\hat{u}_1) = ([0.3, 0.7], 0.8), \mathcal{L}(\hat{u}_2) = ([0.4, 0.7], 0.9), \mathcal{L}(\hat{u}_3) = ([0.5, 0.6], 1.0), \mathcal{L}(\hat{u}_4) = ([0.3, 0.7], 0.2), \mathcal{L}(\hat{u}_5) = ([0.4, 0.6], 0.4), \mathcal{L}(\hat{u}_6) = ([0.5, 0.7], 0.5), \mathcal{L}(\hat{u}_7) = ([0.4, 0.6], 0.8), \mathcal{L}(\hat{u}_8) = ([0.3, 0.7], 1.0)$ and $\mathcal{M}(\hat{u}_1\hat{u}_2) = ([0.2, 0.6], 0.4), \mathcal{M}(\hat{u}_2\hat{u}_3) = ([0.3, 0.5], 0.7), \mathcal{M}(\hat{u}_3\hat{u}_4) = ([0.2, 0.6], 0.4), \mathcal{M}(\hat{u}_4\hat{u}_5) = ([0.3, 0.5], 0.7), \mathcal{M}(\hat{u}_5\hat{u}_6) = ([0.2, 0.6], 0.4), \mathcal{M}(\hat{u}_6\hat{u}_7) = ([0.3, 0.5], 0.7), \mathcal{M}(\hat{u}_7\hat{u}_8) = ([0.2, 0.6], 0.4), \mathcal{M}(\hat{u}_8\hat{u}_1) = ([0.3, 0.5], 0.7)$*

$d_2(\hat{u}) = ([0.4, 1.0], 0.8)$ and $d(\hat{u}) = ([0.5, 1.1], 1.1), \forall \hat{u} \in P$. So, $\mathcal{C}^*$ is a $(([0.5, 1.1], 1.1), 2, ([0.4, 1.0], 0.8))$ - Regular Cubic Fuzzy Graph. Interestingly, this arises despite $\mathcal{M}$ being a non-constant function.

Theorem 6.2. *Let $\mathcal{C}^* : (\mathcal{L}, \mathcal{M})$ be a Cubic Fuzzy Graph on an even cycle $\mathcal{G} : (\mathcal{P}, \mathcal{C})$ with length ≥ 8, If every other edge has the same IVF membership number and fuzzy membership number, then $\mathcal{C}^*$ is a $(([r_\alpha, r_\beta], r_\gamma), 2, ([k_\alpha, k_\beta], k_\gamma))$ - Regular Cubic Fuzzy Graph.*

Proof. Given that every other edge has the same IVF membership number and fuzzy membership number, then $\alpha_1(e_i) = \begin{cases} c_\alpha & \text{for odd } i \\ c_{\alpha_1} & \text{for even } i \end{cases}$, $\beta_1(e_i) = \begin{cases} c_\beta & \text{for odd } i \\ c_{\beta_1} & \text{for even } i \end{cases}$ and $\gamma_1(e_i) = \begin{cases} c_\gamma & \text{for odd } i \\ c_{\gamma_1} & \text{for even } i \end{cases}$
There are 8 possible cases arise:

Case (i). $c_\alpha > c_{\alpha_1}$, $c_\beta > c_{\beta_1}$ and $c_\gamma > c_{\gamma_1}$.
Then, $d_{2\alpha}(\hat{u}) = \{c_\alpha \wedge c_{\alpha_1}\} + \{c_\alpha \wedge c_{\alpha_1}\} = c_{\alpha_1} + c_{\alpha_1} = 2c_{\alpha_1}$, $d_{2\beta}(\hat{u}) = \{c_\beta \wedge c_{\beta_1}\} + \{c_\beta \wedge c_{\beta_1}\} = c_{\beta_1} + c_{\beta_1} = 2c_{\beta_1}$ and $d_{2\gamma}(\hat{u}) = \{c_\gamma \wedge c_{\gamma_1}\} + \{c_\gamma \wedge c_{\gamma_1}\} = c_{\gamma_1} + c_{\gamma_1} = 2c_{\gamma_1}$. So, $d_2(\hat{u}) = ([2c_{\alpha_1}, 2c_{\beta_1}], 2c_{\gamma_1}))$. Also, $d(\hat{u}) = ([c_\alpha, c_\beta], c_\gamma) + ([c_{\alpha_1}, c_{\beta_1}], c_{\gamma_1}) = ([c_\alpha + c_{\alpha_1}, c_\beta + c_{\beta_1}], c_\gamma + c_{\gamma_1})$.

