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NORMS OF CUBIC NEUTROSOPHIC SUBRINGS AND IDEALS

B. ANITHA[1], M. LAVANYA[1*], §

Abstract. In this paper, the notion of cubic neutrosophic subrings is thoroughly investigated, with a particular focus on the role of norms within these structures. The homomorphism theorem associated with cubic neutrosophic subrings is examined in detail, along with several related propositions that explore the properties and behavior of these mathematical objects. Additionally, the paper delves into the development of theorems concerning cubic neutrosophic ideals and level sets.

Keywords: Fuzzy Set, Cubic Set, Neutrosophic Sets, cubic neutrosophic set, Cubic Neutrosophic Ideals of Subring, t_n-t-norm, s_n-s-norm.

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1. Introduction

In 1965, Zadeh [11] introduced the concept of fuzzy set, which allowed for the representation of uncertainty in mathematical terms by assigning each element of a set a membership value between 0 and 1. This model addressed imprecision by enabling the representation of vague or uncertain information. Later, in 1975, Zadeh [12] further extended this concept by presenting interval valued fuzzy set, where instead of a single membership value, each element is assigned an interval. This extension provided a more flexible and nuanced approach to dealing with uncertainty, especially in cases where the exact value of membership is uncertain but falls within a certain range.

Jun et al. [8] introduced the idea of cubic set, which built on fuzzy and interval-valued fuzzy sets. A cubic set is characterized by the use of cubic functions for the membership grades of elements. This approach offers a more sophisticated representation of uncertainty compared to traditional fuzzy set and interval valued fuzzy set. The concepts of fuzzy and interval valued fuzzy set play an essential role in the foundation and development of cubic sets, allowing for a richer and more adaptable model of uncertainty.

Florentin Smarandache [9] introduced neutrosophic set, which generalized the fuzzy set theory further by norms three independent functions for truth, indeterminacy, and falsity. neutrosophic set are particularly suited for handling situations with contradictory,

[1] Department of Mathematics -Annamalai University - India.
e-mail: anibhavan05@gmail.com; ORCID no. https://orcid.org/0009-0001-8185-7318.
e-mail: lavanyamohan3194@gmail.com; ORCID no. https://orcid.org/0009-0007-2430-151X.
[*] Corresponding author.

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incomplete, or uncertain data, as they provide a way to separately model these dif and only iferent types of information.

Later, H. Wang et al. [10] advanced the theory by proposing interval valued neutrosophic set, which integrate the idea of interval values with the neutrosophic framework. This allows for more complex representations of uncertainty, where each of the truth, indeterminacy, and falsity components can be represented by intervals rather than fixed numbers.

Further extending these concepts, Gayen and Sudipta et al. [5] examined interval valued neutrosophic subring under triangular norms. They studied the properties and behaviors of these structures, particularly focusing on the interactions between interval valued neutrosophic sets and triangular norms. Their work contributes to the understanding of how these advanced structures function within algebraic settings, particularly in ring theory.

B. Anitha et al. [2] explored the use of norms in neutrosophic multi-fuzzy subrings, investigating how dif and only iferent norms influence the algebraic structures in neutrosophic and fuzzy contexts. This helped to reveal deeper insights into the interactions between fuzzy logic, neutrosophic set, and algebraic systems like subrings.

The present study extends these previous developments by introducing the concept of cubic neutrosophic set within the framework of ring theory. Specifically, we propose cubic neutrosophic subring, where elements are represented by cubic-valued neutrosophic sets. This new concept enhances traditional neutrosophic subrings by incorporating cubic structures, providing a more detailed and refined approach to the representation of uncertainty. Additionally, we explore the behavior of these cubic neutrosophic subrings when subjected to triangular norms, which are commonly used in fuzzy logic and neutrosophic systems. Through this study, we aim to interpret the various properties and interactions of cubic neutrosophic subring, offering a deeper understanding of their algebraic structures and potential applications in advanced theoretical frameworks.

2. Preliminary

Definition 2.1. [8] *Let X be a non-empty set. A cubic set $\mathbb{C}$ in X is a structure of the form $\mathbb{C} = \{\grave{k}, \tilde{\varrho}(\grave{k}), \varrho(\grave{k})/\grave{k} \in X)\}$ and denoted by $\mathbb{C} =< \tilde{\varrho}, \varrho >$, where $\tilde{\varrho} = [\varrho^L, \varrho^U]$ is an interval valued fuzzy set in X and ϱ is a fuzzy set in X.*

Definition 2.2. [9] *Let X be a universel set. Then a neutrosophic set $\mathfrak{N}$ on X $\mathfrak{N} = \{\grave{k}, \zeta(\grave{k}), \varrho(\grave{k}), \varsigma(\grave{k})/\grave{k} \in X\}$, with $\zeta, \varrho, \varsigma \in [0,1]$.*

Definition 2.3. [5] *An interval valued neutrosophic set of X is denoted as $\tilde{\mathfrak{N}} = \{\grave{k}, \tilde{\zeta}(\grave{k}), \tilde{\varrho}(\grave{k}), \tilde{\varsigma}(\grave{k})/\grave{k} \in X\}$ where $\tilde{\zeta}, \tilde{\varrho}, \tilde{\varsigma} \in [0,1]$.*

Definition 2.4. [1] *Let $t_n : [0,1] \times [0,1] \to [0,1]$ be a t-norm possess the following axioms*

(i) $t_n(\grave{n}, 1) = \grave{n}$
(ii) $t_n(\grave{n}, \grave{k}) = t_n(\grave{n}, m)$ if $\grave{k} \leq \grave{m}$
(iii) $t_n(\grave{n}, \grave{k}) = t_n(\grave{k}, n)$
(iv) $t_n(\grave{n}, t_n(\grave{k}, \grave{m})) = t_n(t_n(\grave{n}, \grave{k}), \grave{m}),\ \forall \grave{n}, \grave{k}, \grave{m} \in [0,1]$.

Definition 2.5. [4] *Let $s_n : [0,1] \times [0,1] \to [0,1]$ be a s-norm possess the following axioms*

(i) $s_n(\grave{n}, 0) = \grave{n}$
(ii) $s_n(\grave{n}, \grave{k}) = s_n(\grave{n}, \grave{m})$ if $\grave{k} \leq \grave{m}$
(iii) $s_n(\grave{n}, \grave{k}) = s_n(\grave{k}, \grave{n})$

(iv) $s_n(\grave{n}, s_n(\grave{k}, \grave{m})) = s_n(s_n(\grave{n}, \grave{k}), \grave{m}),\ \forall \grave{n}, \grave{k}, \grave{m} \in [0,1]$.

Note that if t_n is idempotent function then $t_n(\grave{n}, \grave{n}) = \grave{n}$ also if s_n is idempotent function then $s_n(\grave{n}, \grave{n}) = \grave{n},\ \forall \grave{n} \in [0,1]$.

3. Cubic Neutrosophic subring with respect to t_n and s_n

Definition 3.1. *Let $\mathfrak{X}$ be a non empty set. A cubic neutrosophic set in $\mathfrak{X}$. It can be expressed as $\mathfrak{C} = \{\grave{k}, \zeta^\star(\grave{k}), \varrho^\star(\grave{k}), \varsigma^\star(\grave{k})/\grave{k} \in \mathfrak{X}\}$.*

Where, $\zeta^\star =< \tilde{\zeta}, \zeta >\in [0,1], \tilde{\zeta}$ is an interval valued truth membership function and ζ is a truth membership funtion,

$\varrho^\star =< \tilde{\varrho}, \varrho >\in [0,1], \tilde{\varrho}$ is an interval valued indeterminacy membership function and ϱ is a indeterminacy membership funtion,

$\varsigma^\star =< \tilde{\varsigma}, \varsigma >\in [0,1], \tilde{\varsigma}$ is an interval valued falsity membership function and ς is a falsity membership funtion,

with the condition that $0 \leq \tilde{\zeta} + \tilde{\varrho} + \tilde{\varsigma} \leq 3$ and $0 \leq \zeta + \varrho + \varsigma \leq 3$.

Definition 3.2. *A cubic neutrosophic set $\mathfrak{C} = \{\grave{k}, \zeta^\star_{\mathfrak{C}}(\grave{k}), \varrho^\star_{\mathfrak{C}}(\grave{k}), \varsigma^\star_{\mathfrak{C}}(\grave{k})/\grave{k} \in \mathfrak{R}\}$ of ring $\mathfrak{R}$ is referred to as cubic neutrosophic subring with respect to t_n and s_n of $\mathfrak{R}$ if*

(i) $\tilde{\zeta}_{\mathfrak{C}}(\grave{n} - \grave{k}) \geq t_n\{\tilde{\zeta}_{\mathfrak{C}}(\grave{n}), \tilde{\zeta}_{\mathfrak{C}}(\grave{k})\}; \zeta_{\mathfrak{C}}(\grave{n} - \grave{k}) \leq s_n\{\zeta_{\mathfrak{C}}(\grave{n}), \zeta_{\mathfrak{C}}(\grave{k})\}$

$\tilde{\varrho}_{\mathfrak{C}}(\grave{n} - \grave{k}) \leq s_n\{\tilde{\varrho}_{\mathfrak{C}}(\grave{n}), \tilde{\varrho}_{\mathfrak{C}}(\grave{k})\}; \varrho_{\mathfrak{C}}(\grave{n} - \grave{k}) \geq t_n\{\varrho_{\mathfrak{C}}(\grave{n}), \varrho_{\mathfrak{C}}(\grave{k})\}$

$\tilde{\varsigma}_{\mathfrak{C}}(\grave{n} - \grave{k}) \leq s_n\{\tilde{\varsigma}_{\mathfrak{C}}(\grave{n}), \tilde{\varsigma}_{\mathfrak{C}}(\grave{k})\}; \varsigma_{\mathfrak{C}}(\grave{n} - \grave{k}) \geq t_n\{\varsigma_{\mathfrak{C}}(\grave{n}), \varsigma_{\mathfrak{C}}(\grave{k})\}$

(ii) $\tilde{\zeta}_{\mathfrak{C}}(\grave{n}\grave{k}) \geq t_n\{\tilde{\zeta}_{\mathfrak{C}}(\grave{n}), \tilde{\zeta}_{\mathfrak{C}}(\grave{k})\}; \zeta_{\mathfrak{C}}(\grave{n}\grave{k}) \leq s_n\{\zeta_{\mathfrak{C}}(\grave{n}), \zeta_{\mathfrak{C}}(\grave{k})\}$

$\tilde{\varrho}_{\mathfrak{C}}(\grave{n}\grave{k}) \leq s_n\{\tilde{\varrho}_{\mathfrak{C}}(\grave{n}), \tilde{\varrho}_{\mathfrak{C}}(\grave{k})\}; \varrho_{\mathfrak{C}}(\grave{n}\grave{k}) \geq t_n\{\varrho_{\mathfrak{C}}(\grave{n}), \varrho_{\mathfrak{C}}(\grave{k})\}$

$\tilde{\varsigma}_{\mathfrak{C}}(\grave{n}\grave{k}) \leq s_n\{\tilde{\varsigma}_{\mathfrak{C}}(\grave{n}), \tilde{\varsigma}_{\mathfrak{C}}(\grave{k})\}; \varsigma_{\mathfrak{C}}(\grave{n}\grave{k}) \geq t_n\{\varsigma_{\mathfrak{C}}(\grave{n}), \varsigma_{\mathfrak{C}}(\grave{k})\}$. *Where $\grave{n}, \grave{k} \in \mathfrak{R}$.*

Example 3.1. *Consider Z_2 is a ring. We define a cubic neutrosophic set $\mathfrak{C}$ over t_n and s_n as $\mathfrak{C} = \{\langle 0, < [0.8, 0.9]0.3 >, < [0.6, 0.7], 0.7 >, < [0.5, 0.7], 0.6 >\rangle, \langle 1, < [0.4, 0.5], 0.8 >, < [0.7, 0.9], 0.2 >, < [0.8, 0.9], 0.4 >\rangle\}$.*

Let $t_n(\grave{n}, \grave{k}) = \min(\grave{n}, \grave{k}), s_n = \max(\grave{n}, \grave{k})$ then $\mathfrak{C}$ is cubic neutrosophic subring of Z_2 over t_n and s_n.

Proposition 3.1. *Let $\mathfrak{C}$ be a cubic neutrosophic subring of $\mathfrak{R}$ with respect to t_n and s_n, where t_n, s_n are idempotent then*

(i) $\tilde{\zeta}_{\mathfrak{C}}(0) \geq \tilde{\zeta}_{\mathfrak{C}}(\grave{k}); \zeta_{\mathfrak{C}}(0) \leq \zeta_{\mathfrak{C}}(\grave{k})$

$\tilde{\varrho}_{\mathfrak{C}}(0) \leq \tilde{\varrho}_{\mathfrak{C}}(\grave{k}); \varrho_{\mathfrak{C}}(0) \geq \varrho_{\mathfrak{C}}(\grave{k})$

$\tilde{\varsigma}_{\mathfrak{C}}(0) \geq \tilde{\varsigma}_{\mathfrak{C}}(\grave{k}); \varsigma_{\mathfrak{C}}(0) \leq \varsigma_{\mathfrak{C}}(\grave{k})$

(ii) $\zeta^\star_{\mathfrak{C}}(-\grave{k}) = \zeta^\star_{\mathfrak{C}}(\grave{k}), \varrho^\star_{\mathfrak{C}}(-\grave{k}) = \varrho^\star_{\mathfrak{C}}(\grave{k}), \varsigma^\star_{\mathfrak{C}}(-\grave{k}) = \varsigma^\star_{\mathfrak{C}}(\grave{k})$. $\forall \grave{k} \in \mathfrak{R}$

Proof. Let $\grave{k} \in \mathfrak{R}$. Then we have

(i) $\tilde{\zeta}_{\mathfrak{C}}(0) = \tilde{\zeta}_{\mathfrak{C}}(\grave{k} - \grave{k}) \geq t_n\{\tilde{\zeta}_{\mathfrak{C}}(\grave{k}), \tilde{\zeta}_{\mathfrak{C}}(\grave{k})\} = \tilde{\zeta}_{\mathfrak{C}}(\grave{k})$

$\zeta_{\mathfrak{C}}(0) = \zeta_{\mathfrak{C}}(\grave{k} - \grave{k}) \geq t_n\{\zeta_{\mathfrak{C}}(\grave{k}), \zeta_{\mathfrak{C}}(\grave{k})\} = \zeta_{\mathfrak{C}}(\grave{k})$.

Similarly, $\tilde{\varrho}_{\mathfrak{C}}(0) \leq \tilde{\varrho}_{\mathfrak{C}}(\grave{k})$ and $\varrho_{\mathfrak{C}}(0) \geq \varrho_{\mathfrak{C}}(\grave{k})$

(ii)
$$\begin{aligned}
\tilde{\zeta}_{\mathfrak{C}}(-\grave{k}) &= \tilde{\zeta}_{\mathfrak{C}}(0 - \grave{k}) \\
&\geq t_n\{\tilde{\zeta}_{\mathfrak{C}}(0), \tilde{\zeta}_{\mathfrak{C}}(\grave{k})\} \\
&\geq t_n\{\tilde{\zeta}_{\mathfrak{C}}(\grave{k}), \tilde{\zeta}_{\mathfrak{C}}(\grave{k})\} \\
&= \tilde{\zeta}_{\mathfrak{C}}(\grave{k}) = \tilde{\zeta}_{\mathfrak{C}}(0 - (-\grave{k})) \\
&\geq t_n\{\tilde{\zeta}_{\mathfrak{C}}(0), \tilde{\zeta}_{\mathfrak{C}}(-\grave{k})\} \\
&\geq t_n\{\tilde{\zeta}_{\mathfrak{C}}(0), \tilde{\zeta}_{\mathfrak{C}}(\grave{k})\} \\
&= \tilde{\zeta}_{\mathfrak{C}}(-\grave{k})
\end{aligned}$$

$$\begin{aligned}\zeta_{\mathfrak{C}}(-\grave{k}) &= \zeta_{\mathfrak{C}}(0-\grave{k})\\ &\leq s_n\{\zeta_{\mathfrak{C}}(0),\zeta_{\mathfrak{C}}(\grave{k})\}\\ &\leq s_n\{\zeta_{\mathfrak{C}}(\grave{k}),\zeta_{\mathfrak{C}}(\grave{k})\}\\ &= \zeta_{\mathfrak{C}}(\grave{k}) = \zeta_{\mathfrak{C}}(0-(-\grave{k}))\\ &\leq s_n\{\zeta_{\mathfrak{C}}(0),\zeta_{\mathfrak{C}}(-\grave{k})\}\\ &\leq s_n\{\zeta_{\mathfrak{C}}(0),\zeta_{\mathfrak{C}}(\grave{k})\}\\ &= \zeta_{\mathfrak{C}}(-\grave{k}).\end{aligned}$$

So that $\zeta^\star_{\mathfrak{C}}(-\grave{k}) = \zeta^\star_{\mathfrak{C}}(\grave{k})$.

Similarly, $\varrho^\star_{\mathfrak{C}}(-\grave{k}) = \varrho^\star_{\mathfrak{C}}(\grave{k})$ and $\varsigma^\star_{\mathfrak{C}}(-\grave{k}) = \varsigma^\star_{\mathfrak{C}}(\grave{k})$. $\quad\forall \in \mathfrak{R}$. Hence the result. □

Proposition 3.2. *Let $\mathfrak{C}$ be a cubic neutrosophic subring of $\mathfrak{R}$ under t_n and s_n then*
$\tilde{\zeta}_{\mathfrak{C}}(\grave{n}-\grave{k}) = 1 \Rightarrow \tilde{\zeta}_{\mathfrak{C}}(\grave{n}) \geq \tilde{\zeta}_{\mathfrak{C}}(\grave{k}); \zeta_{\mathfrak{C}}(\grave{n}-\grave{k}) = 0 \Rightarrow \zeta_{\mathfrak{C}}(\grave{n}) \leq \zeta_{\mathfrak{C}}(\grave{k})$
$\tilde{\varrho}_{\mathfrak{C}}(\grave{n}-\grave{k}) = 0 \Rightarrow \tilde{\varrho}_{\mathfrak{C}}(\grave{n}) \leq \tilde{\varrho}_{\mathfrak{C}}(\grave{k}); \varrho_{\mathfrak{C}}(\grave{n}-\grave{k}) = 1 \Rightarrow \varrho_{\mathfrak{C}}(\grave{n}) \geq \varrho_{\mathfrak{C}}(\grave{k})$
$\tilde{\varsigma}_{\mathfrak{C}}(\grave{n}-\grave{k}) = 0 \Rightarrow \tilde{\varsigma}_{\mathfrak{C}}(\grave{n}) \leq \tilde{\varsigma}_{\mathfrak{C}}(\grave{k}); \varsigma_{\mathfrak{C}}(\grave{n}-\grave{k}) = 1 \Rightarrow \varsigma_{\mathfrak{C}}(\grave{n}) \geq \varsigma_{\mathfrak{C}}(\grave{k}), \forall \grave{n},\grave{k} \in \mathfrak{R}.$

Proof. Let $\grave{n},\grave{k} \in \mathfrak{R}$. Then we have
$\tilde{\zeta}_{\mathfrak{C}}(\grave{n}) = \tilde{\zeta}_{\mathfrak{C}}(\grave{n}-\grave{k}+\grave{k}) \geq t_n\{\tilde{\zeta}_{\mathfrak{C}}(\grave{n}-\grave{k}),\tilde{\zeta}_{\mathfrak{C}}(\grave{k}\} = t_n\{1,\tilde{\zeta}_{\mathfrak{C}}(\grave{k}\} = \tilde{\zeta}_{\mathfrak{C}}(\grave{k})$
$\zeta_{\mathfrak{C}}(\grave{n}) = \zeta_{\mathfrak{C}}(\grave{n}-\grave{k}+\grave{k}) \leq s_n\{\zeta_{\mathfrak{C}}(\grave{n}-\grave{k}),\zeta_{\mathfrak{C}}(\grave{k}\} = s_n\{0,\zeta_{\mathfrak{C}}(\grave{k}\} = \zeta_{\mathfrak{C}}(\grave{k})$.

Similarly, $\tilde{\varrho}_{\mathfrak{C}}(\grave{n}) \leq \tilde{\varrho}_{\mathfrak{C}}(\grave{k})$ and $\varrho_{\mathfrak{C}}(\grave{n}) \geq \varrho_{\mathfrak{C}}(\grave{k})$
$\tilde{\varsigma}_{\mathfrak{C}}(\grave{n}) \leq \tilde{\varsigma}_{\mathfrak{C}}(\grave{k})$ and $\varsigma_{\mathfrak{C}}(\grave{n}) \geq \varsigma_{\mathfrak{C}}(\grave{k})$. Hence the result. □

Proposition 3.3. *Let $\mathfrak{C}$ be a cubic neutrosophic subring of $\mathfrak{R}$ with respect to t_n and s_n, where t_n, s_n are idempotent then $\mathfrak{C}(\grave{n}-\grave{k}) = \mathfrak{C}(\grave{k})$ if and only if $\mathfrak{C}(\grave{n}) = \mathfrak{C}(0)$.* $\quad\forall \grave{n},\grave{k} \in \mathfrak{R}$.

Proof. Let $\mathfrak{C}(\grave{n}-\grave{k}) = \mathfrak{C}(\grave{k})$. If $\grave{k} = 0 \Rightarrow \mathfrak{C}(\grave{n}) = \mathfrak{C}(0)$.
Conversely, if $\mathfrak{C}(\grave{n}) = \mathfrak{C}(0)$. Then
$\tilde{\zeta}_{\mathfrak{C}}(\grave{n}) = \tilde{\zeta}_{\mathfrak{C}}(0) \geq \tilde{\zeta}_{\mathfrak{C}}(\grave{n}-\grave{k})$
$\tilde{\zeta}_{\mathfrak{C}}(\grave{n}) = \tilde{\zeta}_{\mathfrak{C}}(0) \geq \tilde{\zeta}_{\mathfrak{C}}(\grave{k})$(by proposition)

$$\begin{aligned}\text{Now, } \tilde{\zeta}_{\mathfrak{C}}(\grave{n}-\grave{k}) &\geq t_n\{\tilde{\zeta}_{\mathfrak{C}}(\grave{n}),\tilde{\zeta}_{\mathfrak{C}}(\grave{k})\}\\ &\geq t_n\{\tilde{\zeta}_{\mathfrak{C}}(\grave{k}),\tilde{\zeta}_{\mathfrak{C}}(\grave{k})\}\\ &= \tilde{\zeta}_{\mathfrak{C}}(\grave{k}) = \tilde{\zeta}_{\mathfrak{C}}(-\grave{k})\\ &= \tilde{\zeta}_{\mathfrak{C}}(\grave{n}-\grave{k}-\grave{n})\\ &\geq t_n\{\tilde{\zeta}_{\mathfrak{C}}(\grave{n}-\grave{k}),\tilde{\zeta}_{\mathfrak{C}}(\grave{n})\}\\ &\geq t_n\{\tilde{\zeta}_{\mathfrak{C}}(\grave{n}-\grave{k}),\tilde{\zeta}_{\mathfrak{C}}(\grave{n}-\grave{k})\}\\ &= \tilde{\zeta}_{\mathfrak{C}}(\grave{n}-\grave{k}).\end{aligned}$$

$$\begin{aligned}\zeta_{\mathfrak{C}}(\grave{n}-\grave{k}) &\leq s_n\{\zeta_{\mathfrak{C}}(\grave{n}),\zeta_{\mathfrak{C}}(\grave{k})\}\\ &\leq s_n\{\zeta_{\mathfrak{C}}(\grave{k}),\zeta_{\mathfrak{C}}(\grave{k})\}\\ &= \zeta_{\mathfrak{C}}(\grave{k}) = \zeta_{\mathfrak{C}}(-\grave{k})\text{(by proposition)}\\ &= \zeta_{\mathfrak{C}}(\grave{n}-\grave{k}-\grave{n})\\ &\leq s_n\{\zeta_{\mathfrak{C}}(\grave{n}-\grave{k}),\zeta_{\mathfrak{C}}(\grave{n})\}\\ &\leq s_n\{\zeta_{\mathfrak{C}}(\grave{n}-\grave{k}),\zeta_{\mathfrak{C}}(\grave{n}-\grave{k})\}\\ &= \zeta_{\mathfrak{C}}(\grave{n}-\grave{k}).\end{aligned}$$

Therefore $\zeta^\star_{\mathfrak{C}}(\grave{n}-\grave{k}) = \zeta^\star_{\mathfrak{C}}(\grave{k})$.