Case (ii). $c_\alpha > c_{\alpha_1}$, $c_\beta < c_{\beta_1}$ and $c_\gamma > c_{\gamma_1}$.
Then, $d_{2\alpha}(\hat{u}) = \{c_\alpha \wedge c_{\alpha_1}\} + \{c_\alpha \wedge c_{\alpha_1}\} = c_{\alpha_1} + c_{\alpha_1} = 2c_{\alpha_1}$, $d_{2\beta}(\hat{u}) = \{c_\beta \wedge c_{\beta_1}\} + \{c_\beta \wedge c_{\beta_1}\} = c_\beta + c_\beta = 2c_\beta$ and $d_{2\gamma}(\hat{u}) = \{c_\gamma \wedge c_{\gamma_1}\} + \{c_\gamma \wedge c_{\gamma_1}\} = c_{\gamma_1} + c_{\gamma_1} = 2c_{\gamma_1}$. So, $d_2(\hat{u}) = ([2c_{\alpha_1}, 2c_\beta], 2c_{\gamma_1}))$. Also, $d(\hat{u}) = ([c_\alpha, c_\beta], c_\gamma) + ([c_{\alpha_1}, c_{\beta_1}], c_{\gamma_1}) = ([c_\alpha + c_{\alpha_1}, c_\beta + c_{\beta_1}], c_\gamma + c_{\gamma_1})$.

Case (iii). $c_\alpha < c_{\alpha_1}$, $c_\beta < c_{\beta_1}$ and $c_\gamma < c_{\gamma_1}$.
Then, $d_{2\alpha}(\hat{u}) = \{c_\alpha \wedge c_{\alpha_1}\} + \{c_\alpha \wedge c_{\alpha_1}\} = c_\alpha + c_\alpha = 2c_\alpha$, $d_{2\beta}(\hat{u}) = \{c_\beta \wedge c_{\beta_1}\} + \{c_\beta \wedge c_{\beta_1}\} = c_\beta + c_\beta = 2c_\beta$ and $d_{2\gamma}(\hat{u}) = \{c_\gamma \wedge c_{\gamma_1}\} + \{c_\gamma \wedge c_{\gamma_1}\} = c_\gamma + c_\gamma = 2c_\gamma$. So, $d_2(\hat{u}) = ([2c_\alpha, 2c_\beta], 2c_\gamma))$. Also, $d(\hat{u}) = ([c_\alpha, c_\beta], c_\gamma) + ([c_{\alpha_1}, c_{\beta_1}], c_{\gamma_1}) = ([c_\alpha + c_{\alpha_1}, c_\beta + c_{\beta_1}], c_\gamma + c_{\gamma_1})$.

Case (iv).$c_\alpha > c_{\alpha_1}$, $c_\beta > c_{\beta_1}$ and $c_\gamma < c_{\gamma_1}$.
Then, $d_{2\alpha}(\hat{u}) = \{c_\alpha \wedge c_{\alpha_1}\} + \{c_\alpha \wedge c_{\alpha_1}\} = c_{\alpha_1} + c_{\alpha_1} = 2c_{\alpha_1}$, $d_{2\beta}(\hat{u}) = \{c_\beta \wedge c_{\beta_1}\} + \{c_\beta \wedge c_{\beta_1}\} = c_{\beta_1} + c_{\beta_1} = 2c_{\beta_1}$ and $d_{2\gamma}(\hat{u}) = \{c_\gamma \wedge c_{\gamma_1}\} + \{c_\gamma \wedge c_{\gamma_1}\} = c_\gamma + c_\gamma = 2c_\gamma$. So, $d_2(\hat{u}) = ([2c_{\alpha_1}, 2c_{\beta_1}], 2c_\gamma))$. Also, $d(\hat{u}) = ([c_\alpha, c_\beta], c_\gamma) + ([c_{\alpha_1}, c_{\beta_1}], c_{\gamma_1}) = ([c_\alpha + c_{\alpha_1}, c_\beta + c_{\beta_1}], c_\gamma + c_{\gamma_1})$.