Similarly, $\varrho^\star_{\mathfrak{C}}(\grave{n}-\grave{k}) = \varrho^\star_{\mathfrak{C}}(\grave{k})$ and $\varsigma^\star_{\mathfrak{C}}(\grave{n}-\grave{k}) = \varsigma^\star_{\mathfrak{C}}(\grave{k})$.

Hence $\mathfrak{C}(\grave{n}-\grave{k}) = \mathfrak{C}(\grave{k}).\forall\grave{k},\grave{k} \in \mathfrak{R}$. □

Theorem 3.1. *If $\mathbf{I}$ and $\mathbf{G}$ are any two cubic neutrosophic subring of $\mathfrak{R}$ then $\mathbf{I}\cap\mathbf{G}$ is also a cubic neutrosophic subring of $\mathfrak{R}$ with respect to t_n and s_n, where t_n and s_n are idempotent.*

Proof. Let **I** and **G** be any two cubic neutrosophic subring of $\mathfrak{R}$. Let $\grave{n}, \grave{k} \in \mathfrak{R}$

(i)
$$\begin{aligned}
\tilde{\zeta}_{\mathfrak{e}I\cap G}(\grave{n}-\grave{k}) &= t_n\{\tilde{\zeta}_{\mathfrak{e}I}(\grave{n}-\grave{k}), \tilde{\zeta}_{\mathfrak{e}G}(\grave{n}-\grave{k})\}\\
&\geq t_n\{t_n\{\tilde{\zeta}_{\mathfrak{e}I}(\grave{n}), \tilde{\zeta}_{\mathfrak{e}I}(\grave{k})\}, t_n\{\tilde{\zeta}_{\mathfrak{e}G}(\grave{n}), \tilde{\zeta}_{\mathfrak{e}G}(\grave{k})\}\}\\
&= t_n\{t_n\{\tilde{\zeta}_{\mathfrak{e}I}(\grave{n}), \tilde{\zeta}_{\mathfrak{e}G}(\grave{n})\}, t_n\{\tilde{\zeta}_{\mathfrak{e}I}(\grave{k}), \tilde{\zeta}_{\mathfrak{e}G}(\grave{k})\}\}\\
&= t_n\{\tilde{\zeta}_{\mathfrak{e}I\cap G}(\grave{n}), \tilde{\zeta}_{\mathfrak{e}I\cap G}(\grave{k})\}\\
\zeta_{\mathfrak{e}_{I\cap G}}(\grave{n}-\grave{k}) &= s_n\{\zeta_{\mathfrak{e}_I}(\grave{n}-\grave{k}), \zeta_{\mathfrak{e}_G}(\grave{n}-\grave{k})\}\\
&\leq s_n\{s_n\{\zeta_{\mathfrak{e}_I}(\grave{n}), \zeta_{\mathfrak{e}_I}(\grave{k})\}, s_n\{\zeta_{\mathfrak{e}_G}(\grave{n}), \zeta_{\mathfrak{e}_G}(\grave{k})\}\}\\
&= s_n\{s_n\{\zeta_{\mathfrak{e}_I}(\grave{n}), \zeta_{\mathfrak{e}_G}(\grave{n})\}, s_n\{\zeta_{\mathfrak{e}_I}(\grave{k}), \zeta_{\mathfrak{e}_G}(\grave{k})\}\}\\
&= s_n\{\zeta_{\mathfrak{e}_{I\cap G}}(\grave{n}), \zeta_{\mathfrak{e}_{I\cap G}}(\grave{k})\}.
\end{aligned}$$

Similarly, $\tilde{\varrho}_{\mathfrak{e}I\cap G}(\grave{n}-\grave{k}) \leq s_n\{\tilde{\varrho}_{\mathfrak{e}I\cap G}(\grave{n}), \tilde{\varrho}_{\mathfrak{e}I\cap G}(\grave{k})\}$ and $\varrho_{\mathfrak{e}_{I\cap G}}(\grave{n}-\grave{k}) \geq t_n\{\varrho_{\mathfrak{e}_{I\cap G}}(\grave{n}), \varrho_{\mathfrak{e}_{I\cap G}}(\grave{k})\}, \tilde{\varsigma}_{\mathfrak{e}I\cap G}(\grave{n}-\grave{k}) \leq s_n\{\tilde{\varsigma}_{\mathfrak{e}I\cap G}(\grave{n}), \tilde{\varsigma}_{\mathfrak{e}I\cap G}(\grave{k})\}$ and $\varsigma_{\mathfrak{e}_{I\cap G}}(\grave{n}-\grave{k}) \geq t_n\{\varsigma_{\mathfrak{e}_{I\cap G}}(\grave{n}), \varsigma_{\mathfrak{e}_{I\cap G}}(\grave{k})\}$.

(ii)
$$\begin{aligned}
\tilde{\zeta}_{\mathfrak{e}I\cap G}(\grave{n}\grave{k}) &= t_n\{\tilde{\zeta}_{\mathfrak{e}I}(\grave{n}\grave{k}), \tilde{\zeta}_{\mathfrak{e}G}(\grave{n}\grave{k})\}\\
&\geq t_n\{t_n\{\tilde{\zeta}_{\mathfrak{e}I}(\grave{n}), \tilde{\zeta}_{\mathfrak{e}I}(\grave{k})\}, t_n\{\tilde{\zeta}_{\mathfrak{e}G}(\grave{n}), \tilde{\zeta}_{\mathfrak{e}G}(\grave{k})\}\}\\
&= t_n\{t_n\{\tilde{\zeta}_{\mathfrak{e}I}(\grave{n}), \tilde{\zeta}_{\mathfrak{e}G}(\grave{n})\}, t_n\{\tilde{\zeta}_{\mathfrak{e}I}(\grave{k}), \tilde{\zeta}_{\mathfrak{e}G}(\grave{k})\}\}\\
&= t_n\{\tilde{\zeta}_{\mathfrak{e}I\cap G}(\grave{n}), \tilde{\zeta}_{\mathfrak{e}I\cap G}(\grave{k})\}\\
\zeta_{\mathfrak{e}_{I\cap G}}(\grave{n}\grave{k}) &= s_n\{\zeta_{\mathfrak{e}_I}(\grave{n}\grave{k}), \zeta_{\mathfrak{e}_G}(\grave{n}\grave{k})\}\\
&\leq s_n\{s_n\{\zeta_{\mathfrak{e}_I}(\grave{n}), \zeta_{\mathfrak{e}_I}(\grave{k})\}, s_n\{\zeta_{\mathfrak{e}_G}(\grave{n}), \zeta_{\mathfrak{e}_G}(\grave{k})\}\}\\
&= s_n\{s_n\{\zeta_{\mathfrak{e}_I}(\grave{n}), \zeta_{\mathfrak{e}_G}(\grave{n})\}, s_n\{\zeta_{\mathfrak{e}_I}(\grave{k}), \zeta_{\mathfrak{e}_G}(\grave{k})\}\}\\
&= s_n\{\zeta_{\mathfrak{e}_{I\cap G}}(\grave{n}), \zeta_{\mathfrak{e}_{I\cap G}}(\grave{k})\}.
\end{aligned}$$

Similarly, $\tilde{\varrho}_{\mathfrak{e}I\cap G}(\grave{n}\grave{k}) \leq s_n\{\tilde{\varrho}_{\mathfrak{e}I\cap G}(\grave{n}), \tilde{\varrho}_{\mathfrak{e}I\cap G}(\grave{k})\}$ and $\varrho_{\mathfrak{e}_{I\cap G}}(\grave{n}\grave{k}) \geq t_n\{\varrho_{\mathfrak{e}_{I\cap G}}(\grave{n}), \varrho_{\mathfrak{e}_{I\cap G}}(\grave{k})\}, \tilde{\varsigma}_{\mathfrak{e}I\cap G}(\grave{n}-\grave{k}) \leq s_n\{\tilde{\varsigma}_{\mathfrak{e}I\cap G}(\grave{n}), \tilde{\varsigma}_{\mathfrak{e}I\cap G}(\grave{k})\}$ and $\varsigma_{\mathfrak{e}_{I\cap G}}(\grave{n}\grave{k}) \geq t_n\{\varsigma_{\mathfrak{e}_{I\cap G}}(\grave{n}), \varsigma_{\mathfrak{e}_{I\cap G}}(\grave{k})\}$.

Hence $\mathbf{I} \cap \mathbf{G}$ is a cubic neutrosophic subring of $\mathfrak{R}$ with respect to t_n and s_n. □

Theorem 3.2. *Let* **I** *and* **G** *be cubic neutrosophic subring of* $\mathfrak{R}$*. If one contains the other, then* $\mathbf{I} \cup \mathbf{G}$ *is a cubic neutrosophic subring of* $\mathfrak{R}$*.*

Proof. Let **I** and **G** be any two cubic neutrosophic subring of $\mathfrak{R}$. Let $\grave{n}, \grave{k} \in \mathfrak{R}$. Then we have

(i)
$$\begin{aligned}
\tilde{\zeta}_{\mathfrak{e}I\cup G}(\grave{n}-\grave{k}) &= s_n\{\tilde{\zeta}_{\mathfrak{e}I}(\grave{n}-\grave{k}), \tilde{\zeta}_{\mathfrak{e}G}(\grave{n}-\grave{k})\}\\
&\geq s_n\{t_n\{\tilde{\zeta}_{\mathfrak{e}I}(\grave{n}), \tilde{\zeta}_{\mathfrak{e}I}(\grave{k})\}, t_n\{\tilde{\zeta}_{\mathfrak{e}G}(\grave{n}), \tilde{\zeta}_{\mathfrak{e}G}(\grave{k})\}\}\\
&= t_n\{t_n\{\tilde{\zeta}_{\mathfrak{e}I}(\grave{n}), \tilde{\zeta}_{\mathfrak{e}G}(\grave{n})\}, t_n\{\tilde{\zeta}_{\mathfrak{e}I}(\grave{k}), \tilde{\zeta}_{\mathfrak{e}G}(\grave{k})\}\}\\
&= t_n\{\tilde{\zeta}_{\mathfrak{e}I\cup G}(\grave{n}), \tilde{\zeta}_{\mathfrak{e}I\cup G}(\grave{k})\}\\
\zeta_{\mathfrak{e}_{I\cup G}}(\grave{n}-\grave{k}) &= s_n\{\zeta_{\mathfrak{e}_I}(\grave{n}-\grave{k}), \zeta_{\mathfrak{e}_G}(\grave{n}-\grave{k})\}\\
&\leq s_n\{s_n\{\zeta_{\mathfrak{e}_I}(\grave{n}), \zeta_{\mathfrak{e}_I}(\grave{k})\}, s_n\{\zeta_{\mathfrak{e}_G}(\grave{n}), \zeta_{\mathfrak{e}_G}(\grave{k})\}\}\\
&= s_n\{s_n\{\zeta_{\mathfrak{e}_I}(\grave{n}), \zeta_{\mathfrak{e}_G}(\grave{n})\}, s_n\{\zeta_{\mathfrak{e}_I}(\grave{k}), \zeta_{\mathfrak{e}_G}(\grave{k})\}\}\\
&= s_n\{\zeta_{\mathfrak{e}_{I\cup G}}(\grave{n}), \zeta_{\mathfrak{e}_{I\cup G}}(\grave{k})\}.
\end{aligned}$$

Similarly, $\tilde{\varrho}_{\mathfrak{e}I\cup G}(\grave{n}-\grave{k}) \leq s_n\{\tilde{\varrho}_{\mathfrak{e}I\cup G}(\grave{n}), \tilde{\varrho}_{\mathfrak{e}I\cup G}(\grave{k})\}$ and $\varrho_{\mathfrak{e}_{I\cup G}}(\grave{n}-\grave{k}) \geq t_n\{\varrho_{\mathfrak{e}_{I\cup G}}(\grave{n}), \varrho_{\mathfrak{e}_{I\cup G}}(\grave{k})\}, \tilde{\varsigma}_{\mathfrak{e}I\cup G}(\grave{n}-\grave{k}) \leq s_n\{\tilde{\varsigma}_{\mathfrak{e}I\cup G}(\grave{n}), \tilde{\varsigma}_{\mathfrak{e}I\cup G}(\grave{k})\}$ and $\varsigma_{\mathfrak{e}_{I\cup G}}(\grave{n}-\grave{k}) \geq t_n\{\varsigma_{\mathfrak{e}_{I\cup G}}(\grave{n}), \varsigma_{\mathfrak{e}_{I\cup G}}(\grave{k})\}$.

(ii)
$$\begin{aligned}
\tilde{\zeta}_{\mathfrak{e}I\cup G}(\grave{n}\grave{k}) &= t_n\{\tilde{\zeta}_{\mathfrak{e}I}(\grave{n}\grave{k}), \tilde{\zeta}_{\mathfrak{e}G}(\grave{n}\grave{k})\}\\
&\geq t_n\{t_n\{\tilde{\zeta}_{\mathfrak{e}I}(\grave{n}), \tilde{\zeta}_{\mathfrak{e}I}(\grave{k})\}, t_n\{\tilde{\zeta}_{\mathfrak{e}G}(\grave{n}), \tilde{\zeta}_{\mathfrak{e}G}(\grave{k})\}\}\\
&= t_n\{t_n\{\tilde{\zeta}_{\mathfrak{e}I}(\grave{n}), \tilde{\zeta}_{\mathfrak{e}G}(\grave{n})\}, t_n\{\tilde{\zeta}_{\mathfrak{e}I}(\grave{k}), \tilde{\zeta}_{\mathfrak{e}G}(\grave{k})\}\}\\
&= t_n\{\tilde{\zeta}_{\mathfrak{e}I\cup G}(\grave{n}), \tilde{\zeta}_{\mathfrak{e}I\cup G}(\grave{k})\}\\
\zeta_{\mathfrak{e}_{I\cup G}}(\grave{n}\grave{k}) &= s_n\{\zeta_{\mathfrak{e}_I}(\grave{n}\grave{k}), \zeta_{\mathfrak{e}_G}(\grave{n}\grave{k})\}
\end{aligned}$$

$$\leq s_n\{s_n\{\zeta_{\mathfrak{C}_I}(\grave{n}), \zeta_{\mathfrak{C}_I}(\grave{k})\}, s_n\{\zeta_{\mathfrak{C}_G}(\grave{n}), \zeta_{\mathfrak{C}_G}(\grave{k})\}\}$$
$$= s_n\{s_n\{\zeta_{\mathfrak{C}_I}(\grave{n}), \zeta_{\mathfrak{C}_G}(\grave{n})\}, s_n\{\zeta_{\mathfrak{C}_I}(\grave{k}), \zeta_{\mathfrak{C}_G}(\grave{k})\}\}$$
$$= s_n\{\zeta_{\mathfrak{C}_{I\cup G}}(\grave{n}), \zeta_{\mathfrak{C}_{I\cup G}}(\grave{k})\}.$$

Similarly, $\tilde{\varrho}_{\mathfrak{C}I\cup G}(\grave{n}\grave{k}) \leq s_n\{\tilde{\varrho}_{\mathfrak{C}I\cup G}(\grave{n}), \tilde{\varrho}_{\mathfrak{C}I\cup G}(\grave{k})\}$ and $\varrho_{\mathfrak{C}_{I\cup G}}(\grave{n}\grave{k}) \geq t_n\{\varrho_{\mathfrak{C}_{I\cup G}}(\grave{n}), \varrho_{\mathfrak{C}_{I\cup G}}(\grave{k})\}, \tilde{\varsigma}_{\mathfrak{C}I\cup G}(\grave{n} - \grave{k}) \leq s_n\{\tilde{\varsigma}_{\mathfrak{C}I\cup G}(\grave{n}), \tilde{\varsigma}_{\mathfrak{C}I\cup G}(\grave{k})\}$ and $\varsigma_{\mathfrak{C}_{I\cup G}}(\grave{n}\grave{k}) \geq t_n\{\varsigma_{\mathfrak{C}_{I\cup G}}(\grave{n}), \varsigma_{\mathfrak{C}_{I\cup G}}(\grave{k})\}$.

Hence $\mathbf{I} \cup \mathbf{G}$ is a cubic neutrosophic subring of $\mathfrak{R}$ with respect to t_n and s_n. □

Definition 3.3. *Let $F : \mathfrak{R} \to \mathfrak{R}_1$. Let $\mathfrak{C} = \langle \zeta^\star, \varrho^\star, \varsigma^\star \rangle$ be a cubic neutrosophic set of $\mathfrak{R}$ and $\mathfrak{C}_1 = \langle \zeta_1^\star, \varrho_1^\star, \varsigma_1^\star \rangle$ be a cubic neutrosophic set of $\mathfrak{R}_1$. Then*

(i) *The pre-image $F^{-1}(\mathfrak{C}_1) = (F^{-1}(\zeta_1^\star), F^{-1}(\varrho_1^\star), F^{-1}(\varsigma_1^\star)$ is a cubic neutrosophic set of $\mathfrak{R}_1$ defined by $F^{-1}(\mathfrak{C}_1)(\grave{k}) = (F^{-1}(\zeta_1^\star)(\grave{k}), F^{-1}(\varrho_1^\star)(\grave{k}), F^{-1}(\varsigma_1^\star)(\grave{k}) = (\zeta_1^\star(F(\grave{k})), \varrho_1^\star(F(\grave{k})), \varsigma_1^\star(F(\grave{k}))$*

(ii) *The image $F(\mathfrak{C}) = (F(\zeta^\star), F(\varrho^\star), F(\varsigma^\star)$ is a cubic neutrosophic set of $\mathfrak{R}_1$ defined by*

$$F(\zeta^\star)(\grave{n}) = \begin{cases} F(\tilde{\zeta})(\grave{n}) = \begin{cases} \vee_{q\in F^{-1}(\grave{n})}\tilde{\zeta}(\grave{k}) & \text{if } F^{-1}(\grave{n}) \neq \phi \\ 0 & \text{otherwise} \end{cases} \\ F(\zeta)(\grave{n}) = \begin{cases} \wedge_{q\in F^{-1}(\grave{n})}\zeta(\grave{k}) & \text{if } F^{-1}(\grave{n}) \neq \phi \\ 1 & \text{otherwise} \end{cases} \end{cases}$$

$$F(\varrho^\star)(\grave{n}) = \begin{cases} F(\tilde{\varrho})(\grave{n}) = \begin{cases} \vee_{q\in F^{-1}(\grave{n})}\tilde{\varrho}(\grave{k}) & \text{if } F^{-1}(\grave{n}) \neq \phi \\ 1 & \text{otherwise} \end{cases} \\ F(\varrho)(\grave{n}) = \begin{cases} \wedge_{q\in F^{-1}(\grave{n})}\varrho(\grave{k}) & \text{if } F^{-1}(\grave{n}) \neq \phi \\ 0 & \text{otherwise} \end{cases} \end{cases}$$

$$F(\varsigma^\star)(\grave{n}) = \begin{cases} F(\tilde{\varsigma})(\grave{n}) = \begin{cases} \vee_{q\in F^{-1}(\grave{n})}\tilde{\varsigma}(\grave{k}) & \text{if } F^{-1}(\grave{n}) \neq \phi \\ 1 & \text{otherwise} \end{cases} \\ F(\varsigma)(\grave{n}) = \begin{cases} \wedge_{q\in F^{-1}(\grave{n})}\varsigma(\grave{k}) & \text{if } F^{-1}(\grave{n}) \neq \phi \\ 0 & \text{otherwise} \end{cases} \end{cases}$$

Theorem 3.3. *Let $F : \mathfrak{R} \to \mathfrak{R}_1$ be an onto homomorphism between $\mathfrak{R}$ and $\mathfrak{R}_1$. If $\mathfrak{C}$ is a cubic neutrosophic subring of $\mathfrak{R}$ with respect to t_n and s_n then $F(\mathfrak{C})$ is a cubic neutrosophic subring of $\mathfrak{R}_1$ under t_n and s_n.*

Proof. Let $\grave{n}_1, \grave{n}_2 \in \mathfrak{R}$ and $\grave{k}_1, \grave{k}_2 \in \mathfrak{R}_1$ if $\mathfrak{R}$ is a cubic neutrosophic subring of $\mathfrak{R}$. Then

(i)
$$F(\tilde{\zeta}_{\mathfrak{C}})(\grave{k}_1 - \grave{k}_2) = \vee_{\grave{n}\in F^{-1}(\grave{k})}\tilde{\zeta}_{\mathfrak{C}}(\grave{n}_1 - \grave{n}_2)$$
$$\geq \vee_{\grave{n}\in F^{-1}(\grave{k})} t_n\{\tilde{\zeta}_{\mathfrak{C}}(\grave{n}_1), \tilde{\zeta}_{\mathfrak{C}}(\grave{n}_2)\}$$
$$= t_n\{\vee_{\grave{n}\in F^{-1}(\grave{k})}\tilde{\zeta}_{\mathfrak{C}}(\grave{n}_1), \vee_{\grave{n}\in F^{-1}(\grave{k})}\tilde{\zeta}_{\mathfrak{C}}(\grave{n}_2)\}$$
$$= t_n\{F(\tilde{\zeta}_{\mathfrak{C}})(\grave{k}_1), F(\tilde{\zeta}_{\mathfrak{C}})(\grave{k}_1)\}$$
$$F(\zeta_{\mathfrak{C}})(\grave{k}_1 - \grave{k}_2) = \wedge_{\grave{n}\in F^{-1}(\grave{k})}\zeta_{\mathfrak{C}}(\grave{n}_1 - \grave{n}_2)$$
$$\leq \wedge_{\grave{n}\in F^{-1}(\grave{k})} s_n\{\zeta_{\mathfrak{C}}(\grave{n}_1), \zeta_{\mathfrak{C}}(\grave{n}_2)\}$$
$$= s_n\{\wedge_{\grave{n}\in F^{-1}(\grave{k})}\zeta_{\mathfrak{C}}(\grave{n}_1), \wedge_{\grave{n}\in F^{-1}(\grave{k})}\zeta_{\mathfrak{C}}(\grave{n}_2)\}$$
$$= s_n\{F(\zeta_{\mathfrak{C}})(\grave{k}_1), F(\zeta_{\mathfrak{C}})(\grave{k}_1)\}.$$

Similarly, $F(\tilde{\varrho}_{\mathfrak{C}})(\grave{k}_1 - \grave{k}_2) \leq s_n\{F(\tilde{\varrho}_{\mathfrak{C}})(\grave{k}_1), F(\tilde{\varrho}_{\mathfrak{C}})(\grave{k}_1)\}$ and $F(\varrho_{\mathfrak{C}})(\grave{k}_1 - \grave{k}_2) \geq t_n\{F(\varrho_{\mathfrak{C}})(\grave{k}_1), F(\varrho_{\mathfrak{C}})(\grave{k}_1)\}, F(\tilde{\varsigma}_{\mathfrak{C}})(\grave{k}_1 - \grave{k}_2) \leq s_n\{F(\tilde{\varsigma}_{\mathfrak{C}})(\grave{k}_1), F(\tilde{\varsigma}_{\mathfrak{C}})(\grave{k}_1)\}$ and $F(\varsigma_{\mathfrak{C}})(\grave{k}_1 - \grave{k}_2) \geq t_n\{F(\varsigma_{\mathfrak{C}})(\grave{k}_1), F(\varsigma_{\mathfrak{C}})(\grave{k}_1)\}$.