Case (v). $c_\alpha < c_{\alpha_1}$, $c_\beta > c_{\beta_1}$ and $c_\gamma > c_{\gamma_1}$.
Then, $d_{2\alpha}(\hat{u}) = \{c_\alpha \wedge c_{\alpha_1}\} + \{c_\alpha \wedge c_{\alpha_1}\} = c_\alpha + c_\alpha = 2c_\alpha, d_{2\beta}(\hat{u}) = \{c_\beta \wedge c_{\beta_1}\} + \{c_\beta \wedge c_{\beta_1}\} = c_{\beta_1} + c_{\beta_1} = 2c_{\beta_1}$ and $d_{2\gamma}(\hat{u}) = \{c_\gamma \wedge c_{\gamma_1}\} + \{c_\gamma \wedge c_{\gamma_1}\} = c_{\gamma_1} + c_{\gamma_1} = 2c_{\gamma_1}$. So, $d_2(\hat{u}) = ([2c_\alpha, 2c_{\beta_1}], 2c_{\gamma_1}))$. Also, $d(\hat{u}) = ([c_\alpha, c_\beta], c_\gamma) + ([c_{\alpha_1}, c_{\beta_1}], c_{\gamma_1}) = ([c_\alpha + c_{\alpha_1}, c_\beta + c_{\beta_1}], c_\gamma + c_{\gamma_1})$.

Case (vi). $c_\alpha > c_{\alpha_1}$, $c_\beta < c_{\beta_1}$ and $c_\gamma < c_{\gamma_1}$.
Then, $d_{2\alpha}(\hat{u}) = \{c_\alpha \wedge c_{\alpha_1}\} + \{c_\alpha \wedge c_{\alpha_1}\} = c_{\alpha_1} + c_{\alpha_1} = 2c_{\alpha_1}$, $d_{2\beta}(\hat{u}) = \{c_\beta \wedge c_{\beta_1}\} + \{c_\beta \wedge c_{\beta_1}\} = c_\beta + c_\beta = 2c_\beta$ and $d_{2\gamma}(\hat{u}) = \{c_\gamma \wedge c_{\gamma_1}\} + \{c_\gamma \wedge c_{\gamma_1}\} = c_\gamma + c_\gamma = 2c_\gamma$. So, $d_2(\hat{u}) = ([2c_{\alpha_1}, 2c_\beta], 2c_\gamma))$. Also, $d(\hat{u}) = ([c_\alpha, c_\beta], c_\gamma) + ([c_{\alpha_1}, c_{\beta_1}], c_{\gamma_1}) = ([c_\alpha + c_{\alpha_1}, c_\beta + c_{\beta_1}], c_\gamma + c_{\gamma_1})$.

Case (vii). $c_\alpha < c_{\alpha_1}$, $c_\beta > c_{\beta_1}$ and $c_\gamma < c_{\gamma_1}$.
Then, $d_{2\alpha}(\hat{u}) = \{c_\alpha \wedge c_{\alpha_1}\} + \{c_\alpha \wedge c_{\alpha_1}\} = c_\alpha + c_\alpha = 2c_\alpha$, $d_{2\beta}(\hat{u}) = \{c_\beta \wedge c_{\beta_1}\} + \{c_\beta \wedge c_{\beta_1}\} = c_{\beta_1} + c_{\beta_1} = 2c_{\beta_1}$ and $d_{2\gamma}(\hat{u}) = \{c_\gamma \wedge c_{\gamma_1}\} + \{c_\gamma \wedge c_{\gamma_1}\} = c_\gamma + c_\gamma = 2c_\gamma$. So, $d_2(\hat{u}) = ([2c_\alpha, 2c_{\beta_1}], 2c_\gamma))$. Also, $d(\hat{u}) = ([c_\alpha, c_\beta], c_\gamma) + ([c_{\alpha_1}, c_{\beta_1}], c_{\gamma_1}) = ([c_\alpha + c_{\alpha_1}, c_\beta + c_{\beta_1}], c_\gamma + c_{\gamma_1})$.