(ii) $F(\tilde{\zeta}_{\mathfrak{C}})(\grave{k}_1\grave{k}_2) = \vee_{\grave{n}\in F^{-1}(\grave{k})}\tilde{\zeta}_{\mathfrak{C}}(\grave{n}_1\grave{n}_2)$
$\geq \vee_{\grave{n}\in F^{-1}(\grave{k})}t_n\{\tilde{\zeta}_{\mathfrak{C}}(\grave{n}_1), \tilde{\zeta}_{\mathfrak{C}}(\grave{n}_2)\}$
$= t_n\{\vee_{\grave{n}\in F^{-1}(\grave{k})}\tilde{\zeta}_{\mathfrak{C}}(\grave{n}_1), \vee_{\grave{n}\in F^{-1}(\grave{k})}\tilde{\zeta}_{\mathfrak{C}}(\grave{n}_2)\}$
$= t_n\{F(\tilde{\zeta}_{\mathfrak{C}})(\grave{k}_1), F(\tilde{\zeta}_{\mathfrak{C}})(\grave{k}_1)\}$
$F(\zeta_{\mathfrak{C}})(\grave{k}_1\grave{k}_2) = \wedge_{\grave{n}\in F^{-1}(\grave{k})}\zeta_{\mathfrak{C}}(\grave{n}_1\grave{n}_2)$
$\leq \wedge_{\grave{n}\in F^{-1}(\grave{k})}s_n\{\zeta_{\mathfrak{C}}(\grave{n}_1), \zeta_{\mathfrak{C}}(\grave{n}_2)\}$
$= s_n\{\wedge_{\grave{n}\in F^{-1}(\grave{k})}\zeta_{\mathfrak{C}}(\grave{n}_1), \wedge_{\grave{n}\in F^{-1}(\grave{k})}\zeta_{\mathfrak{C}}(\grave{n}_2)\}$
$= s_n\{F(\zeta_{\mathfrak{C}})(\grave{k}_1), F(\zeta_{\mathfrak{C}})(\grave{k}_1)\}$.

Similarly, $F(\tilde{\varrho}_{\mathfrak{C}})(\grave{k}_1\grave{k}_2) \leq s_n\{F(\tilde{\varrho}_{\mathfrak{C}})(\grave{k}_1), F(\tilde{\varrho}_{\mathfrak{C}})(\grave{k}_1)\}$ and $F(\varrho_{\mathfrak{C}})(\grave{k}_1\grave{k}_2) \geq t_n\{F(\varrho_{\mathfrak{C}})(\grave{k}_1), F(\varrho_{\mathfrak{C}})(\grave{k}_1)\}$, $F(\tilde{\varsigma}_{\mathfrak{C}})(\grave{k}_1\grave{k}_2) \leq s_n\{F(\tilde{\varsigma}_{\mathfrak{C}})(\grave{k}_1), F(\tilde{\varsigma}_{\mathfrak{C}})(\grave{k}_1)\}$ and $F(\varsigma_{\mathfrak{C}})(\grave{k}_1\grave{k}_2) \geq t_n\{F(\varsigma_{\mathfrak{C}})(\grave{k}_1), F(\varsigma_{\mathfrak{C}})(\grave{k}_1)\}$.

Hence $F(\mathfrak{C})$ is a cubic neutrosophic subring of $\mathfrak{R}_1$ under t_n and s_n. □

Theorem 3.4. *Let $F : \mathfrak{R} \to \mathfrak{R}_1$ be a homomorphism between $\mathfrak{R}$ and $\mathfrak{R}_1$. If $\mathfrak{C}_1$ is a cubic neutrosophic subring of $\mathfrak{R}_1$ under t_n and s_n then $F^{-1}(\mathfrak{C}_1)$ is a cubic neutrosophic subring of $\mathfrak{R}$ under t_n and s_n.*

Proof. Let $\grave{n}, \grave{k} \in \mathfrak{R}$. Let $\mathfrak{C}_1$ is a cubic neutrosophic subring of $\mathfrak{R}_1$. Then

(i) $F^{-1}(\tilde{\zeta}_{\mathfrak{C}_1})(\grave{n} - \grave{k}) = \tilde{\zeta}_{\mathfrak{C}_1}(F(\grave{n} - \grave{k}))$
$= \tilde{\zeta}_{\mathfrak{C}_1}(F(\grave{n}) - F(\grave{k}))$
$\geq t_n\{\tilde{\zeta}_{\mathfrak{C}_1}(F(\grave{n}), \tilde{\zeta}_{\mathfrak{C}_1}(F(\grave{k})\}$
$= t_n\{F^{-1}(\tilde{\zeta}_{\mathfrak{C}_1})(\grave{n}), F^{-1}(\tilde{\zeta}_{\mathfrak{C}_1})(\grave{k})\}$
$F^{-1}(\zeta_{\mathfrak{C}_1})(\grave{n} - \grave{k}) = \zeta_{\mathfrak{C}_1}(F(\grave{n} - \grave{k}))$
$= \zeta_{\mathfrak{C}_1}(F(\grave{n}) - F(\grave{k}))$
$\leq s_n\{\zeta_{\mathfrak{C}_1}(F(\grave{n}), \zeta_{\mathfrak{C}_1}(F(\grave{k})\}$
$= s_n\{F^{-1}(\zeta_{\mathfrak{C}_1})(\grave{n}), F^{-1}(\zeta_{\mathfrak{C}_1})(\grave{k})\}$.

Similarly, $F^{-1}(\tilde{\varrho}_{\mathfrak{C}_1})(\grave{n} - \grave{k}) \leq s_n\{F^{-1}(\tilde{\varrho}_{\mathfrak{C}_1})(\grave{n}), F^{-1}(\tilde{\varrho}_{\mathfrak{C}_1})(\grave{k})\}$ and
$F^{-1}(\varrho_{\mathfrak{C}_1})(\grave{n} - \grave{k}) \geq t_n\{F^{-1}(\varrho_{\mathfrak{C}_1})(\grave{n}), F^{-1}(\varrho_{\mathfrak{C}_1})(\grave{k})\}$,
$F^{-1}(\tilde{\varsigma}_{\mathfrak{C}_1})(\grave{n} - \grave{k}) \leq s_n\{F^{-1}(\tilde{\varsigma}_{\mathfrak{C}_1})(\grave{n}), F^{-1}(\tilde{\varsigma}_{\mathfrak{C}_1})(\grave{k})\}$ and
$F^{-1}(\varsigma_{\mathfrak{C}_1})(\grave{n} - \grave{k}) \geq t_n\{F^{-1}(\varsigma_{\mathfrak{C}_1})(\grave{n}), F^{-1}(\varsigma_{\mathfrak{C}_1})(\grave{k})\}$.

(ii) $F^{-1}(\tilde{\zeta}_{\mathfrak{C}_1})(\grave{n}\grave{k}) = \tilde{\zeta}_{\mathfrak{C}_1}(F(\grave{n}\grave{k}))$
$= \tilde{\zeta}_{\mathfrak{C}_1}(F(\grave{n})F(\grave{k}))$
$\geq t_n\{\tilde{\zeta}_{\mathfrak{C}_1}(F(\grave{n}), \tilde{\zeta}_{\mathfrak{C}_1}(F(\grave{k})\}$
$= t_n\{F^{-1}(\tilde{\zeta}_{\mathfrak{C}_1})(\grave{n}), F^{-1}(\tilde{\zeta}_{\mathfrak{C}_1})(\grave{k})\}$
$F^{-1}(\zeta_{\mathfrak{C}_1})(\grave{n}\grave{k}) = \zeta_{\mathfrak{C}_1}(F(\grave{n}\grave{k}))$
$= \zeta_{\mathfrak{C}_1}(F(\grave{n})F(\grave{k}))$
$\leq s_n\{\zeta_{\mathfrak{C}_1}(F(\grave{n}), \zeta_{\mathfrak{C}_1}(F(\grave{k})\}$
$= s_n\{F^{-1}(\zeta_{\mathfrak{C}_1})(\grave{n}), F^{-1}(\zeta_{\mathfrak{C}_1})(\grave{k})\}$.

Similarly, $F^{-1}(\tilde{\varrho}_{\mathfrak{C}_1})(\grave{n}\grave{k}) \leq s_n\{F^{-1}(\tilde{\varrho}_{\mathfrak{C}_1})(\grave{n}), F^{-1}(\tilde{\varrho}_{\mathfrak{C}_1})(\grave{k})\}$ and
$F^{-1}(\varrho_{\mathfrak{C}_1})(\grave{n}\grave{k}) \geq t_n\{F^{-1}(\varrho_{\mathfrak{C}_1})(\grave{n}), F^{-1}(\varrho_{\mathfrak{C}_1})(\grave{k})\}$,
$F^{-1}(\tilde{\varsigma}_{\mathfrak{C}_1})(\grave{n}\grave{k}) \leq s_n\{F^{-1}(\tilde{\varsigma}_{\mathfrak{C}_1})(\grave{n}), F^{-1}(\tilde{\varsigma}_{\mathfrak{C}_1})(\grave{k})\}$ and
$F^{-1}(\varsigma_{\mathfrak{C}_1})(\grave{n}\grave{k}) \geq t_n\{F^{-1}(\varsigma_{\mathfrak{C}_1})(\grave{n}), F^{-1}(\varsigma_{\mathfrak{C}_1})(\grave{k})\}$.