Case (viii). $c_\alpha < c_{\alpha_1}$, $c_\beta < c_{\beta_1}$ and $c_\gamma > c_{\gamma_1}$.
Then, $d_{2\alpha}(\hat{u}) = \{c_\alpha \wedge c_{\alpha_1}\} + \{c_\alpha \wedge c_{\alpha_1}\} = c_\alpha + c_\alpha = 2c_\alpha$, $d_{2\beta}(\hat{u}) = \{c_\beta \wedge c_{\beta_1}\} + \{c_\beta \wedge c_{\beta_1}\} = c_\beta + c_\beta = 2c_\beta$ and $d_{2\gamma}(\hat{u}) = \{c_\gamma \wedge c_{\gamma_1}\} + \{c_\gamma \wedge c_{\gamma_1}\} = c_{\gamma_1} + c_{\gamma_1} = 2c_{\gamma_1}$. So, $d_2(\hat{u}) = ([2c_\alpha, 2c_\beta], 2c_{\gamma_1}))$. Also, $d(\hat{u}) = ([c_\alpha, c_\beta], c_\gamma) + ([c_{\alpha_1}, c_{\beta_1}], c_{\gamma_1}) = ([c_\alpha + c_{\alpha_1}, c_\beta + c_{\beta_1}], c_\gamma + c_{\gamma_1})$.

In all cases $d_2(x) = ([k_\alpha, k_\beta], k_\gamma)$ and $d(x) = ([r_\alpha, r_\beta], r_\gamma), \forall x \in \mathcal{P}$. Hence C^* is a $(([r_\alpha, r_\beta], r_\gamma), 2, ([k_\alpha, k_\beta], k_\gamma))$ - Regular Cubic Fuzzy Graph. □

Remark 6.2. *If a Cubic Fuzzy Graph $\mathcal{C}^*$ on an even cycle $\mathcal{G}$ of length ≥ 8, where alternate edges share identical IVF membership numbers and fuzzy membership numbers. Surprisingly, $\mathcal{C}^*$ does not necessarily satisfy $(([r_\alpha, r_\beta], r_\gamma), 2, ([k_\alpha, k_\beta], k_\gamma))$ - Total Regular Cubic Fuzzy Graph. Furthermore, since the function $\mathcal{L}$ is non-constant. $\mathcal{C}^*$ is not a Total $(([r_\alpha, r_\beta], r_\gamma), 2, ([k_\alpha, k_\beta], k_\gamma))$ - Regular Cubic Fuzzy Graph.*

Example 6.2. *Consider a graph $\mathcal{G} : (\mathcal{P}, \mathcal{C})$ with $\mathcal{P} = \{\hat{u}_1, \hat{u}_2, \hat{u}_3, \hat{u}_4, \hat{u}_5, \hat{u}_6, \hat{u}_7, \hat{u}_8\}$ and $\mathcal{C} = \{\hat{u}_1\hat{u}_2, \hat{u}_2\hat{u}_3, \hat{u}_3\hat{u}_4, \hat{u}_4\hat{u}_5, \hat{u}_5\hat{u}_6, \hat{u}_6\hat{u}_7, \hat{u}_7\hat{u}_8, \hat{u}_8\hat{u}_1\}$ is defined a Cubic Fuzzy Graph $\mathcal{C}^* : (\mathcal{L}, \mathcal{M})$.* $\mathcal{M}(\hat{u}_1\hat{u}_2) = ([0.4, 0.5], 0.3), \forall \hat{u}_1\hat{u}_2 \in \mathcal{C}$ *and* $\mathcal{L}(\hat{u}_1) = ([0.65, 0.75], 0.45), \mathcal{L}(\hat{u}_2) = ([0.5, 0.7], 0.4), \mathcal{L}(\hat{u}_3) = ([0.7, 0.9], 0.5), \mathcal{L}(\hat{u}_4) = ([0.55, 0.65], 0.35), \mathcal{L}(\hat{u}_5) = ([0.45, 0.55], 0.4), \mathcal{L}(\hat{u}_6) = ([0.6, 0.8], 0.3), \mathcal{L}(\hat{u}_7) = ([0.75, 0.85], 0.5), \mathcal{L}(\hat{u}_8) = ([0.8, 0.9], 0.6)$.