Hence$F^{-1}(\mathfrak{C}_1)$ is a cubic neutrosophic subring of $\mathfrak{R}$ under t_n and s_n. □

4. Cubic Neutrosophic Level Set and Ideals

Definition 4.1. *Let* $\mathfrak{C} = \{\grave{k}, \zeta^{\star}_{\mathfrak{C}}(\grave{k}), \varrho^{\star}_{\mathfrak{C}}(\grave{k}), \varsigma^{\star}_{\mathfrak{C}}(\grave{k})/\grave{k} \in \mathfrak{R}\}$ *be a cubic neutrosophic subring of* $\mathfrak{R}$*. Let* $\alpha^\star, \beta^\star, \gamma^\star \in [0,1]$*. Then the set* $\mathfrak{C}_{\alpha^\star,\beta^\star,\gamma^\star}$ *is referred to as a level set of* $\mathfrak{C}$*, where* $\alpha^\star =< \tilde{\alpha}, \alpha >, \beta^\star =< \tilde{\beta}, \beta >, \gamma^\star =< \tilde{\gamma}, \gamma >$*. If* $\grave{k} \in \mathfrak{C}_{\alpha^\star,\beta^\star,\gamma^\star}$ *then the following inequalities hold.* $\tilde{\zeta}_{\mathfrak{C}}(\grave{k}) \geq \tilde{\alpha}, \zeta_{\mathfrak{C}}(\grave{k}) \leq \alpha; \tilde{\varrho}_{\mathfrak{C}}(\grave{k}) \leq \tilde{\beta}, \varrho_{\mathfrak{C}}(\grave{k}) \geq \beta; \tilde{\varsigma}_{\mathfrak{C}}(\grave{k}) \leq \tilde{\gamma}, \varsigma_{\mathfrak{C}}(\grave{k}) \geq \gamma.$

Theorem 4.1. *If* $\mathfrak{C}$ *is a cubic neutrosophic subring of* $\mathfrak{R}$ *with respect to* t_n *and* s_n *if and only if* $\mathfrak{C}_{\alpha^\star,\beta^\star,\gamma^\star}$ *is a subring of* $\mathfrak{R}$ *with respect to* t_n *and* s_n*.(where* t_n *and* s_n *are idempotent)*

Proof. Since $\tilde{\zeta}_{\mathfrak{C}}(\grave{k}) \geq \tilde{\alpha}, \zeta_{\mathfrak{C}}(\grave{k}) \leq \alpha; \tilde{\varrho}_{\mathfrak{C}}(\grave{k}) \leq \tilde{\beta}, \varrho_{\mathfrak{C}}(\grave{k}) \geq \beta; \tilde{\varsigma}_{\mathfrak{C}}(\grave{k}) \leq \tilde{\gamma}, \varsigma_{\mathfrak{C}}(\grave{k}) \geq \gamma$. $\forall \grave{k} \in \mathfrak{C}_{\alpha^\star,\beta^\star,\gamma^\star}$. Therefore $\mathfrak{C}_{\alpha^\star,\beta^\star,\gamma^\star}$ is non empty. Now, let $\mathfrak{C}$ be a cubic neutrosophic subring of $\mathfrak{R}$ with respect to t_n and s_n and $\grave{n}, \grave{k} \in \mathfrak{C}_{\alpha^\star,\beta^\star,\gamma^\star}$.

To show that $\grave{n} - \grave{k}, \grave{n}\grave{k} \in \mathfrak{C}_{\alpha^\star,\beta^\star,\gamma^\star}$

(i) $\tilde{\zeta}_{\mathfrak{C}}(\grave{n} - \grave{k}) \geq t_n\{\tilde{\zeta}_{\mathfrak{C}}(\grave{n}), \tilde{\zeta}_{\mathfrak{C}}(\grave{k})\} \geq t_n\{\tilde{\alpha}, \tilde{\alpha}\} = \tilde{\alpha}$

$\zeta_{\mathfrak{C}}(\grave{n} - \grave{k}) \leq s_n\{\zeta_{\mathfrak{C}}(\grave{n}), \zeta_{\mathfrak{C}}(\grave{k})\} \geq t_n\{\alpha, \alpha\} = \alpha.$

Similarly, $\tilde{\varrho}_{\mathfrak{C}}(\grave{n} - \grave{k}) \leq \tilde{\beta}, \varrho_{\mathfrak{C}}(\grave{n} - \grave{k}) \geq \beta$ and $\tilde{\varsigma}_{\mathfrak{C}}(\grave{n} - \grave{k}) \leq \tilde{\gamma}, \varsigma_{\mathfrak{C}}(\grave{n} - \grave{k}) \geq \gamma$.

(ii) $\tilde{\zeta}_{\mathfrak{C}}(\grave{n}\grave{k}) \geq t_n\{\tilde{\zeta}_{\mathfrak{C}}(\grave{n}), \tilde{\zeta}_{\mathfrak{C}}(\grave{k})\} \geq t_n\{\tilde{\alpha}, \tilde{\alpha}\} = \tilde{\alpha}$

$\zeta_{\mathfrak{C}}(\grave{n}\grave{k}) \leq s_n\{\zeta_{\mathfrak{C}}(\grave{n}), \zeta_{\mathfrak{C}}(\grave{k})\} \geq t_n\{\alpha, \alpha\} = \alpha.$

Similarly, $\tilde{\varrho}_{\mathfrak{C}}(\grave{n}\grave{k}) \leq \tilde{\beta}, \varrho_{\mathfrak{C}}(\grave{n}\grave{k}) \geq \beta$ and $\tilde{\varsigma}_{\mathfrak{C}}(\grave{n}\grave{k}) \leq \tilde{\gamma}, \varsigma_{\mathfrak{C}}(\grave{n}\grave{k}) \geq \gamma$.

Therefore $\tilde{\zeta}_{\mathfrak{C}}(\grave{k}) \geq \tilde{\alpha}, \zeta_{\mathfrak{C}}(\grave{k}) \leq \alpha; \tilde{\varrho}_{\mathfrak{C}}(\grave{k}) \leq \tilde{\beta}, \varrho_{\mathfrak{C}}(\grave{k}) \geq \beta; \tilde{\varsigma}_{\mathfrak{C}}(\grave{k}) \leq \tilde{\gamma}, \varsigma_{\mathfrak{C}}(\grave{k}) \geq \gamma$.

Thus $\grave{n} - \grave{k}, \grave{n}\grave{k} \in \mathfrak{C}_{\alpha^\star,\beta^\star,\gamma^\star}$ is a subring of $\mathfrak{R}$.

Conversely, Let $\mathfrak{C}_{\alpha^\star,\beta^\star,\gamma^\star}$ be a subring of $\mathfrak{R}$.

To show that $\mathfrak{C}$ is cubic neutrosophic subring of $\mathfrak{R}$ with respect to t_n and s_n.

Let $\grave{n}, \grave{k} \in \mathfrak{R}$ then there exist $\alpha^\star \in [0,1]$ such that

$t_n\{\tilde{\zeta}_{\mathfrak{C}}(\grave{n}), \tilde{\zeta}_{\mathfrak{C}}(\grave{k})\} = \tilde{\alpha}, s_n\{\zeta_{\mathfrak{C}}(\grave{n}), \zeta_{\mathfrak{C}}(\grave{k})\} = \alpha$

so,$\tilde{\zeta}_{\mathfrak{C}}(\grave{n}) \geq \tilde{\alpha}, \tilde{\zeta}_{\mathfrak{C}}(\grave{k}) \geq \tilde{\alpha}; \zeta_{\mathfrak{C}}(\grave{n}) \leq \alpha, \zeta_{\mathfrak{C}}(\grave{k}) \leq \alpha$ also, let there exist $\beta^\star, \gamma^\star \in [0,1]$ such that

$s_n\{\tilde{\varrho}_{\mathfrak{C}}(\grave{n}), \tilde{\varrho}_{\mathfrak{C}}(\grave{k})\} = \tilde{\beta}, t_n\{\varrho_{\mathfrak{C}}(\grave{n}), \varrho_{\mathfrak{C}}(\grave{k})\} = \beta$ and

$s_n\{\tilde{\varsigma}_{\mathfrak{C}}(\grave{n}), \tilde{\varsigma}_{\mathfrak{C}}(\grave{k})\} = \tilde{\gamma}, t_n\{\varsigma_{\mathfrak{C}}(\grave{n}), \varsigma_{\mathfrak{C}}(\grave{k})\} = \gamma$. Then $\grave{n}, \grave{k} \in \mathfrak{C}_{\alpha^\star,\beta^\star,\gamma^\star}$. Since $\mathfrak{C}_{\alpha^\star,\beta^\star,\gamma^\star}$ is a subring of $\mathfrak{R}$.$\grave{n} - \grave{k}, \grave{n}\grave{k} \in \mathfrak{C}_{\alpha^\star,\beta^\star,\gamma^\star}$. Hence,

(i) $\tilde{\zeta}_{\mathfrak{C}}(\grave{n} - \grave{k}) \geq \tilde{\alpha} = t_n\{\tilde{\zeta}_{\mathfrak{C}}(\grave{n}), \tilde{\zeta}_{\mathfrak{C}}(\grave{k})\}; \zeta_{\mathfrak{C}}(\grave{n} - \grave{k}) \leq \alpha = s_n\{\zeta_{\mathfrak{C}}(\grave{n}), \zeta_{\mathfrak{C}}(\grave{k})\}$.

Similarly, $\tilde{\varrho}_{\mathfrak{C}}(\grave{n} - \grave{k}) \leq \tilde{\beta}; \varrho_{\mathfrak{C}}(\grave{n} - \grave{k}) \geq \beta$

and $\tilde{\varsigma}_{\mathfrak{C}}(\grave{n} - \grave{k}) \leq \tilde{\gamma}; \varsigma_{\mathfrak{C}}(\grave{n} - \grave{k}) \geq \gamma$

(ii) $\tilde{\zeta}_{\mathfrak{C}}(\grave{n}\grave{k}) \geq \tilde{\alpha} = t_n\{\tilde{\zeta}_{\mathfrak{C}}(\grave{n}), \tilde{\zeta}_{\mathfrak{C}}(\grave{k})\}; \zeta_{\mathfrak{C}}(\grave{n}\grave{k}) \leq \alpha = s_n\{\zeta_{\mathfrak{C}}(\grave{n}), \zeta_{\mathfrak{C}}(\grave{k})\}$.

Similarly, $\tilde{\varrho}_{\mathfrak{C}}(\grave{n}\grave{k}) \leq \tilde{\beta}; \varrho_{\mathfrak{C}}(\grave{n}\grave{k}) \geq \beta$ and

$\tilde{\varsigma}_{\mathfrak{C}}(\grave{n}\grave{k}) \leq \tilde{\gamma}; \varsigma_{\mathfrak{C}}(\grave{n}\grave{k}) \geq \gamma$.

Hence $\mathfrak{C}$ is a cubic neutrosophic subring of $\mathfrak{R}$ under t_n and s_n. □

Proposition 4.1. *Let* $\mathfrak{C}$ *be a cubic neutrosophic subring of* $\mathfrak{R}$ *with respect to* t_n *and* s_n*, where* t_n, s_n *are idempotent then* $\mathfrak{D} = \{\tilde{\zeta}_{\mathfrak{C}}(\grave{k}) = \tilde{1}, \zeta_{\mathfrak{C}}(\grave{k}) = 0, \tilde{\varrho}_{\mathfrak{C}}(\grave{k}) = \tilde{0}, \varrho_{\mathfrak{C}}(\grave{k}) = 1, \tilde{\varsigma}_{\mathfrak{C}}(\grave{k}) = \tilde{0}, \varsigma_{\mathfrak{C}}(\grave{k}) = 1/\grave{k} \in \mathfrak{R}\}$ *is a subring of* $\mathfrak{R}$*.*

Proof. Let $\grave{n}, \grave{k} \in \mathfrak{D}$. Then

(i) $\tilde{\zeta}_{\mathfrak{C}}(\grave{n} - \grave{k}) \geq t_n\{\tilde{\zeta}_{\mathfrak{C}}(\grave{n}), \tilde{\zeta}_{\mathfrak{C}}(\grave{k})\} = t_n(\tilde{1}, \tilde{1}) = \tilde{1}$

$\zeta_{\mathfrak{C}}(\grave{n} - \grave{k}) \leq s_n\{\zeta_{\mathfrak{C}}(\grave{n}), \zeta_{\mathfrak{C}}(\grave{k})\} = s_n(0, 0) = 0.$

Similarly, $\tilde{\varrho}_{\mathfrak{C}}(\grave{n} - \grave{k}) \leq \tilde{0}, \varrho_{\mathfrak{C}}(\grave{n} - \grave{k}) \geq 1$ and $\tilde{\varsigma}_{\mathfrak{C}}(\grave{n} - \grave{k}) \leq \tilde{0}, \varsigma_{\mathfrak{C}}(\grave{n} - \grave{k}) \geq 1$. Thus $\grave{n} - \grave{k} \in \mathfrak{D}$.