Here $d_2(\hat{u}_1) = ([0.8, 1.0], 0.6), \forall \hat{u}_1 \in \mathcal{P}$ and $d(\hat{u}_1) = ([0.8, 1.0], 0.6), \forall \hat{u}_1 \in \mathcal{P}$. Thus $\mathcal{C}^*$ is a $(([0.8, 1.0], 0.6), 2, ([0.8, 1.0], 0.6))$ - Regular Cubic Fuzzy Graph but $\mathcal{C}^*$ is not a Total $(([r_\alpha, r_\beta], r_\gamma), 2, ([k_\alpha, k_\beta], k_\gamma))$ - Regular Cubic Fuzzy Graph.

Theorem 6.3. *Let a Cubic Fuzzy Graph $\mathcal{C}^* : (\mathcal{L}, \mathcal{M})$ on an even cycle $\mathcal{G} : (\mathcal{P}, \mathcal{C})$ with length ≥ 4. Let $c_{\alpha_1} \geq c_\alpha$, $c_{\beta_1} \geq c_\beta$ and $c_{\gamma_1} \geq c_\gamma$. Let $\alpha_1(e_i) = \begin{cases} c_\alpha & \text{for odd } i \\ c_{\alpha_1} & \text{for even } i \end{cases}$, $\beta_1(e_i) = \begin{cases} c_\beta & \text{for odd } i \\ c_{\beta_1} & \text{for even } i \end{cases}$ and $\gamma_1(e_i) = \begin{cases} c_\gamma & \text{for odd } i \\ c_{\gamma_1} & \text{for even } i \end{cases}$. Then $\mathcal{C}^* : (\mathcal{L}, \mathcal{M})$ is a $(([r_\alpha, r_\beta], r_\gamma), 2, ([k_\alpha, k_\beta], k_\gamma))$ - Regular Cubic Fuzzy Graph.*

Proof. ***Case (i)***. Let a Cubic Fuzzy Graph $\mathcal{C}^* : (\mathcal{L}, \mathcal{M})$ on an even cycle $\mathcal{G} : (\mathcal{P}, \mathcal{C})$ of length ≤ 8. Then, by Theorem 6.2, $\mathcal{C}^*$ is a $(([r_\alpha, r_\beta], r_\gamma), 2, ([k_\alpha, k_\beta], k_\gamma))$ - Regular Cubic Fuzzy Graph.

Case (ii). Let a Cubic Fuzzy Graph $\mathcal{C}^* : (\mathcal{L}, \mathcal{M})$ on an odd cycle $\mathcal{G} : (\mathcal{P}, \mathcal{C})$ of length ≤ 5.

$$d_2(\hat{u}) = ([c_\alpha \wedge c_{\alpha_1},\, c_\beta \wedge c_{\beta_1}], c_\gamma \wedge c_{\gamma_1}) + ([c_\alpha \wedge c_{\alpha_1},\, c_\beta \wedge c_{\beta_1}], c_\gamma \wedge c_{\gamma_1})$$
$$= ([c_\alpha, c_\beta], c_\gamma) + ([c_\alpha, c_\beta], c_\gamma) = ([2c_\alpha, 2c_\beta], 2c_\gamma), \forall \hat{u} \in \mathcal{P}$$
$$d(\hat{u}_1) = ([c_\alpha, c_\beta], c_\gamma) + ([c_\alpha, c_\beta], c_\gamma) = ([2c_\alpha, 2c_\beta], 2c_\gamma)$$
$$d(\hat{u}_i) = ([c_\alpha, c_\beta], c_\gamma) + ([c_{\alpha_1}, c_{\beta_1}], c_{\gamma_1}) = ([c_\alpha + c_{\alpha_1}, c_\beta + c_{\beta_1}], c_\gamma + c_{\gamma_1})$$

So, $d(\hat{u}_i) \neq d(\hat{u}_1)$. Hence, $\mathcal{C}^*$ is not a $(([r_\alpha, r_\beta], r_\gamma), 2, ([k_\alpha, k_\beta], k_\gamma))$ - Regular Cubic Fuzzy Graph. □