(ii) $\tilde{\zeta}_{\mathfrak{C}}(\grave{n}\grave{k}) \geq t_n\{\tilde{\zeta}_{\mathfrak{C}}(\grave{n}), \tilde{\zeta}_{\mathfrak{C}}(\grave{k})\} = t_n(\tilde{1}, \tilde{1}) = \tilde{1}$
$\zeta_{\mathfrak{C}}(\grave{n}\grave{k}) \leq s_n\{\zeta_{\mathfrak{C}}(\grave{n}), \zeta_{\mathfrak{C}}(\grave{k})\} = s_n(0, 0) = 0.$

Similarly, $\tilde{\varrho}_{\mathfrak{C}}(\grave{n}\grave{k}) \leq \tilde{0}, \varrho_{\mathfrak{C}}(\grave{n}\grave{k}) \geq 1$ and $\tilde{\varsigma}_{\mathfrak{C}}(\grave{n}\grave{k}) \leq \tilde{0}, \varsigma_{\mathfrak{C}}(\grave{n}\grave{k}) \geq 1$. Thus $\grave{n}\grave{k} \in \mathfrak{D}$.

Hence $\mathfrak{D} = \{\tilde{\zeta}_{\mathfrak{C}}(\grave{k}) = \tilde{1}, \zeta_{\mathfrak{C}}(\grave{k}) = 0, \tilde{\varrho}_{\mathfrak{C}}(\grave{k}) = \tilde{0}, \varrho_{\mathfrak{C}}(\grave{k}) = 1, \tilde{\varsigma}_{\mathfrak{C}}(\grave{k}) = \tilde{0}, \varsigma_{\mathfrak{C}}(\grave{k}) = 1/\grave{k} \in \mathfrak{R}\}$ is a subring of $\mathfrak{R}$ with respect to t_n and s_n. □

Definition 4.2. *Let $\mathfrak{C}$ be a cubic neutrosophic set of $\mathfrak{R}$ with respect to t_n and s_n. Then $\mathfrak{C}$ is referred to as cubic neutrosophic left ideal with respect to t_n and s_n of $\mathfrak{R}$ if*

(i) $\tilde{\zeta}_{\mathfrak{C}}(\grave{n} - \grave{k}) \geq t_n\{\tilde{\zeta}_{\mathfrak{C}}(\grave{n}), \tilde{\zeta}_{\mathfrak{C}}(\grave{k})\}; \zeta_{\mathfrak{C}}(\grave{n} - \grave{k}) \leq s_n\{\zeta_{\mathfrak{C}}(\grave{n}), \zeta_{\mathfrak{C}}(\grave{k})\}$
$\tilde{\varrho}_{\mathfrak{C}}(\grave{n} - \grave{k}) \leq s_n\{\tilde{\varrho}_{\mathfrak{C}}(\grave{n}), \tilde{\varrho}_{\mathfrak{C}}(\grave{k})\}; \varrho_{\mathfrak{C}}(\grave{n} - \grave{k}) \geq t_n\{\varrho_{\mathfrak{C}}(\grave{n}), \varrho_{\mathfrak{C}}(\grave{k})\}$
$\tilde{\varsigma}_{\mathfrak{C}}(\grave{n} - \grave{k}) \leq s_n\{\tilde{\varsigma}_{\mathfrak{C}}(\grave{n}), \tilde{\varsigma}_{\mathfrak{C}}(\grave{k})\}; \varsigma_{\mathfrak{C}}(\grave{n} - \grave{k}) \geq t_n\{\varsigma_{\mathfrak{C}}(\grave{n}), \varsigma_{\mathfrak{C}}(\grave{k})\}$

(ii) $\tilde{\zeta}_{\mathfrak{C}}(\grave{n}\grave{k}) \geq \tilde{\zeta}_{\mathfrak{C}}(\grave{k}); \zeta_{\mathfrak{C}}(\grave{n}\grave{k}) \leq \zeta_{\mathfrak{C}}(\grave{k})$
$\tilde{\varrho}_{\mathfrak{C}}(\grave{n}\grave{k}) \leq \tilde{\varrho}_{\mathfrak{C}}(\grave{k}); \varrho_{\mathfrak{C}}(\grave{n}\grave{k}) \geq \varrho_{\mathfrak{C}}(\grave{k})$
$\tilde{\varsigma}_{\mathfrak{C}}(\grave{n}\grave{k}) \leq \tilde{\varsigma}_{\mathfrak{C}}(\grave{k}); \varsigma_{\mathfrak{C}}(\grave{n}\grave{k}) \geq \varsigma_{\mathfrak{C}}(\grave{k})$. *Where $\grave{n}, \grave{k} \in \mathfrak{R}$.*

Definition 4.3. *Let $\mathfrak{C}$ be a cubic neutrosophic set of $\mathfrak{R}$ with respect to t_n and s_n. Then $\mathfrak{C}$ is referred to as cubic neutrosophic right ideal with respect to t_n and s_n of $\mathfrak{R}$ if*

(i) $\tilde{\zeta}_{\mathfrak{C}}(\grave{n} - \grave{k}) \geq t_n\{\tilde{\zeta}_{\mathfrak{C}}(\grave{n}), \tilde{\zeta}_{\mathfrak{C}}(\grave{k})\}; \zeta_{\mathfrak{C}}(\grave{n} - \grave{k}) \leq s_n\{\zeta_{\mathfrak{C}}(\grave{n}), \zeta_{\mathfrak{C}}(\grave{k})\}$
$\tilde{\varrho}_{\mathfrak{C}}(\grave{n} - \grave{k}) \leq s_n\{\tilde{\varrho}_{\mathfrak{C}}(\grave{n}), \tilde{\varrho}_{\mathfrak{C}}(\grave{k})\}; \varrho_{\mathfrak{C}}(\grave{n} - \grave{k}) \geq t_n\{\varrho_{\mathfrak{C}}(\grave{n}), \varrho_{\mathfrak{C}}(\grave{k})\}$
$\tilde{\varsigma}_{\mathfrak{C}}(\grave{n} - \grave{k}) \leq s_n\{\tilde{\varsigma}_{\mathfrak{C}}(\grave{n}), \tilde{\varsigma}_{\mathfrak{C}}(\grave{k})\}; \varsigma_{\mathfrak{C}}(\grave{n} - \grave{k}) \geq t_n\{\varsigma_{\mathfrak{C}}(\grave{n}), \varsigma_{\mathfrak{C}}(\grave{k})\}$

(ii) $\tilde{\zeta}_{\mathfrak{C}}(\grave{n}\grave{k}) \geq \tilde{\zeta}_{\mathfrak{C}}(\grave{n}); \zeta_{\mathfrak{C}}(\grave{n}\grave{k}) \leq \zeta_{\mathfrak{C}}(\grave{n})$
$\tilde{\varrho}_{\mathfrak{C}}(\grave{n}\grave{k}) \leq \tilde{\varrho}_{\mathfrak{C}}(\grave{n}); \varrho_{\mathfrak{C}}(\grave{n}\grave{k}) \geq \varrho_{\mathfrak{C}}(\grave{n})$
$\tilde{\varsigma}_{\mathfrak{C}}(\grave{n}\grave{k}) \leq \tilde{\varsigma}_{\mathfrak{C}}(\grave{n}); \varsigma_{\mathfrak{C}}(\grave{n}\grave{k}) \geq \varsigma_{\mathfrak{C}}(\grave{n})$. *Where $\grave{n}, \grave{k} \in \mathfrak{R}$.*

Definition 4.4. *Let $\mathfrak{C}$ be a cubic neutrosophic set of $\mathfrak{R}$ with respect to t_n and s_n. Then $\mathfrak{C}$ is referred to as cubic neutrosophic ideal with respect to t_n and s_n of $\mathfrak{R}$ if*

(i) $\tilde{\zeta}_{\mathfrak{C}}(\grave{n} - \grave{k}) \geq t_n\{\tilde{\zeta}_{\mathfrak{C}}(\grave{n}), \tilde{\zeta}_{\mathfrak{C}}(\grave{k})\}; \zeta_{\mathfrak{C}}(\grave{n} - \grave{k}) \leq s_n\{\zeta_{\mathfrak{C}}(\grave{n}), \zeta_{\mathfrak{C}}(\grave{k})\}$
$\tilde{\varrho}_{\mathfrak{C}}(\grave{n} - \grave{k}) \leq s_n\{\tilde{\varrho}_{\mathfrak{C}}(\grave{n}), \tilde{\varrho}_{\mathfrak{C}}(\grave{k})\}; \varrho_{\mathfrak{C}}(\grave{n} - \grave{k}) \geq t_n\{\varrho_{\mathfrak{C}}(\grave{n}), \varrho_{\mathfrak{C}}(\grave{k})\}$
$\tilde{\varsigma}_{\mathfrak{C}}(\grave{n} - \grave{k}) \leq s_n\{\tilde{\varsigma}_{\mathfrak{C}}(\grave{n}), \tilde{\varsigma}_{\mathfrak{C}}(\grave{k})\}; \varsigma_{\mathfrak{C}}(\grave{n} - \grave{k}) \geq t_n\{\varsigma_{\mathfrak{C}}(\grave{n}), \varsigma_{\mathfrak{C}}(\grave{k})\}$

(ii) $\tilde{\zeta}_{\mathfrak{C}}(\grave{n}\grave{k}) \geq s_n\{\tilde{\zeta}_{\mathfrak{C}}(\grave{n}), \tilde{\zeta}_{\mathfrak{C}}(\grave{k})\}; \zeta_{\mathfrak{C}}(\grave{n}\grave{k}) \leq t_n\{\zeta_{\mathfrak{C}}(\grave{n}), \zeta_{\mathfrak{C}}(\grave{k})\}$
$\tilde{\varrho}_{\mathfrak{C}}(\grave{n}\grave{k}) \leq t_n\{\tilde{\varrho}_{\mathfrak{C}}(\grave{n}), \tilde{\varrho}_{\mathfrak{C}}(\grave{k})\}; \varrho_{\mathfrak{C}}(\grave{n}\grave{k}) \geq s_n\{\varrho_{\mathfrak{C}}(\grave{n}), \varrho_{\mathfrak{C}}(\grave{k})\}$
$\tilde{\varsigma}_{\mathfrak{C}}(\grave{n}\grave{k}) \leq t_n\{\tilde{\varsigma}_{\mathfrak{C}}(\grave{n}), \tilde{\varsigma}_{\mathfrak{C}}(\grave{k})\}; \varsigma_{\mathfrak{C}}(\grave{n}\grave{k}) \geq s_n\{\varsigma_{\mathfrak{C}}(\grave{n}), \varsigma_{\mathfrak{C}}(\grave{k})\}$. *Where $\grave{n}, \grave{k} \in \mathfrak{R}$.*

Theorem 4.2. *Let $\mathfrak{C}$ be a cubic neutrosophic set of $\mathfrak{R}$ with respect to t_n and s_n, where t_n, s_n are idempotent then $\mathfrak{C}$ is a cubic neutrosophic left ideal(right ideal) of $\mathfrak{R}$ with respect to t_n and s_n if and only if $\mathfrak{C}_{\alpha^\star,\beta^\star,\gamma^\star}$ is a left ideal(right ideal) of $\mathfrak{R}$, with $\{\tilde{\zeta}_{\mathfrak{C}}(\grave{k}) = \tilde{\alpha}, \zeta_{\mathfrak{C}}(\grave{k}) = \alpha, \tilde{\varrho}_{\mathfrak{C}}(\grave{k}) = \tilde{\beta}, \varrho_{\mathfrak{C}}(\grave{k}) = \beta, \tilde{\varsigma}_{\mathfrak{C}}(\grave{k}) = \tilde{\gamma}, \varsigma_{\mathfrak{C}}(\grave{k}) = \gamma . \forall \alpha^\star, \beta^\star, \gamma^\star \in [0, 1]$.*

Proof. Let $\mathfrak{C}$ be a cubic neutrosophic left ideal of $\mathfrak{R}$ under t_n and s_n. Let $\grave{n}, \grave{k} \in \mathfrak{C}_{\alpha^\star,\beta^\star,\gamma^\star}$. Then

$\tilde{\zeta}_{\mathfrak{C}}(\grave{n} - \grave{k}) \geq t_n\{\tilde{\zeta}_{\mathfrak{C}}(\grave{n}), \tilde{\zeta}_{\mathfrak{C}}(\grave{k})\} = t_n(\tilde{\alpha}, \tilde{\alpha}) = \tilde{\alpha}$
$\zeta_{\mathfrak{C}}(\grave{n} - \grave{k}) \leq s_n\{\zeta_{\mathfrak{C}}(\grave{n}), \zeta_{\mathfrak{C}}(\grave{k})\} = s_n(\alpha, \alpha) = \alpha.$

Similarly, $\tilde{\varrho}_{\mathfrak{C}}(\grave{n} - \grave{k}) \leq \tilde{\beta}, \varrho_{\mathfrak{C}}(\grave{n} - \grave{k}) \leq \beta$and$\tilde{\varsigma}_{\mathfrak{C}}(\grave{n} - \grave{k}) \leq \tilde{\gamma}, \varsigma_{\mathfrak{C}}(\grave{n} - \grave{k}) \leq \gamma$.
Thus $\grave{n} - \grave{k} \in \mathfrak{C}_{\alpha^\star,\beta^\star,\gamma^\star}$.

Now let $\grave{k} \in \mathfrak{C}_{\alpha^\star,\beta^\star,\gamma^\star}$ and $h \in \mathfrak{R}$.

Then, $\tilde{\zeta}_{\mathfrak{C}}(h\grave{k}) \geq \tilde{\zeta}_{\mathfrak{C}}(\grave{k}) \geq \tilde{\alpha}; \zeta_{\mathfrak{C}}(h\grave{k}) \leq \zeta_{\mathfrak{C}}(\grave{k}) \leq \alpha,$
$\tilde{\varrho}_{\mathfrak{C}}(h\grave{k}) \leq \tilde{\varrho}_{\mathfrak{C}}(\grave{k}) \leq \tilde{\beta}; \varrho_{\mathfrak{C}}(h\grave{k}) \geq \varrho_{\mathfrak{C}}(\grave{k}) \geq \beta, \tilde{\varsigma}_{\mathfrak{C}}(h\grave{k}) \leq \tilde{\varsigma}_{\mathfrak{C}}(\grave{k}) \leq \tilde{\gamma}; \varsigma_{\mathfrak{C}}(h\grave{k}) \geq \varsigma_{\mathfrak{C}}(\grave{k}) \geq \gamma.$
Thus $h\grave{k} \in \mathfrak{C}_{\alpha^\star,\beta^\star,\gamma^\star}$

Hence $\mathfrak{C}_{\alpha^\star,\beta^\star,\gamma^\star}$ is a left ideal of $\mathfrak{R}$.

Similarly, it can be proven RI. Thus $h\grave{k} \in \mathfrak{C}_{\alpha^\star,\beta^\star,\gamma^\star}$.

Conversely, Let $\mathfrak{C}_{\alpha^\star,\beta^\star,\gamma^\star}$ be a left ideal of $\mathfrak{R}$ and $\grave{n}, \grave{k} \in \mathfrak{C}_{\alpha^\star,\beta^\star,\gamma^\star}$ such that

$\zeta^\star_{\mathfrak{C}}(\grave{n}) = \zeta^\star_{\mathfrak{C}}(\grave{k}) = \alpha^\star; \varrho^\star_{\mathfrak{C}}(\grave{n}) = \varrho^\star_{\mathfrak{C}}(\grave{k}) = \beta^\star; \varsigma^\star_{\mathfrak{C}}(\grave{n}) = \varsigma^\star_{\mathfrak{C}}(\grave{k}) = \gamma^\star$.

Since $\grave{n} - \grave{k} \in \mathfrak{C}_{\alpha^\star,\beta^\star,\gamma^\star}$

$\tilde{\zeta}_{\mathfrak{C}}(\grave{n} - \grave{k}) \geq \tilde{\alpha} = t_n(\tilde{\alpha}, \tilde{\alpha}) = t_n\{\tilde{\zeta}_{\mathfrak{C}}(\grave{n}), \tilde{\zeta}_{\mathfrak{C}}(\grave{k})\}$

$\zeta_{\mathfrak{C}}(\grave{n} - \grave{k}) \leq \alpha = s_n(\alpha, \alpha) = s_n\{\zeta_{\mathfrak{C}}(\grave{n}), \zeta_{\mathfrak{C}}(\grave{k})\}$.

Similarly,$\tilde{\varrho}_{\mathfrak{C}}(\grave{n} - \grave{k}) \leq s_n\{\tilde{\varrho}_{\mathfrak{C}}(\grave{n}), \tilde{\varrho}_{\mathfrak{C}}(\grave{k})\}, \varrho_{\mathfrak{C}}(\grave{n} - \grave{k}) \geq s_n\{\varrho_{\mathfrak{C}}(\grave{n}), \varrho_{\mathfrak{C}}(\grave{k})\}$and $\tilde{\varsigma}_{\mathfrak{C}}(\grave{n} - \grave{k}) \leq s_n\{\tilde{\varsigma}_{\mathfrak{C}}(\grave{n}), \tilde{\varsigma}_{\mathfrak{C}}(\grave{k})\}, \varsigma_{\mathfrak{C}}(\grave{n} - \grave{k}) \geq s_n\{\varsigma_{\mathfrak{C}}(\grave{n}), \varsigma_{\mathfrak{C}}(\grave{k})\}$.

Also, since $\grave{n}\grave{k} \in \mathfrak{C}_{\alpha^\star,\beta^\star,\gamma^\star}$ then

$\tilde{\zeta}_{\mathfrak{C}}(\grave{n} - \grave{k}) \geq \tilde{\alpha} = \tilde{\zeta}_{\mathfrak{C}}(\grave{k}); \zeta_{\mathfrak{C}}(\grave{n} - \grave{k}) \leq \alpha = \zeta_{\mathfrak{C}}(\grave{k})$

$\tilde{\varrho}_{\mathfrak{C}}(\grave{n} - \grave{k}) \leq \tilde{\beta} = \tilde{\varrho}_{\mathfrak{C}}(\grave{k}); \varrho_{\mathfrak{C}}(\grave{n} - \grave{k}) \geq \beta = \varrho_{\mathfrak{C}}(\grave{k})$

$\tilde{\varsigma}_{\mathfrak{C}}(\grave{n} - \grave{k}) \leq \tilde{\gamma} = \tilde{\varsigma}_{\mathfrak{C}}(\grave{k}); \varsigma_{\mathfrak{C}}(\grave{n} - \grave{k}) \geq \gamma = \varsigma_{\mathfrak{C}}(\grave{k})$.

Hence $\mathfrak{C}$ is a cubic neutrosophic left ideal of $\mathfrak{R}$ with respect to t_n and s_n are idempotent. In the same way, we can establish that for RI. □

Theorem 4.3. *Let $\mathfrak{C}$ be a cubic neutrosophic set of $\mathfrak{R}$ with respect to t_n and s_n, where t_n, s_n are idempotent then $\mathfrak{C}$ is a cubic neutrosophic ideal of $\mathfrak{R}$ with respect to t_n and s_n if and only if $\mathfrak{C}_{\alpha^\star,\beta^\star,\gamma^\star}$ is an ideal of $\mathfrak{R}$, with $\{\tilde{\zeta}_{\mathfrak{C}}(\grave{k}) = \tilde{\alpha}, \zeta_{\mathfrak{C}}(\grave{k}) = \alpha, \tilde{\varrho}_{\mathfrak{C}}(\grave{k}) = \tilde{\beta}, \varrho_{\mathfrak{C}}(\grave{k}) = \beta, \tilde{\varsigma}_{\mathfrak{C}}(\grave{k}) = \tilde{\gamma}, \varsigma_{\mathfrak{C}}(\grave{k}) = \gamma.\forall \alpha^\star, \beta^\star, \gamma^\star \in [0,1]$.*

Proof. Result follows the above theorem. □

Theorem 4.4. *If **I** and **G** are any two cubic neutrosophic left ideal(right ideal) of $\mathfrak{R}$ with respect to t_n and s_n then $\mathbf{I} \cap \mathbf{G}$ is also a cubic neutrosophic left ideal(right ideal) of $\mathfrak{R}$ with respect to t_n and s_n, where t_n and s_n are idempotent.*

Proof. Let $\grave{n}, \grave{k} \in \mathfrak{R}$. Then we have

(i) By theorem 3.1

$\mathbf{I} \cap \mathbf{G}$ is cubic neutrosophic subring over t_n and s_n of $\mathfrak{R}$.

It is enough to show the following.

(ii)
$$\begin{aligned}
\tilde{\zeta}_{\mathfrak{C}I\cap G}(\grave{n}\grave{k}) &= t_n\{\tilde{\zeta}_{\mathfrak{C}I}(\grave{n}\grave{k}), \tilde{\zeta}_{\mathfrak{C}G}(\grave{n}\grave{k})\} \\
&\geq t_n\{\tilde{\zeta}_{\mathfrak{C}I}(\grave{k}), \tilde{\zeta}_{\mathfrak{C}G}(\grave{k})\} \\
&= t_n\{\tilde{\zeta}_{\mathfrak{C}I\cap G}(\grave{k})\} \\
\zeta_{\mathfrak{C}_{I\cap G}}(\grave{n}\grave{k}) &= s_n\{\zeta_{\mathfrak{C}_I}(\grave{n}\grave{k}), \zeta_{\mathfrak{C}_G}(\grave{n}\grave{k})\} \\
&\leq s_n\{\zeta_{\mathfrak{C}_I}(\grave{k}), \zeta_{\mathfrak{C}_G}(\grave{k})\} \\
&= s_n\{\zeta_{\mathfrak{C}_{I\cap G}}(\grave{k})\}.
\end{aligned}$$

Similarly, $\tilde{\varrho}_{\mathfrak{C}I\cap G}(\grave{n}\grave{k}) \leq s_n\{\tilde{\varrho}_{\mathfrak{C}I\cap G}(\grave{k})\}$ and $\varrho_{\mathfrak{C}_{I\cap G}}(\grave{n}\grave{k}) \geq t_n\{\varrho_{\mathfrak{C}_{I\cap G}}(\grave{k})\}$, $\tilde{\varsigma}_{\mathfrak{C}I\cap G}(\grave{n}\grave{k}) \leq s_n\{\tilde{\varsigma}_{\mathfrak{C}I\cap G}(\grave{k})\}$ and $\varsigma_{\mathfrak{C}_{I\cap G}}(\grave{n}\grave{k}) \geq t_n\{\varsigma_{\mathfrak{C}_{I\cap G}}(\grave{k})\}$.

Hence $\mathbf{I} \cap \mathbf{G}$ is a cubic neutrosophic left ideal of $\mathfrak{R}$ over t_n and s_n.

In the same way we can prove for cubic neutrosophic right ideal of $\mathfrak{R}$ over t_n and s_n. □

Theorem 4.5. *If **I** and **G** are any two cubic neutrosophic ideals of $\mathfrak{R}$ with respect to t_n and s_n then $\mathbf{I} \cap \mathbf{G}$ is also a CNI of $\mathfrak{R}$ with respect to t_n and s_n, where t_n and s_n are idempotent.*

Proof. Results follows from above theorem. □

5. Conclusions

This study looks at special types of mathematical structures called cubic neutrosophic subrings and level sets, along with triangular norms. The research has developed important results in this area. These findings help to understand the properties and structure of these mathematical concepts better. The work provides a solid foundation for future research.

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B.Anitha is an Assistant Professor in the department of Mathematics, Annamalai University, Annamalainagar. She received her Ph.D. degree in Mathematics from Annamalai University in 2016. Her research area in Fuzzy Algebra is Fuzzy Sets and Systems and Fuzzy logics

M.Lavanya completed her undergraduate studies at Sindhi college of arts and science, Chennai. She completed her M.Sc degree at Pachaiyappa's college, Chennai. Currently, she is pursuing her ph.D. at Annamalai University, Annamalai Nagar, Tamil Nadu.