Remark 6.3. *Let a Cubic Fuzzy Graph $\mathcal{C}^* : (\mathcal{L}, \mathcal{M})$ on an even cycle $\mathcal{G} : (\mathcal{P}, \mathcal{C})$ of length ≥ 5. If $\alpha_1(e_i) = \begin{cases} c_\alpha & \text{for odd } i \\ c_{\alpha_1} & \text{for even } i \end{cases}$, $\beta_1(e_i) = \begin{cases} c_\beta & \text{for odd } i \\ c_{\beta_1} & \text{for even } i \end{cases}$ and $\gamma_1(e_i) = \begin{cases} c_\gamma & \text{for odd } i \\ c_{\gamma_1} & \text{for even } i \end{cases}$ Then $\mathcal{C}^*$ is not necessarily a $(([r_\alpha, r_\beta], r_\gamma), 2, ([k_\alpha, k_\beta], k_\gamma))$ - Total Regular Cubic Fuzzy Graph. Furthermore, since $\mathcal{L}$ is non-constant, $\mathcal{C}^*$ also fails to be a Total $(([r_\alpha, r_\beta], r_\gamma), 2, ([k_\alpha, k_\beta], k_\gamma))$ - Regular Cubic Fuzzy Graph.*

7. Results and Discussions

This article builds upon previous research on regular fuzzy graphs, including (2, k)-regular and (r, 2, k)-regular fuzzy graphs, as well as pseudo-regular intuitionistic fuzzy graphs. The novelty of this work lies in analysing these concepts to cubic fuzzy graphs by introducing new regularity classes, namely $(([r_\alpha, r_\beta], r_\gamma), 2, ([k_\alpha, k_\beta], k_\gamma))$ - regular and totally $(([r_\alpha, r_\beta], r_\gamma), 2, ([k_\alpha, k_\beta], k_\gamma))$ - regular cubic fuzzy graphs. This extension generalizes the classical graph regularity into the fuzzy domain by applying constraints to both vertex and edge membership functions. The article defines and analyzes the new degree measures d_2- degree and total d_2- degree of Cubic Fuzzy Graphs and it establishes an important equivalence between the regularity of a Fuzzy Graphs and that of its underlying crisp graphs. Additionally, it examines these regularity conditions in Cyclic Cubic Fuzzy Graphs and enhancing the structural understanding of fuzzy graph models. However, the study is limited to Cubic Fuzzy Graphs with fixed membership intervals and does not address the computational aspects or generalization of the same to the broader classes. It focuses on the specific types of Cubic Fuzzy Graphs so that the results may not be directly applicable to all types of Cubic Fuzzy Graphs warranting further research to extend these concepts and explore their broader implications.

8. Conclusion

This study makes a significant contribution to Fuzzy Graph theory by introducing the novel concepts of $(([r_\alpha, r_\beta], r_\gamma), 2, ([k_\alpha, k_\beta], k_\gamma))$ - regular and total $(([r_\alpha, r_\beta], r_\gamma), 2, ([k_\alpha, k_\beta], k_\gamma))$ - regular cubic fuzzy graphs. Now regularity conditions are established based on vertex d_2 - degree and total d_2 - degree, thereby extending the theoretical framework of Cubic Fuzzy Graphs. Applications of these concepts have been demonstrated through cyclic cubic fuzzy graphs. The results have potential applications in various fields like network network analysis, decision-making and optimization problems. Also, the newly introduced concepts are effective tools in modeling and solving real-world problems characterized by uncertainty and imprecision.

9. Future Directions

The findings of this research work an analysis on $(([r_\alpha, r_\beta], r_\gamma), 2, ([k_\alpha, k_\beta], k_\gamma))$ - Regular and Total Regular Cubic Fuzzy Graphs. It not only contributes to the theoretical foundation of fuzzy graph theory by also open up new avenues for applying cubic fuzzy graphs in diverse domains. The concepts of regularity and total regularity can be extended to higher-dimensional fuzzy graphs, which can provide new insights into the structural properties of complex fuzzy graphs. Future research can be built upon these results to explore more complex structures and applications, underscoring the potential of cubic fuzzy graphs to model and solve real-world problems with uncertainty and imprecision.Investigating the applications of cubic fuzzy graphs in real-world problems, such as image processing, data mining and social network analysis, this concept can uncover novel solutions to the practical problems.

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